SCHLICH'S

MANUAL OF FORESTRY

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VOLUME III.

FOREST MANAGEMENT

BY

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INDEX.

Q.

R.

S.

T.

U.

V.

END OF VOLUME III.

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TABLE OF CONTENTS.

FOREST MANAGEMENT.

INTRODUCTION.

The management of forests depends, apart from local conditions, on the objects which it is proposed to realise. These differ considerably according to circumstances, but, whatever they may be, they can be brought under one of the following two headings:—

(1.) The realisation of indirect effects, such as landscape beauty, preservation or amelioration of the climate, regulation of moisture, prevention of erosion, landslips and avalanches, preservation of game, etc.

(2.) The management of forests on economic principles, such as the production of a definite class of produce, or the greatest possible quantity of produce, or the best financial results.

It rests with the owner of the forest, in so far as his choice is not limited by the laws of the country, to determine in each case what the objects of management shall be, and it then becomes the duty of the forester to see that these objects are realised to the fullest extent and in the most economic manner.

In some cases, the realisation of indirect effects requires a special and distinct management, but in the majority of cases they can be produced in combination with economic working. The present volume deals chiefly with the economic aspect of forest management.

The economic working, whether it aims at the production of a special class or the greatest quantity of produce, or

the best financial results, must be based on the yield of the forest. In order to determine this, the forester must study the laws which govern production; he must be able to measure the produce and the increment accruing annually or periodically, to determine the capital invested in the forest, to regulate the yield according to time and locality, and to organise the systematic conduct of the business.

Accordingly, Forest Management may be divided into the following parts:—

Part I.—Forest Mensuration, dealing with the determination of the dimensions of trees, the volume of trees and whole woods, their age, and increment.

" II.—Forest Valuation, dealing with the determination of the capital employed in forestry, and the financial results produced by it.

" III.—The Foundations of Forest Management.

" IV.—Preparation of Forest Working Plans.

PART IV.

PREPARATION OF FOREST WORKING PLANS.

PREPARATION OF FOREST WORKING PLANS.

In Part III. of this book, the foundations of forest working plans have been dealt with, more particularly the increment, rotation, the growing stock, yield, and the relations existing between increment, growing stock, and yield. Out of the material thus given, the forester constructs for himself an ideal condition which serves him as an object to be aimed at. Just as little as a man striving at perfection ever arrives at it, so does the forester never succeed in bringing his forest in every respect into the ideal or normal state. And if he accidentally should do this, something unforeseen is sure to happen which will produce some imperfection in one way or another. The art of the forester, therefore, consists in bringing his forest as near the normal state as may be possible. In other words, the normal state must serve as a standard towards which the forester is to lead the forest under his charge.

As the management depends, in the first place, on the objects which the proprietor has in view, and as these differ widely, from a protection forest to one worked entirely for financial objects, it follows that the normal state differs with the objects aimed at. Every different object has its own normal state. This having been determined, the forester draws up a working plan or scheme in which he indicates the measures by which he proposes to change the actually existing into the normal state.

Forests may be abnormal in many ways. More particularly the abnormal condition may be due to an abnormal state of

(1.) the increment, or

(2.) the age classes, or

(3.) the growing stock, or

(4.) the whole or parts of the forest may work with a percentage smaller than that which the proprietor has to pay for the money invested in the forest.

Either one or several of the conditions may be abnormal, and in the latter case the question arises, which the forester should take in hand in the first place. Just as a capitalist's first care is to make his capital yield him an appropriate interest, so must the forester see that a full increment takes place in his forest. The increment, above all other considerations, renders the capital invested in a forest active; it replaces, year by year, that part of the growing stock which has been removed by fellings. Without it, the growing stock will diminish until it finally disappears altogether. Hence, it must be the forester's first care to bring the increment up to its full or normal amount. This he does by regulating the cuttings in a suitable manner, followed by efficient regeneration, and rational tending of the growing woods. All parts of the forest with a deficient or undesirable increment must either be filled up if young enough, or utilised and replaced by vigorous young woods. Take, for instance, many if not most of the oak woods in Britain. Quite a respectable number of cubic feet of timber may be standing on the area, but most of the trees are misshapen, or short in the bole, and the increment which they lay on is not only small, but it does not increase the value of the trees in the same degree, as if it was laid on by trees with fine tall stems. In such cases, the undesirable stock should be removed and replaced by a better class of trees. Or, take the majority of Indian forests: they generally contain a considerable number of species, of which only a few, and sometimes even one only, are saleable. Here, the greater part of the increment is laid on by trees without any value. Rational forestry demands that, if not all, at any rate the greater part of the useless material should be removed and replaced by the valuable species.

Next, a proper proportion and distribution of the age classes must be aimed at, in all cases where a regular and normal

return is desired year after year. Without it, the returns will be intermittent, or, even if there are mature woods available, it may not be possible to cut them for fear of damage being done to adjoining woods.

The realisation of a full and regular sustained yield is possible only if both a full increment and a proper proportion of the age classes, as well as their suitable distribution over the forest, have been established.

It has been shown that forests which have been brought into that condition contain also the normal growing stock; the latter comes of itself, if the other two conditions are in order. At the same time, the growing stock is of interest, since it shows the capacity of a forest to yield a certain return for a corresponding period of time; it has also often to be determined, numerically, so as to ascertain tho amount of capital invested in a forest.

Considering all these matters, it is clear that a proper plan or scheme must be devised which lays down the execution of the necessary measures in an orderly manner. A working plan has for its object to lay down, according to time and locality, the entire management of the forest, so that the objects for which the forest is maintained may be as fully as possible realised. And this must be done in an economic manner, for *extravagance has no place in forestry.*

Remembering now what has been said, it will easily be understood that a forest working plan must be based on the principles of silviculture; it must not contravene them. To do justice to this task, the plan must be based upon:—

(1.) an exact and detailed examination of the actual state of the forest in all its component parts;

(2.) the forest must be divided into divisions of workable size;

(3.) the leading principles of management must be indicated, and the yield calculated;

(4.) arrangements must be made for the control of the execution of the plan and the record of all works which

have been executed, so that every succeeding plan may be more accurate;

(5.) the whole material must then be brought together in a working plan report.

Until comparatively recent times, it was the fashion to commence by dealing with forests of some extent as a whole, fixing the yield, and then determining where and when it was to be obtained. Of late years, however, the procedure has been reversed. First of all, the requirements of each part of the forest, or each wood, are determined, and the results are added up, thus giving the measures and works to be executed in the forest as a whole. If each part is brought into a healthy condition, the whole must be the same.

As the objects in view differ, it is impossible to lay down a general rule for the preparation of a working plan. For forests of great value yielding a high revenue, detailed plans are required; under the reverse conditions, simple plans suffice. By way of illustration the following arrangement is given, but it must be understood that it may be too detailed in some cases, while further details may be required in others.

Working Plan Report.

Introduction.

Chapter I.—General Description.

1. Name and situation of forest; name of proprietor.
2. Boundaries.
3. Area.
4. Configuration of the ground.
5. Rock and general character of the soil.
6. Climate.
7. Legal position of forest, rights and privileges.
8. Surrounding population and its requirements.
9. Markets, lines of export.
10. Prices of the several classes of produce.
11. Cost of extraction and transport to markets; supply of labour.

12. General description of forest growth.
13. Injuries to which the crop is exposed.
14. Rate of growth.
15. Yield tables, volume tables, form factors, reducing co-efficients, etc., used in the calculation of the volume and increment of the woods.
16. Organisation and strength of the forest staff.

CHAPTER II.—DETAILED DESCRIPTION OF COMPARTMENTS.

CHAPTER III.—DIVISION AND ALLOTMENT OF AREAS.

CHAPTER IV.—DESCRIPTION OF THE METHOD OF TREATMENT.

1. The objects of management.
2. Choice of species.
3. Choice of silvicultural system.
4. Determination of the rotation.
5. General lines of treatment.
6. Determination and regulation of the yield.

CHAPTER V.—SPECIAL WORKING PLANS.

1. Plans of utilisation.
    a. Final cuttings.
    b. Intermediate cuttings.
    c. Minor produce.
2. Plan of formation.
3. Plan of other works.
4. Maps illustrating the condition of the forest and the proposed treatment.

CHAPTER VI.—MISCELLANEOUS.

1. Reorganisation of the forest staff.
2. Financial forecast.
3. Proposals for the control of the execution of the working plan.
4. Miscellaneous observations.

In discussing the several headings of the working plan report it will be convenient to arrange the remarks into the following chapters:—

Chapter I.—Examination of the Forest, or Collection of Statistics.
" II.—Division and Allotment of the Area.
" III.—Determination of the Method of Treatment and General Lines of Management.
" IV.—Determination of the Yield.
" V.—Control of Execution and Renewal of Working Plans.

The subjects coming under I., II., and III. are not easy to separate, because these chapters overlap to some extent. In practice, they are dealt with simultaneously, more especially Chapters I. and II., but in dealing with them here they must be taken one after the other. It is not possible to put the statistics together in proper order without having divided the forest into a number of working units; nor is it possible to divide and allot the area to its several uses without having previously ascertained what each part of the forest contains. Again, the division and allotment of areas cannot be finally arranged until the method of treatment and the general lines of management have been provisionally laid down. It is for this reason that the division and allotment have been placed between the collection of statistics and the determination of the method of treatment.

The preparation of the special plans enumerated in Chapter V. of the Working Plan Report differs so much according to local conditions that no general patterns can be given. Some examples will be found in Appendices IV. and V.

At one time it was the practice to prepare working plans of high forests for long periods of time, even as much as a whole rotation. Such a procedure is to be strongly deprecated, because the conditions which govern the working of a forest

change from time to time. Although the general lines of action must be determined for some time ahead so as to secure continuity of action, the detailed prescriptions for the management should be laid down only for a short period, say 10 or perhaps 20 years. This is especially desirable where a working plan is prepared for the first time, and where the data upon which it is based aro as yet incomplete. It is desirable, in such cases, to revise the existing arrangements in the light of the experience gained during the actual working of the forest for a limited period.

CHAPTER I.

COLLECTION OF STATISTICS.

The collection of statistics is of the first importance, because the whole fabric of the working plan rests upon the data which have been collected as regards the actual state of the forest, and the notes on the treatment which should be applied to each part. The statistics to be collected must refer, on the one hand, to each wood which forms part of the forest, and, on the other hand, to the general conditions in and around the forest as a whole which are likely to influence the management.

The data to be collected may, therefore, be arranged under the following heads:—

I. Survey and determination of areas.
II. Description of each wood or compartment.
III. Past yields, receipts, and expenses.
IV. General conditions in and around the forest.
V. The statistical report.

The data under II. must be collected separately for each unit of working or compartment; those under III. may be given for each compartment, or each working section, or for the whole forest, according to circumstances.

SECTION I.—SURVEY AND DETERMINATION OF AREAS.

The survey yields the necessary data from which maps can be prepared and the area of the whole forest, as well as of its several divisions, ascertained. It is not intended to describe here the various methods of surveying, as this work must be done by professional surveyors; the following remarks refer only to those points in which the forester must participate.

Before the survey is commenced, various preliminary matters must be attended to, such as :—

(1.) Regulation and demarcation of the boundaries of the forest, and of those parts which are subject to servitudes.

(2.) Demarcation of all areas which are not destined for the production of wood, such as fields, meadows, pastures, swamps, rocky parts, and other areas unfit for growing woods.

(3.) The laying out of a suitable system of roads and rides, in so far as it can be done without a map, or with the help of a sketch map. What cannot be done in this respect before the commencement of the survey, should, if possible, be done during its progress, that is to say, as soon as the necessary data become available. If any part cannot be done until a map becomes available, an additional survey will be necessary.

(4.) Demarcation of the boundaries between woods of different species, ages, or qualities. The latter is necessary only in valuable forests.

The method of survey depends on the value of the forest as represented by its returns; the higher the latter, the more minute should be the survey. Generally speaking, all main lines, such as the boundaries of the property and of the areas subject to servitudes, the roads and principal rides, should be surveyed with the theodolite and chain or measuring staff. The details, such as the limits of woods and of sub-compartments, may be done with the plane table or prismatic compass.

The area of the whole forest and its main parts should be ascertained by the method of co-ordinates; the area of the compartments or woods may be ascertained with the planimeter, or a network of squares each of which represents a fixed area.

Whenever practical, the survey should be based upon a previous triangulation.

The preparation of the maps will be dealt with in the last

section of this chapter. Frequently, general maps of the area are already available. If they are on a sufficiently large scale and reliable, only the additional details required for the management of the forest need be added.

SECTION II.—DESCRIPTION OF EACH WOOD OR COMPARTMENT.

The description of each wood, compartment, or other unit of working is of the first importance, because it gives information on which depends the whole management, viz.:—

(1.) The selection of species to be grown in the future.

(2.) The method of treatment of each wood, and the determination of the rotation.

(3.) The degree of ripeness of each wood.

(4.) The yield capacity of each wood and of the whole forest.

The minuteness of the investigation depends on the value of the forest and the intensity of management. Where these are high, a detailed examination and record are called for; where the returns are likely to be small, or where the demand is considerably below the possible yield, a summary procedure may be indicated. The forester must in each case determine the actual procedure which he considers to be in keeping with the interests of the owner of the forest.

1. THE LOCALITY.

By locality is understood the soil (and subsoil) and the climate, the latter depending on the situation. The agencies which are at work in the soil and the overlying air determine the *yield capacity* or *quality* of the locality.

The details regarding locality in relation to forest vegetation have been given in the fourth edition* of Volume II. of this Manual, pages 7—53. From what has there been said, it will

* All references to Volume II. are meant for the *fourth* edition of that volume.

easily be understood that a description of the soil and climate must form part of the basis upon which a working plan rests.

In describing the climate and soil, the following points deserve attention :—

a. Climate.

(1.) The geographical position of the locality, as indicated by latitude and in many cases also longitude, especially where the vicinity of the sea, large lakes, or high mountains are likely to influence the climate.
(2.) The local peculiarities of the locality, such as altitude, aspect, slope, temperature, moisture in the air, rainfall, exposure to strong, cold, or dry winds, susceptibility to late or early frosts, etc.
(3.) The surroundings of the locality, in so far as they are likely to affect the local climate.

b. Soil.

(1.) The underlying rock.
(2.) The mineral composition of the soil.
(3.) The organic admixtures of the soil.
(4.) The depth of the soil.
(5.) The degree of porosity.
(6.) The degree of moisture.
(7.) The surface covering of the soil.

In forests situated on level ground, the above data may be the same over a considerable portion or the whole of the area, but in the hills they have frequently to be determined for each compartment, or even portions of one compartment, especially if it shows considerable differences of altitude, aspect, or slope.

All these factors combined produce a certain quality or yield capacity of the locality. How this is determinod has been explained in Volume II., page 47, and in Forest Mensuration. Some further remarks on the subject will be found in the last part of this section.

2. The Growing Stock.

The growing wood, or the crop produced on an area, represents the results of the activity of the locality under a certain treatment. All points which have influenced the quantity and quality of the results must be ascertained, to enable the forester to judge of the merits of the treatment hitherto followed and the advisability or otherwise of any changes in it.

a. Method of Treatment, or Silvicultural System.

The different methods of treatment have been described in Volume II., pages 92—117. At this stage, the forester must ascertain the system under which the wood has actually been managed in the past.

b. Species.

Pure woods are indicated by giving the species. In the case of *mixed* woods, the degree of mixture must also be given: this can be done, either by adjectives, such as "some," "a few," or, preferably, by decimals, placing the whole as 1. These decimals should have reference to the area occupied by each species.

Example.—The following description—

Beech = ·5	Oak = ·3
Ash = ·2	Maple = a few,

would mean that ·5 of the area is occupied by beech, ·3 by oak, and ·2 by ash, with a few maples.

In the case of very valuable trees, such as old oaks, or teak trees in Burma, it may be desirable to give their actual number. The manner of admixture is expressed as "in single trees," "in groups," "in strips," or "irregularly distributed."

It is also necessary to state, whether the mixture is permanent or temporary, whether it is of special silvicultural or financial importance, such as a shelter wood (or nurses)

over another tender species, a soil protection wood, standards of valuable species, etc.

The undergrowth, shrubs, herbs, etc., should also be described.

c. Density of the Growing Stock.

To every method of treatment, as determined by the objects of management, corresponds a normal density of the growing stock. The degree of density may be defined as over-crowded, crowded, open, very open, interrupted, irregular, etc. Such terms are indefinite and subject to different interpretations. It is better to place the normal density as equal to 1, and express the actual stocking in decimals of it. The degree of density can be determined by ocular estimate, or, more accurately, by comparing the basal area of the stems with that of a normally stocked wood, or still more accurately, by comparing the volume of the wood with that of a fully stocked wood of the same age. When the density of stocking is insufficient, it should be stated whether the wood is generally open, or whether the deficiency is due to greater or smaller blanks.

By a *blank* is understood an area which, though it belongs to the wood-producing area, has no trees on it or so few that its complete re-stocking is necessary. Areas which are not destined for the production of trees are not included here, as they form part of the areas set aside for other purposes, such as fields, meadows, etc., or are altogether unfit for the production of trees, such as bare rocks, boulder drifts, swampy ground which cannot be drained, etc. As regards the latter, it is not always easy to draw the line between actual blanks and woodland, as they frequently have a thin stocking which may give a small return from time to time.

d. Age.

The methods of determining the age of trees and woods have been given in Forest Mensuration.

An absolutely accurate determination of the age is necessary only when the data are required for the preparation of yield tables or other statistical purposes. Fairly approximate data suffice for the purposes of working plans.

In the case of even-aged or nearly even-aged woods, one or more sample trees are examined. If considerable differences of age exist in a wood, the limits should be given and the wood placed into that age class to which it belongs according to its economic character. If some older or younger groups exist which are not of sufficient extent to be enumerated as separate woods, this should be mentioned. The same holds good for a limited number of standards which are to be held over for a second rotation, or for young growth which has sprung up in an old wood.

In the case of woods which have been kept back in their development, the economic and not the actual age must be given. For instance, a young wood which has stood under heavy shelter and is now 30 years old, but of a development which is ordinarily reached in 10 years, must be entered as 10 years and not as 30 years old.

In the regeneration class, the ages of the overwood and underwood must be given separately.

In selection forests, it suffices to give the limits of the age gradations which are frequently determined by the number of years during which cuttings go once round the forest.

In coppice with standards, the ages of the overwood and underwood are given separately; for the former, the limits of the existing gradations are given.

The age of coppice can generally be easily ascertained from the time when the last cutting occurred.

c. Origin and Past Treatment.

Whenever the necessary data can be ascertained, a short history of each wood should be prepared, giving the method of formation, whether by natural or artificial means, planting or sowing, the manner in which the wood has been tended,

cleanings, thinnings, pruning, natural phenomena which have affected the development, etc. Such a history is very useful in judging the results of the past method of treatment and in determining the future treatment.

f. Volume.

All methods of determining the yield in material require a measurement of the volume, but to a different extent. For some, it is necessary to measure all woods excepting only those which are very young and which are estimated, either direct, or with the assistance of yield tables. For other methods, only those woods require to be measured which will come under the axe during the immediate future of, say, 10 to 20 years.

Where a fine financial management is followed, all woods which are close to ripeness, or of which the ripeness is doubtful, must be accurately measured, so as to calculato the per cent. with which the capital is working.

For the determination of the capital value, an accurate measurement of the volume is indispensable.

The volume should be given separately for the different species, if their value per unit of measurement differs considerably. It is useful to give all volumes in the same measure, as solid cubic feet. The proportion between the different classes of produce need only be given for each working section; best according to local proportionate figures, if such are available.

The different methods according to which the volumo can be measured have been described in Forest Mensuration. The choice of the method of measurement depends on the circumstances of each case.

g. Increment, Capital Value, and Forest Per Cent.

These matters have already been dealt with in full detail.

The determination of the quantity increment is required for the calculation of the yield. It must be done for all

woods, if the increment forms the principal basis for the determination of the yield. In that case, both normal and real increment must be ascertained. When the yield is fixed for only a limited period, the current increment must be ascertained for that number of years, or the mean annual increment of the past is substituted for it.

For financial questions, the volume-, quality-, and price-increment must be determined, as well as the capital invested in the forest, so as to calculate the indicating or forest per cent. The latter is necessary only for woods the financial ripeness of which is doubtful, that is to say, for woods which are approaching the normal final age, and woods which have suffered by injurious agencies, such as wind, snow, fire, insects, game, etc.

3. Determination of the Quality of each Wood.

a. General.

It has already been explained that by the quality of a wood or compartment is understood its yield capacity, as expressed by the quantity of produce which can be derived from it.

The yield capacity depends in the first place on the locality; but injurious influences may have interfered with the full development of the producing factors of the locality, so that abnormal conditions may be the consequence. The forester distinguishes, therefore, between *normal and abnormal quality*. The quality is normal, if no exceptional injurious influences or faulty treatment have affected the development of the wood.

A further distinction must be made between the quality of the "locality" and of the "growing wood" or standing crop. Either of the two can be normal or abnormal. The quality of the locality may be abnormal in consequence of a variety of causes, such as the long-continued removal of litter, or excessive exposure to the effects of sun and air currents which have impoverished the soil; or in consequence of unfavourable

natural phenomena; for instance, if the ground has become swampy, temporarily denuded, or covered with moving sand. An abnormal condition of the growing wood may be produced by faulty treatment, by injurious external agencies, such as drought, frost, wind, fire, insects, diseases of the trees, cattle grazing, etc.

For the preparation of working plans, only the actually existing, or real, quality of the locality should be taken into account, because the restoration of the normal quality is generally a slow process, if it is at all practicable. As regards the growing stock, both values are required, because its normal quality represents the real quality of the locality, and the real quality of the growing stock forms the basis for the calculation of the yield which the forest can give in the immediate future.

On pages 47-48 of Volume II. it has been said that the quality of the locality can be ascertained—

(1.) By an assessment according to the several factors of the locality; or

(2.) By an assessment according to a crop of trees produced on the area in question, or on a similar soil in the vicinity.

It has also been stated that the first of these two methods, however carefully carried out, is always subject to grave errors, because an examination of the chemical composition, the physical properties of the soil, and a determination of the climate, do not indicate the yield capacity of the locality for forestry with any degree of certainty; hence, it should be used only as an auxiliary of the second method, or when the latter is not available.

Thus, it will be seen that the determination of the quality of the locality depends practically on an examination of the wood which it has produced. In fact, a normal growing stock is the true expression for the real quality of the locality; the same investigation gives both the quality of the locality and of the existing crop.

For the purpose of obtaining an actual figure which represents the quality, the best way is to ascertain the volume of the growing stock, including all thinnings, and the number of years in which it has been produced. In dividing the produced volume by the age of the wood, the mean annual increment is obtained which indicates the quality.

It is evident that in reality a multitude of different qualities exist, but for practical work they are grouped into a few, generally not more than five, *quality classes* which are numbered I. to V. Of these I. should represent the lowest and V. the best quality, but unfortunately the reverse numbering has been largely introduced. A still more convenient way is to represent the best quality by 1 and the others in decimals of 1. Each of these quality classes represents a distinct yield capacity which differs with the species and method of treatment.

The quality can be determined with the help of yield tables which represent the progress of volume, or increment, throughout life for each quality class; hence, assessing the quality means, in this case, the selection of the proper yield table. The difficulty is that for every species and silvicultural system a different set of yield tables is required. It may even be desirable to have different sets for different localities, so-called local yield tables; but such a procedure is likely to lead to confusion, as different standards of the quality classes are introduced into the account. Hence, general yield tables are to be preferred, even if the same degree of accuracy is not obtained as in the case of local tables. The difference is, however, not considerable, as experience has shown that, within reasonable limits, general tables give sufficiently accurate data for the preparation of working plans. It has, for instance, been proved that the general yield tables for the Scotch pine prepared for North Germany may safely be used for fairly crowded woods grown in the south of England. The fact is that the sources of inaccuracy, unavoidable in the best methods of measuring the volume of a standing crop, are greater than

those caused by using general yield tables for any particular locality.

Detailed yield tables for oak, beech, Scotch pine, spruce, and silver fir, and summary tables for larch, alder, and birch will be found in Appendix III. In these tables only three quality classes have been distinguished, I. representing the best, III. the least favourable, and II. the average, or middling, quality. Tho great want in Britain are detailed yield tables for the larch.

The quality of young woods cannot be judged by thoir volume, since the factors of the locality may not yet have found full expression in it; here, the quality must be estimated by the general condition of the crop and especially its height growth. Indeed, the latter may be used even in older woods especially as long as it has not ceased.

The determination of the quality from yield tables, in the case of clear cutting in high forest and in coppice, is a simple matter, as previously shown. The regoneration area under tho shelter-wood system gives some trouble, because it is no longer fully stocked, so that tho volume does not reprosent the quality; here, the determination must be based upon an investigation of the quality of tho locality combined with the condition of the shelter-wood and young growth, especially the height growth. A similar proceduro is followed in the case of coppico with standards and in selection forests. The quality of blanks is estimated from the soil and climate, or from that of adjoining woods which have been produced on soil of a similar description.

b. Reduction to One Quality.

Several methods of regulating the yield demand a reduction of the several woods, or working sections, to one quality class. Such a reduction may be made as regards the locality, or the growing wood; in each case as regards the normal or real quality. The method of procedure is the same in all cases.

The reduction is mado by means of the final mean annual

increment, or yield. It can be made under one of the two following conditions:—*Either* the total of the several reduced areas shall be equal to the actual area of the working section; in other words, the reduction is made to the mean quality of the area; *or* the above equality is not required, in which case any quality can be used as the standard, frequently that being chosen which exists over the greater part of the area.

Calculation with the Mean Quality.—By mean quality is understood that which, if it existed throughout the working section, would produce the same total yield as that produced by the several existing qualities in different parts of the working section.

Let $a_1, a_2, a_3 \ldots$ be the several areas,
„ $y_1, y_2, y_3 \ldots$ the corresponding annual yields, or increment, per unit of area,
„ $Y \ldots\ldots$ the mean yield per unit of area, then

$$a_1 \times y_1 + a_2 \times y_2 + a_3 \times y_3 + \ldots = a_1 Y + a_2 Y + a_3 Y + \ldots$$
$$= Y(a_1 + a_2 + a_3 \ldots),$$

and

$$Y = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3 + \ldots}{a_1 + a_2 + a_3 + \ldots} = \frac{\text{total annual yield}}{\text{total area}}.$$

Example:—
A working section of 100 acres contains—

Block (1) 20 acres with 60 cubic feet average increment,
„ (2) 10 „ „ 50 „ „ „
„ (3) 20 „ „ 40 „ „ „
„ (4) 50 „ „ 30 „ „ „ then—

Mean quality

$$Y = \frac{20 \times 60 + 10 \times 50 + 20 \times 40 + 50 \times 30}{100} = 40 \text{ cubic feet.}$$

By reduced area is now understood that which would produce, with a uniform quality $= Y$, the same total yield as the actually existing areas with their varying qualities. The reduced area of each block is obtained by applying, in each case, the inverse proportion of that which exists between the actual and the mean quality:

$$Y : y = a : a' \text{ and reduced area } a' = \frac{a \times y}{Y}.$$

those caused by using general yield tables for any particular locality.

Detailed yield tables for oak, beech, Scotch pine, spruce, and silver fir, and summary tables for larch, alder, and birch will be found in Appendix III. In these tables only three quality classes have been distinguished, I. representing the best, III. the least favourable, and II. the average, or middling, quality. Tho great want in Britain are detailed yield tables for the larch.

The quality of young woods cannot be judged by thoir volume, since the factors of the locality may not yet have found full expression in it; here, the quality must be estimated by the general condition of the crop and especially its height growth. Indeed, the latter may be used even in older woods especially as long as it has not ceased.

The determination of the quality from yield tables, in the case of clear cutting in high forest and in coppice, is a simple matter, as previously shown. The regoneration area under tho shelter-wood system gives some trouble, because it is no longer fully stocked, so that tho volume does not reprosent the quality; here, the determination must be based upon an investigation of the quality of the locality combined with the condition of the shelter-wood and young growth, especially the height growth. A similar proceduro is followed in the case of coppico with standards and in selection forests. The quality of blanks is estimated from the soil and climate, or from that of adjoining woods which have been produced on soil of a similar description.

b. Reduction to One Quality.

Several methods of regulating the yield demand a reduction of the several woods, or working sections, to one quality class. Such a reduction may be made as regards the locality, or the growing wood; in each case as regards the normal or real quality. The method of procedure is the same in all cases.

The reduction is mado by means of the final mean annual

Calculation with any Suitable Quality.—In this case any quality can be used, whether it exists on the area or not.

The total reduced area is obtained by multiplying the several qualities by the corresponding areas and dividing the product by the selected standard quality. It may be greater, equal, or smaller than the actual area, according to the size of the standard quality:—

$$\text{Reduced } A = \frac{a_1 \times y_1 + a_2 \times y_2 + a_3 \times y_3 + \ldots}{Y'}.$$

The reduced areas of the several parts are obtained by the inverse proportion of their qualities to the standard quality; thus:—

$$\text{Reduced } a'_1 = \frac{a_1 \times y_1}{Y'},$$

$$\text{Reduced } a'_2 = \frac{a_2 \times y_2}{Y'},$$

etc.

Example, as above.—Let the standard quality = 50 cubic feet, then total reduced area =

$$\frac{20 \times 60 + 10 \times 50 + 20 \times 40 + 50 \times 30}{50} = 80 \text{ acres},$$

and the reduced area of:—

$$\text{Block (1) } \frac{20 \times 60}{50} = 24 \text{ acres.}$$

$$\text{,, (2) } \frac{10 \times 50}{50} = 10 \text{ ,,}$$

$$\text{,, (3) } \frac{20 \times 40}{50} = 16 \text{ ,,}$$

$$\text{,, (4) } \frac{50 \times 30}{50} = 30 \text{ ,,}$$

$$\text{Total} = 80 \text{ acres.}$$

$$\text{Reduced area of annual coupe} = \frac{80}{10} = 8 \text{ acres},$$

and the size of coupes in the several blocks:—

$$(1)\ x_1 = \frac{50 \times 8}{60} = 6{\cdot}667$$

$$(2)\ x = \frac{50 \times 8}{50} = 8{\cdot}$$

$$(3)\ x_3 = \frac{50 \times 8}{40} = 10$$

$$(4)\ x_4 = \frac{50 \times 8}{30} = 13{\cdot}333\ ;$$

as before.

It is obvious that the last-mentioned method is the more convenient of the two.

4. Notes Regarding Future Treatment.

While drawing up a description of each wood, it is very desirable to note down any observations which may strike the forester regarding the future treatment. Such notes are, of course, only of a preliminary nature, because a final decision on the future treatment to be followed can be arrived at only after the management of the whole forest, or working section, has been laid down. Nevertheless, they are a great help during the progress of the work.

It is not possible to give a complete list of the points which should be attended to, as they differ according to circumstances; the following may, however, be enumerated:—

(*a.*) Filling up the existing wood; if so, the area to be treated and the species to be grown should be given; also the method of sowing, planting, or other cultural operations.

(*b.*) Cleanings, thinnings, or prunings during the period for which the working plan is prepared; the volume to be removed should be estimated.

(*c.*) Degree of ripeness of the principal or final crop, taking into consideration the objects of management; if the latter are financial, the indicating per cent. should be calculated. If it appears advisable that final cuttings should be made, the method of cutting should be given, as well as an estimate of the volume to be removed.

(*d.*) Method of regeneration to be followed and the species to be grown, if this should occur during the period for which the working plan is prepared.

(*e.*) Measures to be taken for the protection of the wood against threatening dangers, especially fire.

(*f.*) Other works to be undertaken, such as construction of roads, draining, irrigation, etc.

(*g.*) Utilisation of enclosures and improvement of boundaries where practicable.

(*h.*) Proposals regarding the formation of sub-compartments, or the abolishment of those which exist, with reasons for such proposals.

SECTION III.—PAST YIELDS, RECEIPTS, AND EXPENSES.

There is no surer basis in estimating future returns than those of the past; hence, it is of importance to ascertain and note down the yield in material, the cash receipts, and costs for as many years as the available data admit. These data will, however, only be forthcoming if records have been kept for some time past.

As far as may be practicable, past yields, receipts, and costs should be given for each unit of working, that is to say each wood or compartment. If the records have not been kept in sufficient detail, the data for each working section should be given; the latter may also be quite sufficient where the management is as yet in a backward condition, or where the receipts are small.

The following notes indicate the class of information which may be required:—

1. Yield of Wood, or Major Produce.

The yield should be given separately—

(*a.*) For the principal species.

(*b.*) For the different classes of timber and firewood, according to size or value.

(*c.*) For final and intermediate returns.

(*d.*) Of cash receipts should be given the total, and the average price of the several classes of material, separated according to species.

(*e.*) The areas over which cuttings extended should, if possible, also be given separately for final and intermediate cuttings.

2. Minor Produce.

Under this heading, the quantity of each article of minor produce which has been removed, and the cash receipts obtained for it should be given.

Receipts derived from areas not used for the production of wood, such as fields, meadows, etc., should be separately recorded.

3. Expenses.

These should be recorded separately for—

(*a.*) Cost of administration and protection.

(*b.*) Taxes, rates, etc.

(*c.*) Formation of woods.

(*d.*) Tending and amelioration.

(*e.*) Maintenance of boundaries.

(*f.*) Construction of roads, drainage, irrigation, and other works.

(*g.*) Cost of harvesting, separated according to major and minor produce.

4. Generally.

For forests worked on financial lines, the receipts and expenses should be so arranged that it is possible to ascertain—

(*a.*) The current forest per cent. of each wood whenever desirable.

(*b.*) The forest rental, being the difference between all receipts and expenses. It is used to determine the percentage which the forest capital has yielded.

(*c.*) The capital value of the forest, being the sum of the value of the soil plus value of the growing stock. For the preparation of working plans based upon financial

principles, these values may be ascertained as follows: the data of receipts and costs of the several woods will enable the forester to calculate a series of expectation values of the soil for the different qualities of locality; besides, all available data referring to actual sales of land, similar to the forest lands in question, should be carefully noted. By combining these data it will be possible to determine tho value of the soil with sufficient accuracy for the purpose of working plans. The capital value of the growing stock is then dotermined as tho cost value, calculated with the valuo of the soil as determined above, or as the utilisation value in the case of woods which are near maturity.

SECTION IV.—GENERAL CONDITIONS IN AND AROUND THE FOREST.

Tho management of a forest depends not only on the state of its several parts, but also on the general conditions which exist in and around it. The latter must, therefore, be ascertained at this early stage, and they should be used for a general description of the forest to be incorporated into the working plan report.

The field of inquiry here indicated is of considerable extent; the following matters may be mentioned:—

(1.) Name and situation of forest, giving tho latitudo and longitude where nocessary.

(2.) Description of boundaries and names of the adjoining properties and their owners.

(3.) Topographical features of the locality.

(4.) General description of the geology, soil, and climate.

(5.) Former and present proprietors; financial position of the latter; whether the funds for formation, tonding, administration, amelioration, etc., are available; whether specially hoavy cuttings must be made to meet the demands of the proprietor.

(6.) Nature of proprietorship: whether full and unfettered

property, or whether servitudes and privileges rest on it ; in the latter case their extent should be recorded.

(7.) Rights enjoyed by the proprietor of the forest elsewhere, such as rights of way or floating, or rights over other lands, etc.

(8.) Requirements of the surrounding population and condition of the market for forest produce generally; special industries in the vicinity which require forest produce, such as mines, smelting works, saw mills ; imports which compete with the local supply; substitutes for wood available in the vicinity.

(9.) Extent of forest offences; their causes; effect upon the forest; suggestions for their prevention.

(10.) Labour available in the vicinity ; rate of wages.

(11.) Past system of management; changes introduced from time to time; prescriptions of former working plans and their effect upon the forest.

(12.) Natural phenomena which have affected the condition of the forest, such as storms, snow, frost, fire, insects, fungi, etc.

(13.) Conditions of game and cattle grazing; their effect upon the forest.

(14.) Past seed years of the more important species.

(15.) Opportunities for consolidating the property, either by exchange or purchase; conversion of fields, meadows, etc., into forest, or the reverse.

(16.) The staff of the forest, its organisation and efficiency.

SECTION V.—THE STATISTICAL REPORT.

The data which have been collected in the manner indicated in the previous four sections must be brought together in a statistical report, accompanied by maps to illustrate it. The form of this report depends entirely on the circumstances of each case. In one instance, it will be necessary to go into minute details, in another, a more summary treatment is

indicated. The following documents will ordinarily form part of the report:—

1. Register of Boundaries.

This should give—

(*a.*) The boundary marks in consecutive numbers.

(*b.*) The angles backwards and forwards at each point.

(*c.*) The horizontal distance between every two boundary marks.

(*d.*) The nature of the boundary line, whether a road, water-course, waterparting, ditch, cleared line, etc.

(*e.*) The names of adjoining properties and of their owners.

The value of the register of boundaries is considerably enhanced, if its correctness has been acknowledged by the adjoining owners before the proper court of law.

Description of

Locality.			Area, in Acres.			Boundaries.	Locality.
Working Section of Block.	Com-partment.	Sub-com-partment.	Stocked	Blank	Total		
Cæsar's Camp	1	*a*	24	2	26	*North & East:* Sir A. Hayter's land. *South:* Compartments 13 and 12. *West:* Roman road.	*Elevation:* 420 feet above sea-level, sloping towards the east with moderate gradient, down to 380 feet. *Geological Formation:* Middle Bagshot sands. *Soil:* Loamy sand, fairly good in upper part of compartment, and good in lower part; no pan to 4 feet depth.

2. Table of Areas.

The following form of this table is given as an illustration; it serves as a summary of all areas, and shows how each part is utilised:—

Locality.			Grand Total. In Acres.	Area used for the Production of Wood. In Acres.	Area not used for the Production of Wood. In Acres.				
Working Section or Block.	Compartment.	Sub-compartment.			Roads and Rides.	Fields.	Meadows.	Water, etc.	Total.
Cæsar's Camp.	1	*a*	29·2	26	·4	2·2	—	·6	3·2
		Etc.							

Compartments.

Growing Stock.					Quality of		Remarks, and Notes regarding Future Treatment.
Silvicultural System.	Species.	Age, in Years.	Mean Height in Feet.	Volume in solid Cubic Feet.	Locality.	Growing Stock.	
High Forest	Oak = ·4 Chestnut = ·2 Beech = ·1 Scotch pine = ·3	70	Oak, Chestnut & Beech = 54 Scotch pine = 60	Broad-leaved = 34,000 Scotch pine = 28,800 62,800	II. (or middling)	·8	The mixture of species is uneven; the Scotch pine is chiefly found in the northern part and the broad-leaved trees in the South; the northern part is well-stocked, the southern part is open; most of the oaks and chestnuts are frost-cracked. *Future Treatment:* The southern part is so increment-poor that it should be cut over during the next ten years, leaving the best oaks and chestnuts underplanted with beech. In the northern part only dead or dying trees should be removed during the next ten years.

3. Description of Compartments.

This description may be drawn up in a tabular form, or otherwise; the former is preferable, as it presents a more intelligible picture of the forest, and gives greater security that nothing has been overlooked. It is quite impossible to recommend any particular form for this table, but by way of illustration the appended form is given. (See pages 266, 267.)

In this table, the quality of locality indicates that which corresponds to the normal quality of the growing stock. The real quality of the growing stock is given in decimals, the normal quality being placed equal to 1.

If the forest is worked on financial principles, further columns must be added for the quantity and quality increment, wherewith to calculate the forest per cent.

4. Table of Qualities of Locality.

Locality.			Total Area. Acres.	Quality Classes of Locality. Area in Acres.			Silvicultural System and Species.
Working Section.	Compartment.	Sub-compartment.		I. Best.	II. Middling.	III. Lowest.	
Cæsar's Camp	1	a	26		26		High forest of Scotch pine with some broad-leaved trees here and there.
"	1	b	14			14	
"	2		38		32	6	
"	3		17	17			
"	4		31		16	15	
"	5		33	12	17	4	
"	6		31		25	6	
"	7		27	7		20	
	Etc.						
		Total	217	36	116	65	

Whenever the management is of a certain intensity, it is useful to prepare this table, as it enables the forester to calculate the total yield capacity of the area. In the table, each working section must be recorded separately, as the yield capacity depends on the species, silvicultural system, and rotation.

Assuming that the yield tables for Scotch pine given in Appendix III. apply to the woods in question, and that the latter are worked under a rotation of 80 years, the normal mean production per acre and year will be as follows :—

For the I. Quality class = 146 cubic feet.
II. " " = 90 "
III. " " = 37 "

The mean annual increment, or the yield capacity, of the area shown in the above table would therefore be—

Yield capacity $= 36 \times 146 + 116 \times 90 + 65 \times 37 = 18{,}101$ cubic feet,

or average yield capacity per acre $= \dfrac{18{,}101}{217} = 83$ cubic feet.

This figure represents the normal yield; the real or actual yield depends on the quality of the growing stock and the ages of the several woods. Assuming that the mean quality (density of stocking) of all woods were equal to ·7, the actual yield would, for the present, be equal to

$$18{,}101 \times \cdot 7 = 12{,}671 \text{ cubic feet},$$

or 58 cubic feet per acre and year, while measures would have to be taken to increase the yield capacity in the future by growing more completely stocked woods.

5. Table of Age Classes.

This table is of great importance, as it gives a correct idea of the proportion of the different age classes, a matter which affects the determination of the yield in the immediate future. It may be prepared in the following form :—

Table of Age Classes.

Locality.			Present Mean Age.	Age Classes, Areas in Acres.						
Working Section.	Compartment.	Sub-compartment.		I. 1-20.	II. 21-40.	III. 41-60.	IV. 61-80.	V. Over 80.	Blanks.	Total.
Cæsar's Camp	1	a	70				24		2	26
	1	b	35		14					14
	2		95					32	6	38
	3		54			17				17
	4		16	31						31
	5		46			33				33
	6		24		31					31
	7		86					27		27
			Total...	31	45	50	24	59	8	217

6. Table of Past Yields.

This table should give the past yields in produce for as many years as possible, and the mean annual yield calculated from these data, as in the example below:—

Table of Past Yields.

Material cut in Past Years, in solid Cubic Feet.

Year.	Conifers.								
	Final.			Intermediate.			Total.		
	Timber	Fire-wood.	Total.	Timber	Fire-wood.	Total.	Timber.	Fire-wood.	Total.
1881	7,200	1,750	8,950	4,700	1,300	6,000	11,900	3,050	14,950
1882	.								
.	.								
.	.								
.	.								
1890	.								
Total in 10 Years	77,600	16,400	94,000	36,400	12,200	48,600	114,000	28,600	142,600
Annual Average	7,760	1,640	9,400	3,640	1,220	4,860	11,400	2,860	14,260

Remarks.—The area set aside for the production of wood amounted, in the beginning of 1881, to 217 acres.
The annual yield was fixed at 15,000 solid cubic feet: or 150,000 for the period of 10 years; hence, the average cuttings were below the fixed yield by 740 cubic feet annually.

Similar statements are prepared for the different species, or groups of species, and a summary of the whole drawn up.

Where specially valuable timber has been cut, like oak standards, teak, etc., it can be entered separately.

7. Maps.

It is most useful to represent on maps the data required for the preparation of a working plan, so far as this can be done. Such maps give at a glance a clear picture of the forest which impresses itself more readily on the mind than a lengthy description. As it is not possible to represent

everything on one map, it is usual to prepare different sets, such as the—

(*a.*) Topographical map.
(*b.*) Geological map.
(*c.*) Soil map.
(*d.*) Detailed map on a large scale.
(*e.*) Map showing the nature and age of the growing woods, called the stock map.
(*f.*) Map showing the working sections and cutting series.
(*g.*) Detailed road map.
(*h.*) Map showing the qualities of locality.

There is, however, no need for so many separate maps, as several of them can be combined into one. Ordinarily three maps suffice, namely :—

a. The Geological Map.

This map should show the geological formation of the upper layers, on which the nature of the soil depends. In it can also be shown the general topography of the area; the limits of the various qualities of locality can be entered by lines of a distinguishing colour, the quality being indicated by a number.

b. The Detailed Map.

The scale of this map depends on circumstances. In India, the ordinary scale is 4 inches = 1 mile. In a few cases, maps on a scale of 8 inches = 1 mile, and in others of 2 inches = 1 mile, have been prepared.

The map should show, amongst other items :—

(1.) Name of forest and year of survey.
(2.) Boundaries, all boundary marks being indicated on the map and numbered; boundaries between free property and parts subject to servitudes.
(3.) Names of adjoining properties and their owners.
(4.) Area, total as well as of the main divisions.
(5.) Areas not used for the production of wood.
(6.) Contour lines, or height curves.

(7.) The system of roads and rides, watercourses, and other natural lines, with their names.

(8.) The boundaries of working sections, blocks, compartments, and sub-compartments, with their names and numbers.

c. *The Stock Map.*

This has for its principal object to give a picture of the manner in which the area is stocked with wood; a smaller scale than 4 inches = 1 mile generally suffices for it. The map should contain, apart from the necessary details, a representation of the existing species, silvicultural systems, and distribution of the age classes. This can be done in a variety of ways, as for instance in the following:—

In *high forest* the principal species are shown by different washes; the age classes by different shades of the same wash, the youngest being given the lightest, and the oldest the darkest shade; the regeneration class receives some distinguishing mark.

Mixed woods may receive a separate wash, or they may be distinguished by the addition of small trees or marks of various colours.

Coppice woods may receive a separate wash, if shown on the same sheet.

Coppice with standards may be distinguished from coppice by the addition of miniature trees.

Selection forest may be indicated by colouring it with the wash of the principal species, and indicating other species by special marks.

Blanks remain uncoloured.

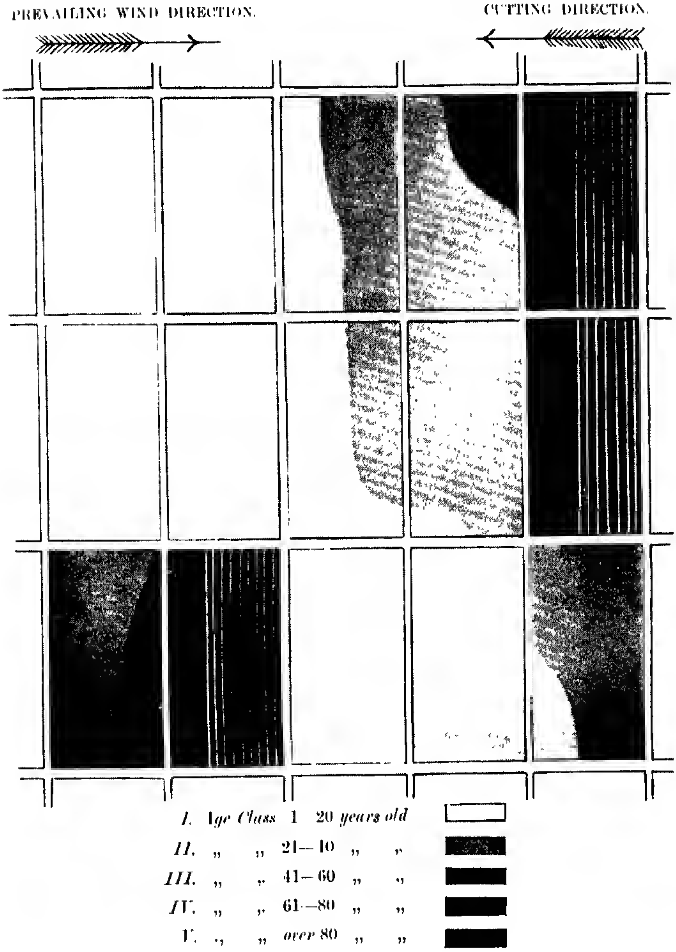
The stock map should be renewed whenever a new working plan is prepared; if this is done, it gives, in the course of time, an excellent representation of the history of the forest.

By way of illustration, the appended Fig. 51 is added. It illustrates two cutting series, a number of these constituting a working section, or a complete series of age gradations.

Fig. 51.

REPRESENTATION OF TWO CUTTING SERIES

(THE UPPER CONSISTING OF 10 COMPARTMENTS, THE LOWER OF 5)



Rotation = 80 years.

The White Lines indicate the coupes to be cleared during the next 10 years.

[To face p. 272.]

CHAPTER II.

DIVISION AND ALLOTMENT OF THE FOREST AREA.

1. The Working Circle.

By a working circle is understood the area which is managed under the provisions of one and the same working plan.

The area of a working circle depends on local conditions. It is intimately connected with the general organisation of a forest property. Assuming that the unit of administration is represented by a *Range*, or executive charge, a working circle may comprise all areas included in one range. Sometimes the conditions differ so much that different working plans are drawn up for parts of one range, but, as a rule, this should be avoided, unless several small properties are comprised in one range. This occurs frequently in many European States, where Government forest officers manage both State and Communal forests. Hence, it may be said that the minimum size would be the area of a property belonging to the same owner; the maximum will ordinarily be the area forming one executive charge or range.

The division of an extensive property into ranges depends chiefly on :—

(1.) The situation, and

(2.) The intensity of management.

In the case of scattered blocks, in hilly country, or where means of rapid locomotion are wanting, a range will comprise a smaller area than if the property is consolidated, situated on level ground, or where railways and other means of locomotion enable the range officer to move rapidly from one part of his charge to the other.

In forests which yield a small return the ranges may be large; where the money yield is high, it pays best to make the ranges small, so that an intense and detailed management may be possible.

Each working circle or range, as the case may be, must be further divided. The unit of that division is the *compartment*. A number of compartments are grouped together into *cutting series*, and a number of the latter form a *working section*, consisting of a complete series of age gradations or age classes. In some cases a working section is identical with a working circle, or the latter may contain several of the former. The whole of this division is effected by utilising, in addition to the outer boundaries, interior natural lines, such as waterpartings, watercourses, precipices, etc., and artificial lines, as roads, already constructed or projected, and rides.

Although the division of the working circle depends chiefly on the system of roads and rides, it is desirable, before indicating how they should be laid out, to explain more fully what is understood by compartment, sub-compartment, cutting series, and working section.

2. The Compartment.

By compartment is understood the unit of working; it forms, therefore, the unit of the division of the forest.

The above definition should never be lost sight of. If the boundaries of a compartment can be made to coincide with those of a wood showing a certain composition or age, so much the better, but it is a mistake to insist upon such an arrangement; the main point is that each compartment should be of a certain size, so as to fulfil its objects as the unit of working. If that area includes two or more different kinds of growing woods, they may be distinguished as sub-compartments; but the boundaries of the compartment should never be twisted out of shape for the sake of including only one kind of growing stock.

The formation of compartments is necessary—

(1.) For general orientation, so as to enable the forester to define accurately any particular part of the area.

(2.) To render all parts of the forest easily accessible, since one or more sides of the compartment should abut on roads or rides.

(3.) To assist in the prevention of fires, and to enable the forester to stop any which may have broken out.

(4.) For the location of the annual or periodic coupes.

(5.) To facilitate the transport of forest produce.

(6.) To obviate the necessity for repeated surveys of the coupes.

(7.) In some cases, to facilitate hunting and shooting.

The shape of compartments depends on the configuration of the ground. In the plains, a rectangular shape (with sides 2 : 1, or 3 : 2) is most suitable. On hilly ground, such a shape is not always practicable; but the actual shape should, as far as possible, approach that of a rectangle.

The size of compartments cannot be laid down; it depends chiefly on—

(1.) The size of the working circle.

(2.) The intensity of management.

(3.) The extent of danger from fire.

3. The Sub-compartment.

If, within the limits of a compartment, considerable differences exist in respect of species, silvicultural system age of growing stock, quality of locality, etc., it may be divided into two or more sub-compartments; the latter may be temporary, if the differences will disappear after some time, or permanent. Sub-compartments may be marked by shallow ditches or other cheap boundary marks.

The forester should not go too far in the formation of sub-compartments, as it is accompanied by additional expenditure. As a rule, sub-compartments should be formed only if the additional income derived from different treatment at least

covers the additional expense involved thereby. The formation of sub-compartments depends on the intensity of management.

4. The Working Section.

A part of a working circle which forms a separate series of age classes is called a working section. If a working circle consists of only one series of age classes, it is identical with a working section. In working circles of some extent, however, different conditions may demand the establishment of two or more series of ago classes, or a division of the working circle into two or more working sections. The principal causes which demand tho formation of working sections are the following:—

a. Species.

When several species appear in a working circle as pure woods, they must be placed into different working sections if they require essentially different treatment, or if a certain quantity of material of each species has to be cut annually. When, on the other hand, the several species appear in mixed woods, such a separation is neither practicable nor necessary.

b. Silvicultural System.

Each silvicultural system may demand the formation of a separate working section. If, for instance, part of a high forest is treated under the compartment system, and another part as a selection forest, each part must be formed into a separate working section. Coppice woods and coppice with standards always must form separate working sections.

c. Rotation.

Even in the case of the same species and silvicultural system, areas worked under different rotations must be placed into different working sections whenever an even or approximately even annual yield is expected. Unless this is done,

it will happen, either that the annual yield is uneven, or, if the same quantity is cut every year, that the different rotations merge into one.

d. Servitudes.

If part of a working circle is subject to servitudes, it should be placed into a separate working section; this is necessary to protect the interests of the owner, as well as of the right holder.

e. Differences in the Quality of the Locality.

Differences in the quality of the locality cause the establishment of different working sections, if they necessitate the growing of different species, or the adoption of different treatment or rotations.

f. Distribution of Cuttings.

If cuttings must be made annually in different parts of the working circle, so as to supply local demands, it is often advisable to form different working sections, though this is not absolutely necessary.

g. Size of the Working Circle.

When the area of a working circle exceeds a certain limit, it may be more convenient to divide it into several working sections, although no difference in the character of the growing stock and the management exists. In this way, better arrangements can be made for the execution of the work.

h. Generally.

A working circle, consisting of several working sections, is said to be normal if each separate working section is in a normal state.

Although the formation of working sections is in certain cases unavoidable, the forester should not go to extremes in

this respect. A separate record must be kept for each working section, and they cause extra trouble and expense in other ways; hence, moderate differences of conditions, especially in the rotation, should not induce the forester to introduce separate working sections.

The question may be asked, why a separate working plan should not be drawn up for each working section, thus making the latter always identical with a working circle. Such a procedure is not desirable, because it involves extra labour and repetitions in the working plan report. It is preferable, whenever practicable, to have one working plan for each executive charge, because the management of the different working sections can be so arranged that they supplement each other, thus enabling the forester to provide for a proper allotment of work amongst the staff and a proper distribution of the yield. Where the areas managed on different lines are mixed up with each other, the division of a working circle into two or more working sections becomes an absolute necessity. It need hardly be pointed out that the areas belonging to one working section need not form a consolidated block; they may be scattered amongst other areas forming another working section.

5. The Cutting Series.

A working section in its simplest shape should consist of a series of age gradations equal to the number of years (or periods) in the rotation, so arranged that cuttings commence in the oldest age gradation and proceed steadily towards the youngest in the direction which is determined by the circumstances of each case. It has, however, been pointed out on page 219 that such a simple arrangement is, in the case of high forest, not always admissible, and that every working section in such a forest must be further divided into several parts which are called "*cutting series.*" Only such a further division gives the necessary order and elasticity to the arrangement of the coupes.

Each cutting series should comprise a number of gradations, the ages of which differ by a certain number of years (see diagram on page 218); it can be regarded as a working section in which cuttings are made periodically instead of annually; ordinarily, however, a certain number of cutting series together form one complete series of age gradations, or a working section.

The number of age gradations to be included into one cutting series depends on local circumstances. On the whole, small cuting series are desirable, as each gives a point of attack where cuttings can be made. Amongst the advantages of small cutting series the following may be mentioned:—

(1.) The special requirements of each wood can be met at the right time; if a cutting is desirable at a given time, it can be made without interfering with the safety of adjoining woods.

(2.) A suitable change of coupes can be arranged, so as to protect the forest against the dangers which may make themselves felt if two or more annual coupes adjoin each other.

(3.) The establishment of small cutting series assists the forester in distributing the yields to meet local demands.

In order to realise these advantages, it is necessary that each cutting series should receive a shape and be so situated that the coupes can be suitably arranged, and that cutting in one series does not interfere with the requirements of adjoining series; in other words, each cutting series must be independent of its neighbour. Where these conditions do not exist, they must be specially provided by the clearance of broad rides between the cutting series, called severance cuttings.

6. Severance Cuttings.

By a severance cutting is understood a cleared strip of varying breadth by which two woods are separated in the

general direction of the cuttings, at a place where some time afterwards regular cuttings are to commence.

Severance cuttings are necessary whenever an existing cutting series is too long, and when it is desirable to divide it into two or more series. Their object is to accustom the edge trees of the wood on the leewardside to a free position, so that they may develop into storm-firm trees, and be able to withstand the effects of strong winds when the wood on the windward side has been cut. An example will explain this. A wood comprising *a* and *b* (Fig. 52) is to be divided into two cutting series I. and II. To prevent the trees in II. being thrown by wind when I. is cut, a strip *c* is cleared some time

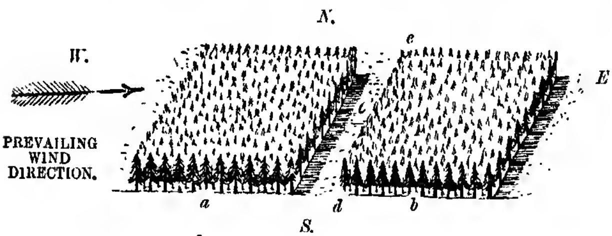


Fig. 52.

before cuttings in I. are commenced, so that the edge trees along the line *d*—*e* may become storm-firm.

Severance cuttings need not be straight; they may, if necessary, be curved, or run along two or three sides of a wood. The latter is necessary where the prevailing wind direction is not constant but oscillates, say, from north-west to south-west. The breadth of severance cuttings differs according to species, their height growth, and the strength of threatening winds; it will ordinarily range between 30 and 60 feet.

Severance cuttings must be made while the wood to be protected is still young and capable of developing firm edge trees; such a development is generally no longer possible after the trees have passed middle age. They must be made, in the windward wood, some 15 to 20 years before the regular cut-

tings are commenced. Where danger from windfalls is great, it is desirable first to clear a narrow strip, and to widen it a few years afterwards in one or more instalments, so as gradually to accustom the edge trees to the effects of strong winds. If the severance cutting is not to form a road or ride it is at once re-stocked, so as to avoid loss of increment, and because the existence of a young wood in front of that to be protected is an additional safeguard against windfalls. When a severance cutting is made along an existing road or ride, it is, of course, placed to the windward of it.

If the proper time for making a severance cutting is past, and the wood to be protected is too old, it would be a dangerous procedure to make such a cutting. In that case, it is better to make a series of thinnings in the strip along the edge of the wood to be protected, before cuttings in the windward wood are commenced. Whether this measure will have the desired effect is doubtful, but it is better than to risk a regular severance cutting.

7. The System of Roads and Rides.

As already indicated, working sections, cutting series, and compartments must be separated from each other by natural or artificial lines. Apart from suitable natural boundary lines, such as waterpartings, watercourses, precipices, fields, meadows, etc., roads are the best boundaries of compartments and cutting series, because they facilitate the transport of the produce. It is, therefore, desirable that in the first instance a suitable network of roads should be designed and marked on the ground. Roads alone, however, rarely suffice. In some cases, roads already exist which are not suitable for boundaries, in others even new roads must be so laid out that they cannot be used as boundaries, because they must lead in the direction of the places of consumption. Besides, in hilly or swampy ground they often follow a direction which renders them unfit to serve as boundaries.

The missing division lines are provided by a system of

rides, that is to say by cleared strips of various breadths. A distinction is made between major and minor rides.

a. Major Rides.

In so far as roads or natural lines are not available, cutting series, and in many cases working sections, should be bounded by major rides. These should run in the direction of the cuttings, that is parallel to the prevailing wind direction, whenever the configuration of the ground does not necessitate deviations.

In coppice and coppice with standards, the major rides need not be more than 6 to 8 feet broad, unless they are used as roads for the transport of the material. In high forest, they must be much broader, because they are used as severance cuttings to induce the edge trees of adjoining woods to become wind-firm and accustomed to other climatic influences. In the case of woods consisting of species which are easily thrown by wind, they should not be less than 30 feet broad, and if the major ride is also used as a fire line, it may be still broader.

The edges of the woods consisting of species easily thrown by wind, if bordering on major rides, should be strongly thinned from an early age onward, so as to produce strongly developed trees.

Major rides may be utilised for stacking wood. Their area is entered as non-productive of trees; in many cases, however, they produce grass.

In young woods, the major rides should be cut at once while the edge trees are capable of producing a strong root system; in woods which are past middle age, only 6 to 8 feet broad lines should be cleared in the first instance, which are widened to the required breadth when the adjoining woods are cut over.

b. Minor Rides.

Minor rides should run more or less at right angles to major rides; they complete the delimitation of the compart-

ments. The coupes will, therefore, run parallel to the minor rides and stand at right angles to the major rides. Minor rides need not be more than 6 to 8 feet broad, unless they are used as fire lines.

c. The Network of Rides.

Major and minor rides together form the network or system of rides. The laying out of it depends, especially in the case of shallow rooted species, chiefly on the prevailing wind direction. In the plains, the latter can generally be determined without much trouble. In mountainous districts, the matter is frequently beset by difficulties, because the configuration of the ground may produce a local direction which differs from the general direction. No rule can be laid down for such deviations; the question must be studied on the spot. The direction can frequently be recognised by the shape of the crowns of trees, by a slanting position of the stems, and, above all, by the direction in which trees have been thrown. As regards the latter, it must not be overlooked that local storms sometimes throw trees in a direction which differs from the ordinary direction of gales. In many cases, reliable information can be obtained from local people who have lived for some time in the locality.

The laying out of the system of rides is of great importance, because it is used in the protection of the woods against natural phenomena, and it leads to order in the management. These advantages outweigh the loss of productive area which is, after all, very limited. Regular networks of rides with right angles at the corners are practicable only in the plains; on hilly ground they must accommodate themselves to the configuration of the ground. The example on the next page will illustrate this. The forest occupies a ridge, the slope of which is indicated by dotted contour lines - - - - . The top of the ridge, being much exposed, must be treated as a separate working section under the selection system; it is

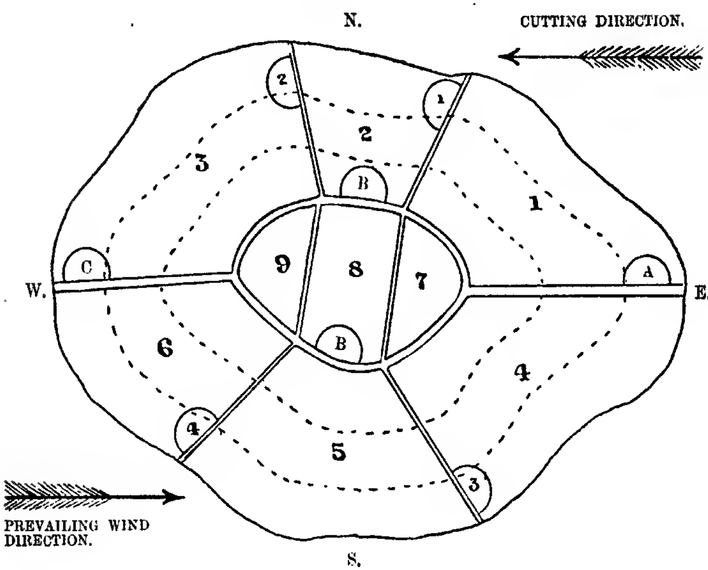


Fig. 53.

separated from the rest by a major ride (B). The slopes are treated under the compartment system, and they are divided into two parts by the major rides (A) (B) and (C). The numbers (1) (2) . . . indicate the minor rides, and 1, 2, 3 . . . the compartments. The prevailing wind blows from the west.

The division would probably be somewhat on the following lines :—

Working Section I. = *Compartment System.*

Cutting	Series	*A*	comprises	compartments	1 & 2
,,	,,	*B*	,,	,,	3
,,	,,	*C*	,,	,,	4
,,	,,	*D*	,,	,,	5 & 6

Working Section II. = *Selection System.*
Comprises compartments 7, 8, and 9.

The cutting direction would be from east to west, a direction which is indicated by the numbering of the compartments.

The coupes in compartments 1 to 6 would run at right angles to the major rides (A) and (C), or up and down the hill side, as it is generally objectionable to let the coupes run horizontally.

8. Demarcation of the Divisions of a Forest.

It is generally desirable that all interior boundary lines should be demarcated by boundary marks, so that they can be recognised if they should have become obliterated in consequence of cuttings, windfalls, etc. For this purpose, boundary marks may be placed at all points where rides cross, or where they form an angle. If straight rides are very long, it is useful to place intermediate marks at suitable distances. Such marks are placed on one side of the rides, so that they may not interfere with the transport of the produce; it is useful to choose always the same side, say the north side of the major rides, and the east side of the minor rides.

9. Naming and Numbering of the Divisions of a Forest.

The methods of naming and numbering the divisions differ much. Judeich recommends the following:—

(*a.*) Working sections receive Roman numbers = I., II., . . .
(*b.*) Cutting series receive names and slanting capital letters = *A*, *B* . . .
(*c.*) Compartments receive Arabic numbers = 1, 2 . . .
(*d.*) Sub-compartments receive small Roman letters = 1_a, 1_b . . .
(*e.*) Major rides receive upright capital letters surrounded by a circle = (A), (B) . . .

(*f.*) Minor rides receive small Arabic figures surrounded by a circle = 1, 2 . . .

(See illustration on page 284.)

Sometimes a number of compartmonts are joined into a "block"; if so, the latter should receive a name.

The numbering of compartments must be consecutive throughout the working circle, unless the latter consists of two or more entirely separated blocks; it leads to confusion to have a separate series for each working section. The numbering should be done so as to indicate the cutting direction.

In the French State forests, the following system of numbering the divisions was prescribed until a short time ago:—

Working sections aro numbered = I., II., III., . . .
Periodic blocks „ „ = 1, 2, 3 . . .
Compartments „ „ = *A*, *B*, *C* . . . with the addition of the number of the poriodic block; thus IV. C_2 means Working Section IV., compartment *C* in the 2nd periodic block.

CHAPTER III.

DETERMINATION OF THE METHOD OF TREATMENT.

Before the yield can be calculated and the general plan of operations laid down, the method of treatment must be determined. The questions here involved have already been discussed in this and in Volume II.; the following references will be useful:—

(1.) For choice of species, see Volume II., fourth edition, page 122.

(2.) For choice of silvicultural system, see Volume II., page 112.

(3.) For choice of method of formation, see Volume II., page 275.

(4.) For choice of method of tending, see Volume II., pages 287—323.

(5.) For choice of rotation, see pages 191 to 202 of this volume.

(6.) The financial aspect of forestry has been explained at pages 149 to 164 of this volume.

The determination of the method of treatment depends chiefly on—

(*a.*) The objects of management.

(*b.*) The locality.

(*c.*) The growing stock actually existing in the forest.

(*d.*) The dangers to which the growing stock and the soil are exposed.

(*e.*) The conditions of demand and supply of forest produce.

(*f.*) The supply of labour.

(*g.*) The legal position of the forest, existence of rights, privileges, etc.

The nature of these determining factors shows that no general rule can be laid down, but that the method of treatment and the general lines of management must be determined locally on the merits of each case. In the following pages only a few general hints are given.

1. Choice of Species.

The first step should always be to examine carefully how far the existing species meet the objects of management and suit the locality. If the existing species do not answer, then the forester should not hesitate to change them. On the other hand, the introduction of a new species should not be lightly undertaken, as it generally involves some loss, and frequently introduces uncertainty as to future results. Only species which have been tried within a reasonable distance and succeeded should be chosen. The cultivation of untried and especially exotic species is, in the first place, justified only on a small scale. Above all, personal fancies must be banished from the forester's mind.

The formation of mixed woods is of great importance, and its advisability should always be considered. In this respect attention is invited to pages 74 to 91 of Volume II.

2. Silvicultural System.

This depends, of course, on the species which exist on the area, or which have been selected for introduction.

High forest yields the greatest quantities and generally also the most valuable classes of major produce, as well as various kinds of minor produce; it is, except in the case of clear cutting, the best system for the preservation of the quality of the locality. On the other hand, it requires in the case of broad-leaved species at least fairly good soil; it also requires a greater capital than other systems, owing to its greater growing stock, accompanied in some cases by a low mean annual forest per cent. The growing stock is also exposed to

many dangers, such as snow, ice, wind, insects, and fire. High forest must be adopted in the case of all species which cannot be regenerated by stool shoots or root suckers.

As to the different kinds of high forest, clear cutting, shelter-wood compartment system, group system, selection forest, or their modified forms, it should be remembered that clear cutting is most easy to carry out, and that the selection system is most favourable for the protection of the quality of the locality, while the modified forms stand between these two extremes. Again, clear cutting exposes the growth most to dangers; at the same time, it is generally obligatory in the case of light-demanding species. Its drawbacks can, to some extent, be reduced by making the coupes of small extent. Still, it should only be followed in the case of hardy species.

The question as to which of the forms of high forest gives the highest returns has been much discussed. On the whole, a carefully carried-out system of clear cutting, with a suitable species, may be expected to give the best financial results; the ordinary shelter-wood system approaches it in this respect, while even higher returns have been claimed for some of its modifications, such as the group eystem, the selection system, the two-storied high forest, or the system of isolated trees in conjunction with an underwood.

In how far a locality can maintain the higher returns (if any) under the clear cutting system, depends on the nature of the soil and the climate. Where these are not favourable, one of the ehelter-wood systems is certainly to be preferred. The latter are absolutely essential in the case of species which are tender during early youth.

Coppice woods are only possible in the case of those species which reproduce from the stool. To do well, they require a fairly mild climate and protected position. Coppice woods make smaller demands on the chemical and physical properties of the soil and give quicker returns; they require a much smaller capital; owing to the shortness of the rotation,

the danger of damage from outside (excepting frost and game) is much smaller, and the whole management can be more regular and is safer than in the case of high forest. On the other hand, coppice gives smaller quantities and lower quality, except in special cases, as, for instance, where tanning bark, hop-poles, or vine-stakes are produced. Until comparatively recent times, coppice woods paid very well, but, owing to the fall in prices of late years, this is no longer the case in Europe; in fact, special cases excepted, they will have to be converted into high forest.

Pollarding frequently gives fair returns, and it can be carried on in conjunction with agriculture.

Coppice with standards holds an intermediate position between high forest and coppice woods as regards the preservation of the quality of locality and the capital value of the growing stock. The comparatively free-growing standards reach a certain diameter in a shorter time than trees in high forest; on the other hand, the stems are shorter and less clean than in regular high forest, and they are more subject to frost cracks and scorching of the bark. The yield is smaller than in high forest, but larger than in coppice; financially, it generally holds a position between the two.

3. Method of Formation.

This depends chiefly on the silvicultural system, the species, and the nature of the locality.

Natural regeneration is cheaper in itself, and, if successful, leads to fully stocked young woods; but in the case of high forest, seed-years do not always come when they are wanted, so that a regular progress of the regeneration may be seriously disturbed; the loss of time and other accompanying disadvantages may more than outweigh the smaller original outlay. Natural regeneration must frequently be augmented by a certain amount of planting or sowing. To conduct the process of natural regeneration by seed demands skill and attention on the part of the forester.

Artificial regeneration generally accompanies the system of clear cutting in high forest, but sowing and planting may also be done under shelter-woods; it is necessary in the case of first afforestations, or where a change of species is contemplated.

Whether direct sowing or planting is preferable depends chiefly on the species and local conditions.

4. Method of Tending.

To sow or plant an area is a comparatively simple business when once the most suitable species has been selected. The process of natural regeneration requires considerable skill. The most important part of a forester's work is, however, the application of a suitable method of tending, and more especially the system of thinning. Views on this subject differ much. Until some 25 years ago, the method of early and heavy thinnings prevailed in Britain with anything but satisfactory results. On the Continent, on the other hand, the system of fully stocked woods had been elaborated, leading to much higher financial results than were obtained in Britain. During the last 25 years, a change has taken place in both cases, but in opposite directions. The system of lighter thinnings has commenced to find favour in Britain, while a considerable portion of Continental foresters now advocate heavy thinning. Until quite recently, the volume removed in thinnings on the Continent may, perhaps, be placed at 40 per cent. of the final yield; now it is advocated to make thinnings so strong that the volume removed in them is equal to that of the final yield, and even more. Formerly, the stems removed in thinnings were dead, dying, suppressed, and, at the outside, dominated trees; now it is advocated to leave suppressed trees and to remove instead some of the dominating trees. It is said that such a system leads to better financial results, but it appears to the author that all these calculations are based on the assumption that no decrease in the value per cubic foot of the final yield takes place. If the heaviness of the thinnings passes a certain

limit, the shape and height growth of the trees comprising the final crop must be interfered with, accompanied by a reduction in the value. This consideration limits the heaviness of the thinnings; they must never be made so heavy that they interfere with the most favourable development of the trees constituting tho final crop, otherwise any advantage derived from the early and heavy thinnings will be more than vitiated by a reduction in the value of the final crop.

The most favourable proportion between thinnings and final crop can be determined only by the collection of extensive statistics, and the financial values based upon them.

5. The Rotation.

The rotation is chiefly governed by the species, silvicultural system, and the objects of management. Though the financial aspect should never be lost sight of, there are many cases where a departure from the financial rotation is fully justified. This occurs where short rotations interfere with the preservation of the yield capacity of locality, where considerations for the production of a certain class of produce or the dictates of political economy are of paramount importance.

CHAPTER IV.

DETERMINATION AND REGULATION OF THE YIELD.

As long as the owner of a forest is satisfied with intermittent returns, the regulation of the yield is chiefly governed by silvicultural considerations, that is to say, every wood is cut over when it is just ripe according to the objects of management, while thinnings are made when they are necessary. If the owner desires a sustained annual yield of equal, or nearly equal, quantity, although the forest is not in a normal state, the various cuttings may have to be made at other times. All such deviations demand certain sacrifices on the part of the owner which differ according to the actual condition of the forest and the objects of management.

These sacrifices are due to the fact that the final cuttings must be made at an age other than the normal age, as determined by the objects of management; even thinnings may have to be postponed instead of being made when the condition of the woods demands such cuttings. This deviation may be brought about by a surplus or deficiency of mature woods, or by their being so situated that they cannot be cut over at the proper time out of consideration for the safety of adjoining woods.

The task of the forester in such cases is to secure a sustained annual yield, and yet to lead the forest, with the smallest possible loss to the owner, gradually over into the normal state as described in Part III. of this volume. Many different methods have been elaborated with the view of achieving that task, which approach the subject from various points of view. All the older methods started by considering, in the first place, the forest as a whole, determining the yield,

and then seeing in which parts of the forest it had best be cut. Moreover, working plans were generally prepared for a long period of time, usually a whole rotation. Gradually, a method which considered in the first place each wood rather than the whole was developed. It is, according to the author's views, the most rational of all methods, and it will, therefore, be described first.

SECTION I.–SELECTION OF WOODS FOR CUTTING IN ACCORDANCE WITH SILVICULTURAL REQUIREMENTS AND THE OBJECTS OF MANAGEMENT.

The method now to be described has been gradually developed since Heinrich Cotta was called to Saxony in 1811; it advanced by steps, until it was finally put into a precise form by Judeich; hence, it is now known as "*Judeich's Bestandswirthschaft.*" The German word "Bestand" can be rendered into English by the word "wood," meaning part of a forest forming a unit of fairly the same description. The main character of the method lies in the fact that first of all the requirements of each wood are considered, and that the management of the whole forest represents a summing-up of the treatment laid down for each wood.

The first step under Judeich's method is a suitable division of the forest into units, called woods, or compartments; next, a general plan is sketched, to indicate the manner in which a normal arrangement of the age classes, both as regards size and grouping, is to be effected; then the treatment of each wood is determined for a limited number of years, with due consideration of its silvicultural requirements; the cuttings thus indicated are added up, and they represent the yield during the first, say, 10 years, unless considerations for a sustained yield in the future demand certain modifications.

It will thus be seen that the method chiefly aims at the establishment of a full increment and a proper arrangement of the age classes. Not a word is said about the normal

growing stock, because this must come of itself, if the other two conditions of the normal state have been established.

Finally, an important part of the method is to fix the working only for a limited number of years, and to have revisions at regularly recurring periods, when the requirements of each wood are reconsidered with due regard to the introduction of a suitable arrangement of the age classes, and especially the introduction of small cutting series. The latter point is of great importance, because it secures a perfectly free hand to the manager to take each wood in hand at the right moment.

In describing the method, the author of this manual has, however, introduced a few changes. Judeich bases the ripeness of each wood upon financial considerations; he ascertains the forest or indicating per cent., and calls a wood ripe if that per cent. has sunk below the general per cent. *p*. This the author considers too narrow a view to take; the ripeness should, in his opinion, be determined by the "objects of management" as laid down by the owner of the forest, of which the financial ripeness may be one, or not.

1 Application of the Method to Clear Cutting in High Forest and the Shelter-wood Compartment System.

a. The General Plan of Operations.

This is represented by a plan which gives the division of the forest into units of working or compartments. The latter are marked off by natural or artificial boundary lines, as described in Chapter II. This plan enables the forester to determine, in a general way, the order and direction in which the cuttings should proceed, and the grouping together of compartments into suitable cutting series. The latter may be a definite arrangement, but in many cases temporary cutting series must be designed, which will, at the time of subsequent revisions, be gradually led over into more permanent groupings.

b. Determination of the Rotation.

The next step is to determine the rotation in accordance with the objects to be aimed at, as laid down by the owner of the forest. How this is done has already been explained.

The financial rotation is ascertained by calculating the soil rental, and the forest per cent. for a number of characteristic woods; in this way it is possible to determine the financial rotation within 10 or 20 years. The rotation actually decided on, as determined by the objects of management, may then be compared with the financial rotation, with a view to bringing out the financial sacrifice involved in a departure from the latter.

c. Determination of the Final Yield.

The next step is to select, with due consideration of the desired cutting direction and the establishment of suitable cutting series, the woods where final cuttings are called for during the period for which the working plan is to be prepared, say for the next 10 years. Special care is taken not to put down for cutting any wood the removal of which would expose the adjoining woods to windfall, or where difficulties of transport would be encountered. Subject to the modifications caused by these considerations, the following areas would he selected:—

(1.) All areas which must be cut to meet silvicultural or protective necessities, such as the establishment of severance cuttings, woods which must be sacrificed in order to work up to a proper grouping of age classes and arrangement of cutting series.

(2.) All decidedly ripe or over-ripe woods, the ripeness to be determined by the objects of management. In the case of a financial management, this would comprise all woods the current forest per cent. of which has sunk below the general per cent. p.

(3.) All woods the ripeness of which is doubtful, and which

may be situated in the direction of the cuttings. This includes the woods which will become ripe during the working plan period.

The sum total of the cuttings indicated under these three headings represents the final yield to be assigned to the period for which the working plan is being prepared.

For small forests, or those where a sustained annual or periodic yield is not called for, nothing further is required. It is different in the case of extensive areas, especially those where considerations for a steady annual income, for the regular supply of markets, or the occupation of the staff and workmen necessitate an approximately even annual out-turn. Here, the yield, as determined above, must be subjected to a *modifying regulator*, either as regards the area to be cut or the volume to be removed during the working plan period.

This regulator can take any suitable shape, such as the size of the mean annual or periodic coupe, or the yield calculated according to volume. Judeich prefers the mean annual coupe, as obtained by dividing the total area by the fixed rotation. If a forest has an area of 2,000 acres and is worked under a general rotation of 80 years, the mean annual coupe would be equal to $\frac{2000}{80} = 25$ acres. During a working plan period of 10 years, the normal cutting area for it would amount to $25 \times 10 = 250$ acres. In other words, during a period of 10 years, 250 acres should be cut over, and the areas selected for cutting should be brought within that limit. This, however, is only desirable if the proportion of the age classes is fairly normal. In all cases where considerable deviations from the normal proportion exist, such a narrow limit cannot be drawn, because in some cases it is highly desirable to cut more than the normal area, if, for instance, too large a proportion of old or defective woods exists. In other cases, the cuttings should be below the normal area, if, for instance, the area of mature woods is deficient. Hence, the

regulator should give merely the maximum and minimum area to be dealt with. In the above case the area might be given as 200 to 300 acres, or 225 to 275 acres.

As long as the total area determined under (1.) to (3.) falls between these limits, it may be accepted as the area to be dealt with during the first 10 years. If it is larger than the maximum, then some of the most suitable areas enumerated under (3.) should be held over until the second period of 10 years; if smaller than the minimum, then possibly some further woods may be found which could be added to those already placed under (3.). In extreme cases, the yield may be kept for a number of years below the proper minimum.

Example.—Let the age classes in the above example be as follows:—

Age Class.		Normal Distribution of Areas. Acres.	Real Distribution of Areas. Acres.	
1—20	—	500	400	700
21—40	—	500	300	
41—60	—	500	700	1,300
Over 60	—	500	600	
	Total =	2,000	2,000	

As there is a considerable excess of old woods, the area to be cut, or taken under regeneration, every 10 years should be more than $\frac{2,000}{8} =$ 250 acres. Indeed, for the next 20 years up to 300 acres might be cut during every 10 years. The result would be as follows:—

	After 10 Years. Acres.		After 20 Years. Acres.	
1—20	= 500	850	= 550	975
21—40	= 350		= 425	
41—60	= 500	1,150	= 425	1,025
Over 60	= 650		= 600	
Total =	2,000		2,000	

After that, 250 acres might be cut during every 10 years, so that, at the end of 30 years in all, the distribution of the age classes would be:—

Age Class.		After 30 Years. Acres.	
1—20	=	525	1,012
21—40	=	487	
41—60	=	425	988
Over 60	=	563	
		2,000	

The deviations from the normal distribution still existing will disappear by continuing to cut about 250 acres every 10 years.

d. The Intermediate Yields.

The limit between final and intermediate yields is not always quite clear. In a general way it may be said that—

(1.) *Final yields* comprise—

(*a.*) All returns obtained from woods which are put down for regeneration during the first period.

(*b.*) All returns from other woods which, in consequence of unforeseen causes, are so large that the regeneration of the woods becomes necessary, whether the final cutting over is done during the working plan period or later on.

(2.) *Intermediate yields* comprise all other returns derived from—

(*a.*) Cleanings.

(*b.*) Ordinary thinnings.

(*c.*) Pruning, cutting of standards, etc.

(*d.*) Accidental cuttings, such as dry wood cuttings, windfalls, etc., in so far as they do not occur in the areas put down for regeneration during the working plan period, or are of such extent that they come under sub-head (1 *b.*), above.

The woods to be cleaned and thinned are put down according to their areas. The quantity of intermediate yields is best estimated according to past local experience with due consideration of the condition of the several woods. Where the necessary local data are not available, the most suitable average data obtained elsewhere must be used, or an estimate made in accordance with the condition of the woods.

The question whether the regulation of the yield should refer to the final cuttings only, or include the intermediate cuttings, has been much discussed. There can be no doubt that the systematic working of a forest should, in the first

place, be regulated by the final cuttings. At the same time, the intermediate yields may be utilised to equalise any unavoidable inequalities of the final yield. Under any circumstances, both classes of yields must be estimated, so as to ascertain the probable quantities of produce which will be placed upon the market, and to prepare the annual budgets.

The total yield of the working plan period is suitably divided amongst the years comprised in it.

e. Separation of Yield into Classes of Produce.

The yield should be separatod according to classes of produce as it is brought into the market, say as timber and firewood, or large timber, poles, mining props, fagots, etc., each being given in solid cubic feet. This separation should be based upon locally obtained proportional figures.

It is also desirable to give separately the yield of the important species, as for instance oak, other broad-leaved species, larch, other conifers, etc. In India, teak, sál, deodar, and some other valuable species should always be given separately.

2. Coppice Woods.

In the first place, the rotation must be determined. By dividing the total area (real or reduced) by the rotation, the size of the annual coupe is obtained.

Next, the area is divided into as many coupes as the rotation contains years, taking into consideration all matters influencing a proper arrangement of the age gradations, more especially the requirements of transport.

If a coppice forest is so extensive that it is desirable to cut in several places in each year, while the rotation remains the same throughout, equal to r, it is first divided into a corresponding number of working sections, and then each of the latter into r annual coupes.

If the several working sections are treated under different rotations, a separate account must be kept for each; for

instance, oak coppice worked for bark alongside of coppice of an entirely different nature, such as alder coppice.

In order to obtain, as far as practicable, equal annual returns, the calculations should be made with reduced areas, though it is not necessary to go into very minute details. The different coupes should be marked on the ground.

The final yield is ascertained by estimating the returns which may be expected from the areas to be cut over during the working plan period, and dividing them by the number of years in the period.

Intermediate returns consist of all cuttings made on areas not put down for cutting over during the working plan period. As a rule, they are not of much importance. Their amount should be estimated according to average local figures.

3. Coppice with Standards.

The first step is to lay down a division into annual coupes in the same way as for simple coppice; this division regulates the yield of the underwood.

The normal yield in overwood, as given on page 232, can serve only as a very general guide; in reality, the management of the overwood partakes of the character of forest gardening or selection fellings. Hence, this silvicultural system offers considerable difficulties if the areas are extensive. Any but a very elastic method of fixing the yield would be out of place.

The determination of the final yield in overwood is effected by estimating, on the areas to be dealt with during the working plan period, the probable amount of material to be taken out. In doing this, the forester is guided by silvicultural considerations and the degree of ripeness of the several standards. Whenever fairly equal annual, or periodic, returns are aimed at, the forester should see that not more is cut during the working plan period than is likely to be produced again by increment during the same space of time, allowance being made for any surplus or deficiency of growing stock.

The sum total of the quantity of overwood thus ascertained and of the underwood makes up the expected yield during the working plan period. The executive officer should, however, not be forced to abide absolutely by that estimate, but be permitted to modify it within certain limits, in accordance with requirements as they may become manifest in the course of the period for which the working plan is drawn up.

Intermediate cuttings occur on the areas not put down for final cuttings. Their amount should be estimated in a summary way on the basis of local experience.

4. The Selection Forest.

The selection forest resembles the coppice with standard forest, since the several age classes of standards are mixed on the area in a similar manner. In the case of selection forests, it is desirable to go round the whole area within a moderate number of years, that is to say, to select again trees for felling over the same portion of the area after a moderate interval, thus avoiding having to cut too much at one time.

The area to be taken in hand annually is obtained by dividing the total area by the number of years, l, fixed as above. By multiplying the quotient by 10, the area to be dealt with during the next 10 years is obtained. On the area thus fixed, all mature trees are cut and the necessary thinnings in the younger age classes made. The age of maturity, or the rotation, is fixed as in the case of clear cutting in high forest or the shelter-wood compartment system.

Example :—

Area of a selection forest = 600 acres,
Rotation = 120 years,
l = 20 years,

$$\text{Annual cutting area} = \frac{600}{20} = 30 \text{ acres.}$$

Area to be dealt with during the first 10 years = 300 acres.

On this area all 120 years old trees are cut, as well as the necessary number of younger trees, so as to reduce them to

suitable proportions. As it is a laborious matter to ascertain the age of the trees, the diameter (or girth) at maturity is substituted for the age. For instance, instead of an age of 120 years, it may be laid down that trees with a diameter of, say, 2 feet at chest-height above the ground shall be considered mature. At the same time, it must be remembered that some trees will never grow beyond a certain size. These must be cut, even if they have not reached the minimum diamoter (or girth) laid down for maturo trees.

The areas to be dealt with in each period of, say, 10 years should be marked off on the ground ; in some cases even the annual coupes may be marked.

A distinction between final and intermediate returns is very difficult in the case of selection forests.

To attempt a regulation of the expected roturns by volume seems of doubtful utility. At any rate, to fix the yield in material only, whether in cubic feet or number of trees, is a risky procedure, which may lead either to over- or under-working of the forest. By far the best plan is to fix the yield by area, and to determine the minimum size of the mature trees to be cut. This area should not be exceeded. With this reservation, the probable amount of final returns to be cut may be estimated according to Brandis' or von Mantel's methods to be described hereafter. The same precaution as in the case of coppice with standards is here neccssary; care must be taken not to cut more than will be replaced by increment, whenever fairly even annual or periodic returns are expected, always allowing for a surplus or deficiency of growing stock.

5. Change from one Silvicultural System to another called a Conversion.

As a general rule, the returns during the period of conversion are likely to be uneven in amount. If the new system requires a higher rotation than that to be abolished, the returns will be smaller, until the conversion has been

completed, and possibly even longer. Hence, before a change of system is undertaken, it should be carefully considered whether the advantages expected from the change are likely at least to compensate for the unavoidable disadvantages.

The number of conversions from one silvicultural system to another which are conceivable is considerable, and it is impossible to give any general rules of procedure. Whatever the nature of the conversion may be, the only sure basis for the determination of the expected yield is the annual cutting area. Hence, the consideration of a few special cases will bring out the essential points to be considered in each conversion:—

a. Conversion of the High Forest Selection System into the Uniform, or Compartment System.

This conversion necessitates the substitution of even-aged for uneven-aged woods, and it frequently involves the cutting over of trees at an age differing from that which is most advantageous. To justify this sacrifice, the compartment system must offer decided advantages over the selection system.

It is usual to fix one rotation for the conversion, to divide the rotation into periods of even lengths, and to allot to each period a corresponding portion of the total area, with due consideration of the condition of the several woods. As a rule, the several age classes are not evenly distributed over a selection forest; generally, more old wood is found in some parts of the area, and more young wood in others. This fact is taken advantage of in the allotment, that is to say Period I. receives those woods which contain most old trees, especially those with little increment. Period II. receives the woods which are richest in middle-aged trees, and so on. In effecting this allotment, a proper grouping of the future age classes must not be overlooked.

Example:—Assuming the rotation to be 120 years and the whole area allotted to three periods of 40 years each, the working during the first rotation would be as follows:—

During Period I.—Part A, approximately equal to one-third of the total

area, will be regenerated, either naturally or artificially, or by a combined method. From part B any over-mature trees are removed by selection, the necessary thinnings made, and blanks, if any, stocked. In part C over-mature trees are removed, blanks stocked, incomplete young woods filled up, and others thinned.

During Period II.—Part B will be regenerated. In part A any remaining shelter trees will be removed and probably thinnings commenced. In part C over-mature trees will be cut and thinnings made wherever necessary.

During Period III.—Part C will be regenerated. In part B any remaining shelter trees will be cut and thinnings commenced. In part A the necessary thinnings will be made.

The following table will further illustrate the procedure:—

Period.	Part A.	Part B.	Part C.
I. 1—40 years.	*Regenerated.*	Over-mature trees removed. Thinnings made. Blanks stocked.	Over-mature trees removed. Thinnings made. Blanks stocked. Incomplete woods filled up.
II. 41—80 years.	Any shelter trees removed. Probably thinnings commenced.	*Regenerated.*	Over-mature trees removed. Thinnings made.
III. 81—120 years.	Thinnings made.	Any shelter trees removed. Probably thinnings commenced.	*Regenerated.*

b. Conversion of Coppice with Standards into the Compartment System.

This conversion is easiest effected by gradually growing so much overwood in each coupe that it represents a high forest. For this purpose, the coppice with standard system is continued for a time, but as little as possible overwood cut, and as many poles as possible are left standing, until the area is fully stocked with overwood. The poles thus left should, if possible, be seedling trees and not stool shoots. Another method is to grow the high forest direct out of the underwood,

provided the latter contains a sufficient number of seedling trees, and has not suffered by too much cover overhead. In either case, a good deal of planting may be necessary.

To prevent a great unevenness of returns during the first rotation, the conversion will be effected only gradually, as indicated under *a*, Conversion of a Selection Forest.

Example.—If the future rotation of the high forest be 120 years, the work would be distributed as follows:—

During Period I., of, say, 40 years.—Convert one-third of the area; cut very sparingly in the second part of the area; cut as usual in the third part.

During Period II., of 40 years.—Convert the second part; cut sparingly in the third part. Thinnings will be commenced in the first part.

During Period III., of 40 years.—Convert the third part. Thinnings in the first part will be in full swing. Thinnings will be commenced in the second part.

c. Conversion of a Forest of Broad-leaved Species into a Forest of Conifers.

An irregularly stocked forest of broad-leaved species, partly high forest, partly coppice, and partly coppice with standards, shall be converted into a coniferous forest, a conversion which is indicated by the special conditions of the locality.

The first and most important step is to divide the forest into a suitable number of compartments by laying out a system of roads and rides suitable to the locality. These compartments are then grouped into a suitable number of cutting series, without taking into consideration the present conditions of the several woods, but merely future requirements.

It would be problematic to determine the rotation to be adopted for the future coniferous forest. On the other hand, the age should be determined which the oldest coniferous wood should have reached when the conversion has been concluded, so as to have, from that moment forward, woods of sufficient age to cut and supply the market. This age determines the period during which the conversion is to be effected, called the "conversion period." The latter must not

be too short, or else there would be no final cuttings for a number of years after the conversion had been completed. Supposing 60 years were chosen for the period of conversion, then at its close the oldest coniferous wood would have an age of 60 years.

By dividing the total area by 60, the area is ascertained which should be converted annually.

In selecting the areas to be taken in hand year by year, two considerations present themselves :—

(1.) A suitable arrangement of the future cutting series.

(2.) To begin with cutting over the woods which are poorest in increment.

A consideration of both decides the allocation of the annual coupes over the forest area.

Example.—A coppice with standard forest of 1,200 acres shall, in the course of 60 years, be converted into a coniferous forest. Every 10 years $\frac{1,200}{6} = 200$ acres must be taken in hand for conversion. In that case, the yield would consist of :—

During the first 10 years—

(1.) The clearing of 200 acres.

(2.) The treatment of 1,000 acres as coppice with standards.

During the second 10 years—

(1.) The clearing of 200 acres.

(2.) The treatment of 800 acres as coppice with standards.

During the third 10 years—

(1.) The clearing of 200 acres.

(2.) The treatment of 600 acres as coppice with standards.

(3.) Thinnings in the oldest coniferous woods.

And so on.

It is evident that the returns fall off from period to period, in so far as the reduction is not made good by thinnings in the young coniferous woods. This can to some extent be modified by not making any cuttings in the area of coppice with standards which will come under conversion during the next period of 10 years; in other words, to let the material become 10 years older than it otherwise would.

The expected yield is determined by estimating the returns

from the area to be converted during the first period and adding thereto the necessary cuttings on the rest of the area; the latter should be sparingly done, so as to equalise the cuttings as much as possible.

SECTION II.—OTHER METHODS OF DETERMINING AND REGULATING THE YIELD OF FORESTS.

The number of other methods is so large that it is not possible, or necessary, to describe them all in this place. The most important methods may, according to their principal characteristics, be grouped in the following manner:—

A. Division of the forest into fixed annual coupes.

B. Allotment of woods to the periods of a rotation.

(1.) According to area.

(2.) „ „ volume.

(3.) „ „ area and volume combined.

C. Regulation of the yield according to increment and growing stock.

(1.) The Austrian method.

(2.) Hundeshagen's „

(3.) Von Mantel's „

(4.) Brandis' „

D. Regulation of the yield according to increment and growing stock, combined with the allotment of areas to the several periods of a rotation (Heyer's method).

A. Division of Forest into Fixed Annual Coupes.

Under this method, the area of the forest is divided into as many annual coupes as there are years in the rotation, and each coupe marked on the ground. Every year one coupe is cut over, giving the annual yield of final returns, to which must be added the necessary thinnings in the other coupes.

The size of each annual coupe is $= \frac{A}{r}$, if the area is at once re-stocked, or $= \frac{A}{r+s}$, if each coupe lies fallow for s years. In either case, A may represent the actual or reduced area.

The merits of this method are small. It aims more directly than any other method at the establishment of a regular series of age gradations which becomes normal after one rotation, if the division of the area is based upon the reduced area of the several parts; but it achieves this object only by heavy sacrifices, because the returns during the first rotation must be very uneven, unless at the outset a proper proportion and distribution of the age classes existed. The method takes no notice of disturbances, nor of the state of the market; hence, it is very rigid. Above all, it neglects to a considerable extent the fundamental principle that the most important measure must always be the establishment of the normal increment within the shortest possible period of time.

The method is applicable to coppice woods, coppice with standards, and, with modifications, to selection forests. For all other methods of high forests it is unsuited, except perhaps for clear cutting with a very short rotation.

B. Allotment of Woods to the Different Periods of One Rotation.

In order to remove the great rigidity of the fixed annual coupes and to obtain a method which is suitable for the treatment of high forest, especially if managed under the shelter-wood systems, the several woods comprising a forest are allotted to a number of periods. The latter are generally from 3 to 6 in number, and each comprises from 10 to 30 annual coupes. In this way, the forest is divided into as many lots as there are periods in the rotation; during each period one of these lots is dealt with. Thus, operations extend over the whole area once in each rotation. Deviations from this arrangement occur occasionally, for instance if a sub-compartment is not cut over, or twice cut over, during the first rotation in order to make the compartment uniform.

It is evident that during the first rotation the total yield is represented by the growing stock which happens to stand in the forest at the commencement of operations, plus that part

of the increment which is added to it during the course of the first rotation; it may be equal to, smaller or larger than, the normal yield.

An essential part of this method of regulating the yield is the preparation of a framework or general working plan, drawn up for one rotation and divided into a number of periods, showing during which period each wood is to be cut over. The allotment can be made according to area, volume, or the two combined, so that practically three different methods exist which must be described separately.

1. The Method of Periods by Area.

a. Description of the Method.

The woods of a forest are so allotted to the several periods of one rotation that each contains the same or approximately the same area, called the periodic coupe.

Where few or no differences exist in the quality of the locality in the different parts of the forest, the size of each periodic coupe will be $= \frac{A}{t}$, where t represents the number of periods in the rotation. If such differences exist, tho areas must be reduced to one common quality standard, and the size of the periodic coupe becomes $= \frac{\text{red. } A}{t}$. Unless this is done, the periodic yields in the second and following rotations will not be equal.

In allotting the woods to the several periods, that to be dealt with first receives the oldest woods and those with the most deficient increment, taking into consideration a suitable arrangement of the cutting series; the allotment to the other periods is made according to the age of the woods, with due consideration to a suitable grouping of the age classes. If then the totals in the several periods differ, shiftings are made by moving certain areas backward or forward, until each period contains the same or approximately the same area.

The woods placed into the first period are measured, their volume calculated, and the increment for half the number of years in the period, $\frac{n}{2}$, added. The total of the volume thus obtained is divided by the number of years in the period n, so as to obtain the average of the final annual yield during the first period. To this amount the thinnings must be added.

For an example see Appendix IV., A, at page 380, where the working plan for the communal forest of Krumbach, a village in Hesse-Darmstadt, has been given. This working plan is being actually followed.

b. Merits of the Method.

The method is simple and can be applied by any intelligent manager. It establishes the normal state within one rotation, if no disturbing events occur. At the same time, it may yield very uneven returns during the first rotation, though this can to some extent be avoided by suitable shiftings. Although the method is much less rigid than that of fixed annual coupes, it is often difficult to produce during the first rotation a proper grouping of the age classes.

Another disadvantage is that a surplus of growing stock may be dragged over a whole rotation, whereas it should be removed as quickly as possible; or, on the other hand, it may take a whole rotation to make good any deficit of growing stock.

For a financial management, the method is only moderately adapted, except in so far that it introduces order into the management. It gives only a limited latitude to the forester to hold over vigorous woods, or to cut over at an early date those which are deficient in increment.

2. The Method of Periods by Volume.

a. Description of the Method.

The woods of a forest are so allotted to the several periods of a rotation that each yields the same or approximately the

same volume. In some cases, only the final returns are thus regulated; in others, the intermediate returns are utilised to equalise the yields of the several periods.

The allotment is based upon the table of age classes; then shiftings are made, so as to bring woods which have a poor increment early under the axe and establish, as far as practicable, a suitable grouping of age classes; then further shiftings are made, so as to equalise the periodic returns. The result represents the general working plan for the first rotation. It will be observed that, in the majority of cases, the areas placed into the several periods will be uneven, resulting in uneven returns during the second rotation unless a fresh allotment is made.

Example.—In Appendix IV., B, page 386, only the final returns have, for simplicity's sake, been equalised. The data are those of the Krumbach communal forest given in Appendix IV., A.

As the future returns have to be estimated for a whole rotation, it is evident that yield tables must be used; accordingly, the above general working plan has been based upon the returns for beech high forest, given at page 356. After making the shiftings indicated in the general working plan, the volumes allotted to the several periods stand as follows:—

	Periodic Yield. Cubic feet.	Annual Yield. Cubic feet.	Area in the Period. Acres
Period I.	= 201,740	10,087	34·
„ II.	= 207,421	10,371	32·5
„ III.	= 210,932	10,547	30·5
„ IV.	= 206,180	10,309	30·
„ V.	= 212,286	10,614	33·
Total	= 1,038,559		160·
		Mean periodic area =	32·

An attempt to equalise the returns further would necessitate the cutting up of compartments, which is not desirable.

The areas placed into the several periods are uneven, and fresh shiftings will have to be made later on, so as to equalise the returns during the second rotation.

b. Merits of the Method.

The method has this advantage over the method by area, that it gives during the first rotation equal or approximately

equal periodic returns; it considers the interests of the present generation more fully. On the other hand, the estimate of the future returns is more or less problematic, so that the oquali-sation of the returns for a whole rotation ahead is a very uncertain operation. It shares with the method by area the disadvantage that a proper grouping of age classes is generally beset by difficulties. It may also drag a surplus or deficit of growing stock over a whole rotation.

Whereas the method by area establishes the normal state of the forest within one rotation, the method by volume takes generally several rotations to accomplish this.

As regards its financial aspect, it stands on about the same footing as tho method by area.

3. The Method of Periods by Area and Volume Combined.

The woods of a forest are so allotted to the several periods of a rotation, that each contains the same area and yields the same or approximately tho same volume.

The equalisation of the periodic areas and returns is effected, either by adding columns for the volume to the general working plan used for the method by area, or by adding columns showing the reduced areas to the general working plan used for the method by volume. Shiftings are made, until both area and yield are the samo or approximately the same in each period.

It will easily be understood that such an equalisation is a difficult operation, especially in a very abnormal forost; hence, more than an approximate equalisation cannot be attempted.

The method shows some of the advantages and disadvantages of the two previous methods of which it is a combination. Its principal disadvantage is that a suitable grouping of age classes is still more difficult than in the case of each of the two component methods.

In practice, various modifications of the above three methods have been evolved which sometimes partake more of one and sometimes more of another of the methods.

C. Regulation of the Yield according to Increment and Growing Stock.

The methods coming under this heading calculate the yield by means of a formula based on the increment laid on and any difference which may exist between the real and normal growing stock. Having thus determined the yield, the woods for cutting are selected from time to time in accordance with silvicultural considerations. There is no necessity for drawing up a general working plan for a longer period than suits the special requirements of each case.

Of a considerable number of methods coming under this heading, only the following need be mentioned here, as the others are of moderate practical importance.

1. The Austrian Method.

(*Die Oesterreich'sche Cameral Taxation.*)

a. Description of the Method.

In the year 1788 (during the reign of the Emperor Joseph II., one of the most enlightened Sovereigns known in history) the Austrian Government issued instructions regarding the assessment of forests for the purpose of taxation. In these instructions reference was made to the difference which may exist between the real and normal growing stock of a forest. This led to the knowledge that a forest, which is expected to give permanently an annually equal return of the normal age and amount, must contain the normal growing stock corresponding to the rotation and method of treatment. Foresters speedily applied this principle to the regulation of the yield of forests by saying that, in order to lead an abnormal forest over into the normal state, it is necessary to establish the normal growing stock, in other words, to remove a surplus or to save up any deficit as the case might be. The method developed upon this basis is called the Austrian assessment method. Authors differ as to the details

of the original method, but a general survey of the literature on the subject gives the following rule for determining the yield :—

"If the normal growing stock is present in a forest, then the actual, or real, increment must be utilised; if the real growing stock is greater than the normal, more than the real increment must be removed for a time; if the real growing stock is smaller than the normal, less than the real increment must be utilised, until the deficiency has been made good."

In carrying this excellont idea into effect, however, errors were introduced, which are still upheld by some foresters of the present day.

The procedure is described as follows :—

(1.) The increment is calculated as the mean annual increment of a series of years.

(2.) The normal growing stock is placed equal to the *normal* final mean annual increment, corresponding to the normal rotation, multiplied successively by the ages of all age gradations; the sum of all these products gives the value $G_n = I \times \frac{r}{2}$, calculated for the middle of the growing season. Here I represents the normal annual increment of all age gradations, which is equal to the volume of the oldest age gradation.

(3.) The real growing stock is obtained by multiplying the *real* final mean annual increment by the present age of each age gradation. For this purpose it is nocessary to determine for each wood the real final age and the volume at that age.

(4.) The difference between the real and normal growing stock is removed during such period as the owner, or forester, may determine according to the circumstances of each case.

(5.) The general formula for calculating the yield, if the

deficiency or surplus of growing stock is to be removed in the course of a years, runs as follows :—

$$\text{Annual Yield} = \text{real Increment} + \frac{\text{real Gr. Stk.} - \text{norm. Gr. Stk.}}{a}$$

$$Y = I_{real} + \frac{G_{real} - G_{normal}}{a}.$$

If $G_r < G_n$, then the last position becomes negative.

b. Merits of the Method.

The method was the first which based the calculation of the yield upon a knowledge of the increment and the growing stock. It has the advantages over the methods previously described in this section that—

(1.) It teaches the proportion between the real and normal growing stock and enables the owner to remove any surplus or deficiency at his pleasure.

(2.) It assures to the owner the utilisation of the full real increment, whenever the normal growing stock is present.

(3.) It distinguishes in the yield between increment and growing stock; in other words, between the removal of genuine annual increment and that of surplus capital.

On the other hand, the method, as above described, has serious drawbacks :—

(1.) The calculation of the real and normal growing stock, based upon the final mean annual increment, is not correct and not even safe. As, however, both are calculated in the same manner and one is deducted from the other, the error is to some extent eliminated.

(2.) As the yield is determined by a formula, the method, if rigidly applied, may lead to absurd results : for instance, it may happen that a full increment takes place, that numerically the real growing stock is equal to the normal growing stock, and yet there may not be a single mature wood in the forest fit to cut.

If the method is applied judiciously, that is to say, if—

(1.) the real growing stock is taken as that actually existing in the forest, and the normal growing stock calculated from a suitable yield table;

(2.) the yield, as calculated with the formula, is modified to suit the special conditions of each forest;

then the method is one of considerable merit. It enables the forester to arrange the grouping of the age classes and cutting series in the most desirable way, and to do justice to all other silvicultural requirements, since it leaves him an entirely free hand in the selection of the woods to be cut.

The method is applicable to all silvicultural systems, but the determination of the increment involves much labour, if it is to be accurate. Under it, a forest is gradually led over into the normal state, though perhaps not for a considerable period of time; the difference between the real and normal state will, after the first rotation is passed, be so small, that it can be neglected.

The sample working plan given in Appendix V., page 388, has been based upon the formula of this method.

2. Hundeshagen's Method.

a. Description of the Method.

Hundeshagen's method of determining the yield is based upon the idea that the real yield must bear the same proportion to the real growing stock as that existing between the normal yield and normal growing stock; he thus obtains the equation—

$$Y_{real} : G_{real} = Y_{norm.} : G_{norm.}$$

and

$$Y_r = G_r \times \frac{Y_n}{G_n}.$$

In words, the real yield is equal to the real growing stock multiplied by the normal yield and divided by the normal growing stock. Hundeshagen calls the quotient $\frac{Y_n}{G_n}$, by which the

real growing stock is multiplied, the "utilisation per cent." (More correctly this indicates only the rate of utilisation, whereas the utilisation per cent. is $\frac{Y_n}{G_n} \times 100$.)

The normal yield is placed equal to the normal increment, or equal to the contents of the oldest age gradation in a normal series of age classes. The normal growing stock is obtained by adding up the volumes given in a suitable yield table; by the real growing stock Hundeshagen understands that which is actually standing in the forest.

In applying the method, Hundeshagen does not ask for a general working plan, except for a limited number of years; he is satisfied with determining the species, silvicultural system, general lines of management, the rotation and general rules for the grouping of the age classes; he leaves it to the manager to select the woods for cutting from time to time, say every 5 or 10 years.

As the yield is determined by the growing stock which happens to exist, and as this practically changes from year to year, it would, theoretically speaking, be necessary to re-measure the growing stock every year, but as the changes are slow, Hundeshagen considers it sufficient if the re-measuring is done once every 20 or 30 years.

Hundeshagen determines, in the manner above described, only the final returns; he adds the intermediate returns, estimated in a summary manner, or calculated according to average data obtained locally.

b. Merits of the Method.

The principal assumption of Hundeshagen is not quite correct; at any rate, there is no justification for maintaining that the real yield bears the same proportion to the real growing stock as the normal yield to the normal growing stock, because the rate of increment is not determined by the quantity of growing stock which stands in a forest. On the contrary, a large growing stock consisting of defective old

woods may give a small increment, while a small growing stock consisting of vigorous young woods may show a large increment.

The method, if applied rigorously, may lead to absurd measures, just in the same way as the original Austrian method; it prescribes a definite annual yield, while not a single mature wood may be present; or it prescribes too small a yield, whenever a considerable portion of the area is stocked with decrepit old woods which ought to be cut over as quickly as possible and replaced by vigorous young woods. In all such cases, the yield, as fixed by the formula, must be modified in accordance with the requirements of each case.

The method does not distinguish in the yield between increment and surplus growing stock, and in this respect it stands below the Austrian method. Moreover, it may drag a surplus of growing stock over an undefined period.

Hundeshagen assumes that, with the yield calculated according to his method, the normal growing stock will be established naturally, as the yield bears a fixed proportion to the real growing stock; if the latter is greater than the normal amount, more than the increment will be removed, and *vice versâ*. This is ordinarily the case, but not under all circumstances. If, for instance, both the increment and growing stock are deficient, the yield may be greater than the increment, so that the growing stock is still further reduced, at any rate for a time; hence, the establishment of the normal state may be considerably delayed.

On the other hand, Hundeshagen's method has this great advantage, that the increment need not be determined, such a determination being at all times beset by difficulties and uncertainty. All that the method requires is a suitable yield table, and the measurement of the growing stock actually standing in the forest. Hence, the method is by no means to be despised, if a general plan is added indicating the grouping of the age classes to be aimed at. For the rest, it leaves a free

hand to the manager to shape the management in accordance with the requirements of each case, as long as the volume determined by the formula need not be rigorously cut. It may reasonably be assumed that Hundeshagen himself expected this.

3. Von Mantel's Method.

Von Mantel, in arranging for the speedy determination of the yield of certain forests in Bavaria, laid it down that this should be done according to the formula—

$$\text{Annual Yield} = \frac{\text{Real Growing stock of the forest}}{\text{Half the number of years in the rotation}} = \frac{G_{real}}{\frac{r}{2}}$$

This formula rests upon the same basis as Hundeshagen's method, if for the latter the normal growing stock is calculated with the final mean annual increment as in the Austrian assessment method. Hundeshagen's formula—

$$Y = G_{real} \times \frac{Y_n}{G_n}$$

goes, in that case, over into—

$$Y = G_{real} \times \frac{I_n}{I_n \times \frac{r}{2}} = \frac{G_{real}}{\frac{r}{2}}.$$

The cutting of the yield according to von Mantel's formula will gradually lead to the establishment of the normal growing stock, as the following considerations will show :—

Supposing the real growing stock, by which von Mantel understands that actually present in the forest, is equal to the normal growing stock, then his formula goes over into—

$$Y = \frac{\frac{r \times I}{2}}{\frac{r}{2}} = I_{norm}.$$

The formula gives, therefore, the correct yield, provided the increment is normal.

If the actual growing stock is smaller than the normal, say $G_{real} = \frac{r \times I}{2} - x$, then the

$$Y = \frac{\frac{r \times I}{2} - x}{\frac{r}{2}} = I - \frac{2 \times x}{r} = I - \frac{x}{\frac{r}{2}},$$

which means that less than the increment is cut.

Supposing that the real growing stock is greater than the normal: $G_{real} = \frac{r \times I}{2} + x$; then,

$$Y = \frac{\frac{r \times I}{2} + x}{\frac{r}{2}} = I + \frac{x}{\frac{r}{2}};$$

more than the increment will be cut, so that the surplus of growing stock will gradually disappear.

All these assumptions depend, however, on the supposition that the normal increment is laid on. If the increment is deficient, the abnormal state may be further increased until the increment has reached its normal size.

The merits of the method are approximately those of Hundeshagen's method. It introduces an additional inaccuracy by being based on the assumption, that the normal growing stock is $= \frac{I \times r}{2}$. On the other hand, the normal growing stock and normal yield need not be determined; in other words, the method can do without yield tables. It is only necessary to measure the growing stock and to determine the rotation.

The method is very simple, and it is specially suited for determining the yield of selection forests.

4. Brandis' Method.

The method to be described under this head will be best understood by indicating the circumstances which lead to its elaboration.

Sir Dietrich Brandis, on being appointed Superintendent of the Pegu forests in Burma in 1856, found himself confronted by enormous areas of teak forests in danger of being heavily overworked. These forests contained teak in varying proportions, but on the whole to a limited extent which has since been ascertained to amount to perhaps 10 per cent., while about 90 per cent. of the growing stock consisted of species which at that time had no market value. Indeed, the latter were allowed to be removed free of chargo without let or hindrance. Moreover, even teak trees required to be of a certain size to make their extraction really remunerative. At that time, it was considered desirable that no teak tree should be removed, unless it had reached a circumference of 6 feet, or roughly a diameter of 2 feet, measured at 6 feet from the ground. Trees of that girth and above were called trees of the first class.

Under those circumstances, Brandis' object was to ascertain, as quickly as possible, tho number of first class teak trees which might be removed annually, without exposing the forests to deterioration. For this purpose, he designed a method, by which he ascertained—

(1.) the number of first class teak trees in the forests;

(2.) the time which it takes to replace thom.

By dividing the number of first class trees ascertained under (1.) by the number of years ascertained under (2.), he calculated the maximum number of trees which it was permissible to cut annually.

It will thus be seen that the volume of markotable growing stock was ascertained, and that this was removed at such a rate as at least to maintain it; in other words, the maximum yield was fixed at that quantity of marketable timber which reached maturity every year, thus maintaining the mature growing stock in the forest and utilising the actual increment. With the view to utilising an excess of mature material it was laid down that, if the proportion of first class trees appeared excessive as compared with the younger classes, extra cuttings might temporarily be made, and *vice versâ*; hence, the method

is one based on increment and growing stock. Various safeguards were added, such as an allowance for trees which it did not pay to extract; where few second and third class trees existed, some first class trees were left standing to provide seed for regeneration; immediately along the banks of streams cuttings were made very sparingly, etc.

For the rest, the method leaves a free hand to the forester, who arranges the cuttings with due regard to silvicultural requirements and a proper succession of the different coupes.

The number of trees of the several size classes wore originally ascertained by counting, or measuring, them along narrow strips, generally 100 feet broad, laid through the forest along the line of march (called "linear valuation surveys"). From the contents of these sample strips (or plots) the contents of the blocks, or forest, were calculated. The rate of increment was determined by counting the concentric rings on a sufficient number of stumps, thus ascertaining the average number of years which a teak tree takes to reach the limits of the several size classes.

The original method was subsequently further elaborated, so that the sample plots are now systematically arranged over the area, with the view to obtaining correct data for the number of trees in the several blocks of the forest. The cuttings, based on these data, were also localised: in other words, an area check was added to the calculated yield, so as to guard against over or under cutting.

The method does not claim to be theoretically quite correct, but it is correct enough wherever large areas have to be dealt with in a short time. It works expeditiously, and, if judiciously applied, prevents a deterioration of the forest. Had it not been for this method, the valuable teak forests of Lower Burma might have been exhausted before their sustained yield capacity had been ascertained. It is a method to be strongly recommended for adoption in countries where systematic forest administration is in its earlier stages, and where only a limited number of species are as yet of commercial value.

Example:—

This examplo is based upon data containod in the working plan for the East Yoma, Salsuwa, and Tindaw Resorves in tho Thayetmyo Division of Burma, drawn up by Mr. A. Rodger, Deputy Conservator of Forests.

The productivo area of the forests amounts to 84,022 acres, divided into 51 compartments. Of these, one was counted out altogether, while at least two sample plots in each of tho other 50 compartments were marked, and the trees counted according to size classes. On the basis of the data thus obtained, the contents of the forosts in sound teak trees over 1 foot 6 inches girth wore calculatod. They are as follows:—

Class	I. ovor 7 feet	girth	=	31,523
,,	II. 6 foot to 7 feet	,,	=	18,114
,,	III. 4 feet 6 inches to 6 feot	,,	=	42,768
,,	IV. 3 feet to 4 foot 6 inches	,,	=	101,737
,,	V. 1 foot 6 inches to 3 feet	,,	=	150,910

To determine the rate of growth, countings on 198 trees and logs wore made, which gave the following averages:—

Girth.	Age in Years.	Years required to pass through each Class.
1 foot 6 inches	31	
		29
3 feet	60	
		33
4 feet 6 inches	93	
		37
6 feet	130	
		26
7 foot	156	
		125

Honce, the rotation was fixod at 160 years, and dividod into five periods of 32 years each.

From observations made in this and other forests in Burma, it was ascertained that the following percentages of sound trees are likely to survive, and be available for utilisation:—

Class	I. over 7 feet	girth	=	95	per cent.
,,	II. 6 feet to 7 feet	,,	=	85	,,
,,	III. 4 foot 6 inches to 6 feot	,,	=	70	,,
,,	IV. 3 feet to 4 feet 6 inches	,,	=	50	,,
,,	V. 1 foot 6 inchos to 3 feot	,,	=	25	,,

giving the following numbers of treos availablo for utilisation:—

Class	I. =	29,947	
,,	II. =	15,397	133,392 (II.–V.)
,,	III. =	29,938	
,,	IV. =	50,869	
,,	V. =	37,728	
	Total	163,879	

As it requires 125 years to pass a tree of 1 foot 6 inches girth into the first class, the average number of trees passing annually into the first class would be

$$\frac{133,932}{125} = 1,071 \text{ trees a year.}$$

All the same, there is evidently a surplus of trees over 7 feet girth, as it is hardly necessary to keep more than half the average yield of a period of such trees standing in the forest, or $1,071 \times 16 = 17,136$. The balance, $29,947 - 17,136 = 12,811$ trees, might be removed in addition to the actual increment. Assuming now that this surplus of growing stock were removed during the first period of 32 years, or 400 trees on an average annually, the theoretically correct yield would amount to:—

Yield representing the annual increment	=	1,071 trees
„ „ removal of surplus stock	=	400 „
Total permissible yield annually	=	1,471 trees.

As a rule, it is desirable, for other reasons, not to mark up to the full theoretical yield. On the one hand, certain trees never reach a girth of 7 feet, while on the other hand trees of 8 and 9 feet girth yield much higher prices per cubic foot than smaller trees. In many cases, mature trees must be left standing to provide seed for natural regeneration, and other matters demand consideration. Hence, the actual yield has, in this instance, been fixed at 1,000 trees annually, to begin with.

D. Regulation of the Yield according to Increment and Growing Stock, combined with the Allotment of Areas to the several Periods of a Rotation.

This method was originally elaborated by Carl Heyer, and classed under C., as it rested on the Austrian method. Subsequently it was further developed, especially by Gustav Heyer, until it became the combination indicated in the above heading: it is generally known as "Heyer's method."

1. Theory of the Method.

The theory of Heyer's method is as follows:—

(*a.*) To arrange all woods into a general working plan according to periods, so that each period contains the same or approximately the same area. The object of this arrangement is to equalise the increment during the second and subsequent rotations.

(*b.*) To equalise the real and normal growing stock, if any difference should exist, in such manner and within such time as may be indicated in each case and approved by the owner.

(*c.*) To utilise the real increment, calculating the mean for a series of years, plus or minus the quota of growing stock determined under (*b*).

It is obvious that these objects can be realised only by a complicated procedure, and even then only approximately, because changes in one direction disturb the balance in another.

2. Practical Application of the Method.

(*a.*) The first step is to allot, by means of the table of age classes, all woods to the several periods, and to equalise the areas by suitable shiftings, as indicated under the method of periods by area; care being taken to allot the woods, as far as this is practicable, with due consideration to silvicultural requirements, and a proper distribution of age classes.

(*b.*) The real increment is placed equal to the real final mean increment, for which purpose it is necessary to determine the final age of each wood (which may differ from the normal final age) and its probable volume at that age; the latter divided by the former gives the mean annual increment. In order to avoid having to calculate the increment year by year, it is generally calculated for a number of years, which may be called a'. If an abnormal wood is cut over during the a' years at an age differing from the normal, and a normal wood grows up in its place, the increment must be calculated separately for each part of a' years.

(*c.*) The normal and real growing stocks are calculated as for the Austrian method; the former is placed $= \frac{I \times r}{2}$, where I represents the normal final mean

increment; the latter is obtained by multiplying the real final mean increment of each wood by its age. The difference between the real and normal growing stock is removed as may be approved by the owner, say in equal amounts in the course of a years.

(*d.*) The theoretical yield is then fixed by the formula—

$$Y = \frac{\text{Real Increment of } a' \text{ years}}{a'} + \frac{G_{real} - G_{norm}}{a}.$$

If a' is placed equal to a, that is to say if the real increment is calculated for the number of years during which any difference between the real and normal growing stock is to be removed, the above formula goes over into—

$$Y = \frac{G_{real} + Ia_{real} - G_{norm.}}{a}.$$

(*e.*) The next step is to ascertain, whether the woods preliminarily placed into the several periods are sufficient to meet the yield during each period as calculated by the formula under (*d*), or whether they contain too much or too little volume; in the latter case, suitable shiftings must be made which necessitate, of course, fresh calculations of the increment and real growing stock, as the final ages of some of the woods are thereby altered. This process is continued until the requirements of the method are realised, that is to say until each period contains the same area, and at the same time the volume necessary to meet the yield as calculated under (*d*). As already indicated, the forester must, in this respect, be satisfied with approximate results.

(*f.*) The regulation of the yield is restricted to final returns. The intermediate returns are estimated only for the first period, or part of it, by means of yield tables, or past experience, and added to the final yield.

3. Merits of the Method.

The method is one of great precision. On the other hand, it is very complicated, and it calculates the increment, as well as the normal and real growing stock, incorrectly, as in the case of the Austrian method. The latter objection could be removed by using suitable yield tables, instead of the final mean annual increment, for the calculation of the increment and normal growing stock, and by measuring the growing stock actually standing in the forest. Nevertheless, the method involves great labour, and the necessary calculations are of an uncertain nature.

CHAPTER V.

CONTROL OF EXECUTION AND RENEWAL OF WORKING PLANS.

It is not sufficient to prepare a working plan; it is also necessary to see that its provisions are carried out; and when the period for which it lays down tho management of a forest has come to an end, a new, or rather a revised, plan must be prepared.

As the preparation of a working plan is to some extent based upon incomplete data, it is of importance to keep a careful record during its execution, so as to eliminate in the course of time all doubtful elements. Apart from this, changes in areas or in other respects may occur which must be noted. The work of control and renewal comprises, therefore, three distinct operations:—

(1.) The record of changes as they occur.

(2.) The record of works.

(3.) The preparation of revised working plans from time to time, or renewals.

1. Record of Changes.

(*a.*) All changes in the areas must be recorded. Part of the area may be sold or exchanged, or additional areas bought; areas hitherto used for the production of wood may be set aside for other purposes, or *vice versâ*. The progress of the cuttings may cause alterations in the allotment of areas; natural phenomena may produce changes, such as floods, landslips, fires, etc. All such changes should be noted at the close of each year, in the maps as well as in the tables of areas.

(*b.*) All final cuttings should be entered on the record and the maps.

2. Record of Works.

The record of works has for its object—

(*a.*) To give a general view of all cuttings in the forest and their distribution over the several woods or compartments.

(*b.*) To give the means of comparing the provisions of the working plan with the execution or actual results.

The special shape to be adopted depends on local circumstances, but information on the following points is required:—

(1.) Result of each cutting according to quantity and amount realised by its sale.

(2.) A comparison of the estimate with the actual results.

(3.) The harvest of minor produce according to receipts and, if possible, quantity.

(4.) Tho data showing the net results of management. For a sample see the table below.

(5.) The means of following up the history of each wood or compartment, as illustrated in Appendix V., page 394.

Table of Yield, Receipts

Year.	Area, Acres.	Wood Sold, in Solid Cubic Feet.				Receipts, Shillings.			Expenses,	
		Timber.	Fire-wood.	Bark.	Total.	From Wood.	From Minor Produce.	Total.	Harvesting of Wood.	Harvesting Minor Produce.
1891	253	12,300	7,000	100	19,400	7,600	400	8,000	1,200	100
1892										
.										
.										
1900										
Total...		130,000	70,000	1,000	201,000	75,000	4,000	79,000	14,000	900
Annual Average	253	13,000	7,000	100	20,100	7,500	400	7,900	1,400	90

3. Renewal of Working Plans.

The renewal may in some cases amount to an entirely new plan; but in the majority of cases much of the work done on the first occasion can be used again, only subsequent changes being noted.

The most important part of what remains from the provisions of the first working plan is the allotment of areas, or the order of cuttings then initiated; but even this frequently requires modification.

The task at a renewal is, strictly speaking, the same as on the first occasion, except that a good portion of the work need not be done over again, and that the experience gained during the past period makes that task a much easier one than on the previous occasion. Hence it may be indicated as follows:—

(*a.*) Investigation of the manner in which the provisions of the former working plan have been carried out, whether there were reasons for departing from them, and, if so, what they were.

(*b.*) Investigation of the extent to which the provisions of the former working plan were judicious and appropriate.

and Expenses.

Shillings.					Net Result, in Shillings.		Forest Capital, Shillings.			Percentage given by Forest Capital during Year.	Remarks.
Formation and Improvement.	Administration and Protection.	Taxes, etc.	Miscellaneous.	Total.	Total.	Per Acre.	Soil.	Growing Stock.	Total.		
200	400	200	100	2,200	5,800	22·92	31,100	128,700	159,800	3·63	
2,100	4,000	2,000	1,000	24,000	55,000						
210	400	200	100	2,400	5,500	21·74	31,100	128,700	159,800	3·44	

(*c*.) Preparation of a new working plan, based upon—

(1.) The old working plan.

(2.) The corrected records and maps.

(3.) The results of past yields in material and money.

(4.) The account of past works of formation, tending, and improvement.

PART III.

THE FOUNDATIONS OF FOREST MANAGEMENT.

THE FOUNDATIONS OF FOREST MANAGEMENT.

Forest working plans regulate, according to time and locality, the management of forests in such a manner that the objects of the industry are as fully as possible realised. As the latter differ widely, it follows that working plans cannot be drawn up according to any uniform pattern. The working plan for a protection forest or a park-like forest is altogether different from that of a forest which is managed on economic principles. In this volume, only forests of the latter class will be considered, that is to say, it will be explained how forests should be managed so as to produce the best financial results, or the greatest volume, or the most suitable class of produce for a specific purpose.

The yield (or the return) of a forest consists of major or principal, and minor produce. By the former, timber, firewood, and bark are understood. It is in the nature of things that forests should yield chiefly such articles; at the same time, articles of minor produce (such as turpentine, fodder, grazing, fruits, caoutchouc, etc.) are frequently of considerable importance, and demand modifications of that management which would be indicated by considering only the realisation of major produce.

Major produce is derived from the final and intermediate yields. The latter comprise the thinnings and other cuttings which are made from time to time during the course of the life of a wood, while the former is the return yielded by the final cutting of the wood to be followed by a new crop, whether the old crop is removed in one cutting or by a number of successive cuttings, as in the case of natural regeneration under a shelter wood.

The major produce of forests, wood, is one of the indispensable articles of life, but it is bulky and not adapted for a long transport by land. Hence, it must in many cases be produced locally. To this must be added that long periods of time elapse between the planting and harvesting of woods. Both these matters make it desirable that the yield of forests should be continuous and brought into the market in annually equal or approximately equal quantities, necessitating a management based upon the principle of a sustained yield.

Generally speaking, a *sustained yield* is secured, if all areas which have been cleared are re-stocked within a reasonable time and the young woods which spring up properly tended, so that the soil continues to produce crops of wood. At the same time, a distinction must be made between—

(1.) The intermittent working, if the successive final returns are separated by a varying number of years.

(2.) The annual working, if final cuttings occur in each year. If the latter are approximately equal in quantity year by year, the method is called the "equalised annual working."

The regulation of the yield of forests worked intermittently is very simple. It is only necessary to ascertain the most suitable rotation, taking into consideration the objects of management, and to make the intermediate cuttings whenever they are necessary. The matter becomes more difficult when an annually equal yield is expected.

Although the method of annual working, and especially of the equalised annual working, is not an absolute necessity, still it is in the majority of cases highly desirable, more especially where extensive areas are under treatment, or where a steady market has to be regularly supplied. It has considerable advantages, of which the following may be mentioned :—

(1.) It is best adapted to meet the requirements of the market, and therefore favours the development of a

regular and steady demand with a sustained competition of purchasers.

(2.) It affords equal employment year after year, and enables the administration to maintain a regular number of workmen and to instruct them thoroughly in their work.

(3.) It secures to the owner an equal, or approximately equal, annual income, and facilitates budget arrangements.

On the other hand the method has disadvantages, such as :—

(1.) It cannot be introduced without cutting certain woods at an age differing from that which is most desirable, in all cases where a regular series of age gradations does not exist, or where the age gradations are irregularly distributed over the area.

(2.) Owing to the necessity of bringing annually the same quantity of produce into the market, it interferes with the complete utilisation of special demands for forest produce, or the omission of cuttings when the demand is slack.

These remarks show that each of the two methods of working possesses peculiar advantages, and that the choice depends on circumstances. In the majority of cases, the annual working will be found more suitable and profitable, without, however, strictly adhering to it when it would involve sacrifices out of proportion to the general advantages of the method.

Correctly speaking, in order to have equal annual returns it would be necessary to regulate the intermediate cuttings as well as the final returns. Against such an arrangement the following reasons may be given :—

(1.) Areas, which yield equal final returns, do not always give equal intermediate returns.

(2.) Thinnings depend much more than final cuttings on the method of formation and tending; they must be made when they are necessary, so that the time for

their execution can, in many cases, only be determined a short time before they become necessary.

(3.) The yield of intermediate cuttings depends frequently on events which do not occur regularly, or which cannot be foreseen, so that it is almost impossible to estimate it correctly beforehand; for instance in the case of wind-break, snow-break, death caused by disease or insects, etc.

Hence, it is desirable to confine the regulation of the annual yield in the first place to the final cuttings, and to be satisfied with an approximate equalisation of the intermediate returns, such as will naturally happen, if the final cuttings are systematically equalised; provided always that the thinnings are not made so heavy as to affect the subsequent final returns to an extent which would neutralise the advantages of heavy thinnings.

If a forest is to yield a return, either annually or periodically, it must be in a certain state. In order to determine what this state should be under a given set of conditions, it is useful to construct an ideal pattern, such as would be presented by a forest which has grown up uninfluenced by external interfering circumstances. The ideal state differs, of course, for every different method of treatment, in accordance with the objects at which the management aims. In all these cases, a forest which corresponds in every way to the objects of management, is called a *normal forest*. It enables the forester to study the laws which must govern the management, and it serves as an ideal to be aimed at, though it may never be altogether reached, and at any rate not permanently maintained.

The normal state of a forest, under a given set of conditions, depends chiefly on the presence in it of—

(1.) A normal increment.

(2.) A normal distribution of the age classes.

(3.) A normal growing stock.

By *normal increment* is understood that which is *possible*,

given a certain locality, species, and rotation. An abnormal ncrement may be caused by faulty formation, faulty treatment, injurious external influences, and also, for the time being, by a preponderance of certain age classes.

By a *normal distribution of age classes* is understood a series of age gradations so arranged that at all times, when cuttings are to be made, woods of the normal age are available of sufficient extent and in such a position that no obstacles to their cutting exist.

The *normal growing stock* is that which is present in a forest, in which the age gradations are normally arranged and show the normal increment. It can, however, also be present (in quantity) in an abnormal forest, if the deficiency of some woods is made good by a surplus in others.

For the strictly annual working and the clear cutting system, a forest is, therefore, normal, if it consists of a series of fully-stocked woods equal in number to the number of years in the rotation and of the same yield capacity, so that each year a wood of the normal age can be cut, and that the returns are equal, at any rate in quantity if not in value.

From a financial point of view, the further condition must be added that there should be no woods in the forest, the current per cent. of which has sunk below the general per cent. p (see p. 157).

In accordance with these definitions, the following matters demand special attention :—

(1.) The increment.

(2.) The rotation, or the normal age at which woods should be cut over.

(3.) The normal age classes.

(4.) The normal growing stock.

(5.) The normal yield.

(6.) The relations which exist between increment, growing stock, and yield.

CHAPTER I.

THE INCREMENT.

Every tree or wood may lay on three different kinds of increment, namely:—

(1.) Quantity or volume increment.

(2.) Quality increment.⎫
(3.) Price increment. ⎭ Also called the value increment.

SECTION I.—VOLUME INCREMENT.

By volume increment is understood the increase in the volume caused by the growth of a tree or a wood. It is measured by the solid cubic foot, or the stacked cubic foot. The different kinds of volume increment and the modes of measuring them have been explained in Forest Mensuration. For the purpose of working plans, it must be added that for short periods, say 5—10 years, the periodic mean annual increment can be put equal to the current annual increment without any appreciable error.

The calculations of increment may refer to the final yield only, or to the intermediate yields, or to both together. In the tablos in Appendix III., the various classes of increment have been calculated accordingly.

1. Progress of Volume Increment.

a. Of Single Trees.

The volume increment is produced by an annual extension of the crown and roots, and by the addition of a new layer between wood and bark all over the stem, branches and roots. As a general rule, the stem, or trunk, is the most important

part of the tree; hence, the forester is specially interested in the height and diameter growth.

It has been explained in Volume II. of this Manual, fourth edition, p. 56, that the energy of *height growth* differs not only according to species, but also, in the case of one and the same species, according to the age of the tree, the productive power of the locality, and the method of treatment; besides, there is in this respect a great difference between seedlings and coppice shoots.

The subjoined table and Fig. 39 exhibit the height growth

Age, Years.	Height in Feet.					Age, Years.
	Spruce.	Silver Fir.	Scotch Pine.	Beech.	Oak.*	
10	8	3	12	6	10	10
20	20	9	29	18	25	20
30	35	18	41	31	37	30
40	51	30	55	45	48	40
50	65	45	65	56	58	50
60	76	58	73	67	67	60
70	85	71	80	76	73	70
80	93	82	86	85	79	80
90	99	91	91	92	83	90
100	104	98	95	98	87	100
110	109	104	99	104	91	110
120	112	109	103	107	94	120

* The dates for the oak in this and the following statements are not taken from the Yield Tables in Appendix III., which refer only to oak grown on alluvial soil.

for some of the more important species of Europe, taking in each case a locality of the first quality class, and assuming that the trees have grown up in fully-stocked woods.

Generally speaking, in the case of *seedlings*, the height growth during earliest youth is, in temperate climates, comparatively slow; it then increases rapidly, remains steady for a time, then decreases, and ceases altogether or nearly so.

The periods, when the current annual and mean annual height increment show their maxima, are of special interest to the forester, but the data at present available give wide

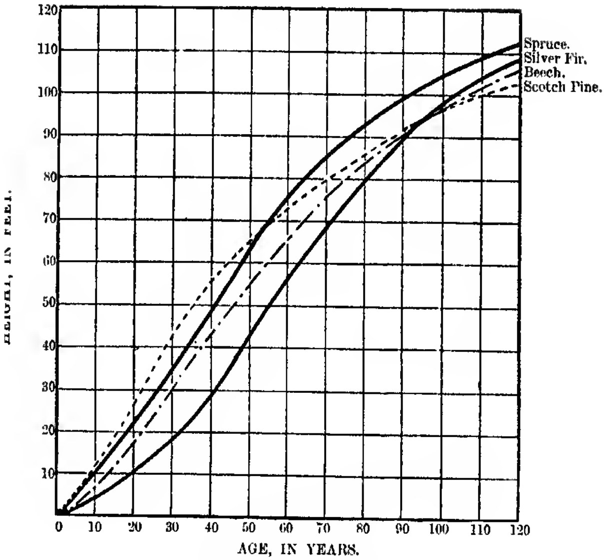


Fig. 39.—Diagram illustrating the Relative *Height* Growth of Spruce, Silver Fir, Beech and Scotch Pine on Localities of the First Quality.

imits for those periods. The following table gives approximately the average ages at which the current annual and mean ınnual height growth culminate :—

Species.	Current Annual Height Increment. Average Year when Maximum occurs.	Mean Annual Height Increment. Average Year when Maximum occurs.
Oak	20	40
Scotch Pine	25	40
Spruce	35	50
Beech	38	50
Silver Fir	50	80

On the whole, the culmination occurs earlier—

(1.) In the case of light-demanding species, and
(2.) In the better localities.

The latter point will be illustrated by giving the following data referring to Scotch pine:—

Age Period, in Years.	Height Increment, in Feet.		
	I. or Best Quality.	II. or Middling Quality.	III. or Lowest Quality.
1—10	12	8	4
11—20	**17**	9	4
21—30	15	**12**	6
31—40	11	9	**7**
41—50	10	8	5
51—60	8	6	4
61—70	7	6	4
71—80	6	5	4
81—90	5	5	3
91—100	4	4	3
101—110	4	4	3
111—120	4	3	

The first quality culminates about the year 15.
The second „ „ „ „ „ 25.
The third „ „ „ „ „ 35.

In the case of teak, the current annual height increment generally reaches its maximum during the first five years of the tree's life, frequently in the second or third year. Deodar shows a height growth similar to that of spruce. Sál shows, as far as is known at present, a remarkably even rate of height growth up to an age of 80 or 100 years.

Coppice shoots show, generally, the greatest height growth during the first few years of their existence; the rate of increment begins to fall off early, nor do such shoots, rare cases excepted, reach the same ultimate height as seedling trees.

The *lateral increment* of the trunk of a tree, *i.e.*, diameter or sectional area increment, depends on the surface of the leaf canopy and on its activity. Hence, free growing trees increase more rapidly in diameter than those grown in fully-stocked woods. At the same time, the position of the leaf surface is

of importance. Trees with a crown coming close to the ground are comparatively more tapering, while those with the crown restricted to the upper part of the stem show a more cylindrical shape. The form or shape of the stem depends, therefore, on the distribution of the crown. If, with advancing age, the crown of trees in fully-stocked woods moves higher up the stem, the difference in diameter increment between the lower and upper part of the stem decreases, and this is accompanied by what may be called the "form increment"; in other words, the tree becomes less tapering. The forester expresses this, as explained at p. 36, by the "form factor," or the co-efficient, by which the volume of a cylinder of the same base and height as the tree must be multiplied, in order to obtain the volume of the stem of tho tree.

It has been stated, at p. 38, that in practice only the form factors based on a measurement of the base at height of chest, or 4¼ feet above the ground, are used, and that the form factors there given refer to trees grown in fairly crowded woods.

In the case of trees grown in coppice with standards, form factors are as a rule out of the question.

b. Volume Increment of Whole Woods.

The increment of a wood consists, during the first period of life, of the full increment of the individual trees. As soon as the trees close overhead, the extension of the crowns is interfered with, followed by a decrease in the diameter increment. As long as the degree of crowdedness is not too great, the height growth is not reduced; on the contrary, a moderate degree of density of the leaf canopy encourages height growth. Although, during this period, the individual tree has less increment than it would have in a free position, a fully-stocked wood can have, and generally has, a larger increment per unit of area than an open wood, because the total increment is equal to the mean increment per tree

multiplied by the number of trees. What degree of density of a wood gives the maximum increment is a question which awaits solution. In the meantime it must not be forgotten that a fairly crowded condition encourages height growth, decreases the tapering of the stems, and kills off the lower branches, thus producing most valuable trunks.

While the loss of material is very small in trees grown in the open, it becomes considerable in the case of fully-stocked woods. Not only do all the lower branches die off, but the

Age.	Current Annual Increment. Cubic Feet per Acre.					Mean Annual Increment. Cubic Feet per Acre.					Age.
	Spruce.	Silver Fir.	Scotch Pine.	Beech.	Oak.	Spruce.	Silver Fir.	Scotch Pine.	Beech.	Oak.	
10	50	30	80	15	30	50	30	80	15	30	10
20	130	58	151	39	86	90	44	115	27	39	20
30	**370**	88	180	126	177	173	58	137	60	79	30
40	327	166	**186**	191	**211**	212	85	149	93	110	40
50	297	245	178	**228**	203	229	117	155	120	129	50
60	273	292	154	196	183	236	146	**155**	133	140	60
70	241	364	126	199	169	**237**	178	151	142	114	70
80	210	**407**	112	188	151	231	206	146	148	**146**	80
90	185	399	97	176	134	228	228	140	151	144	90
100	165	340	83	165	120	222	239	135	152	143	100
110	147	255	76	154	109	215	**240**	129	**152**	140	110
120	130	210	66	143	97	208	238	124	152	137	120

greater number of the trees of which the wood originally consisted must be removed by degrees, because they are gradually overtopped, suppressed, and finally killed; these form, ordinarily, the material of which the thinnings consist.

In fully-stocked woods, especially in those treated as high forest, a distinction must be made between the dominating and suppressed trees; the former may be called the major or primary part of the growing stock, and the latter the minor or secondary part. Not only the latter, but also a considerable portion of the former will be removed in the thinnings, in the same degree as, with the advancing age of the wood, they lose their dominating character and join the secondary part of

the growing stock. Exceptions to this may occur, which have been explained in Volume II.

The progress of the increment in whole woods has by no means been determined for all important species, though much material bearing on this question has been collected of late years on the continent of Europe. In India, matters are still more backward.

The data on page 177 refer to European species, taken from woods growing on land of the first quality class. They give

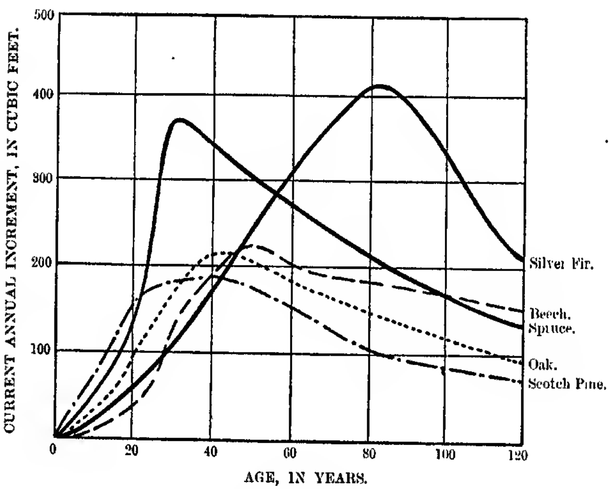


Fig. 40.—Diagram showing the *Current* Annual Increment.

the total production of timber and firewood in fully-stocked woods.

These tables and illustrations justify the following conclusions:—

(1.) The current annual increment rises rapidly after the first youth is passed, and reaches its maximum, in the case of many species, about the time when the height growth culminates; it then falls and reaches zero at the death of the wood. In the case of some species, the

maximum is reached some time after the culmination of the height growth, as in the case of silver fir.

(2.) The mean annual increment keeps below the current annual increment, until the two become equal; after that period, the mean annual increment is greater than the current annual increment.

(3.) The mean annual increment reaches its maximum at the precise moment when it is equal to the current annual increment.

(4.) When the mean annual increment culminates, the current annual increment must, naturally, already be

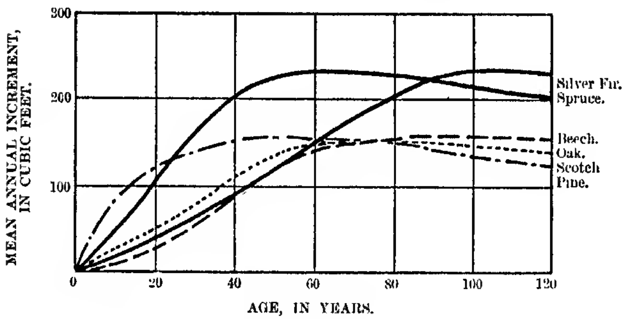


Fig. 41.—Diagram showing the *Mean* Annual Increment.

past its maximum, and be falling; hence, the former culminates later than the latter. During the intermediate period between the two culminations, the mean annual increment is still rising, whereas the current annual increment is already falling.

These points are further illustrated by tho diagram (Fig. 42 on next page), giving the current annual and mean annual increment for Scotch pine.

(5.) Whenever the object of management consists in the realisation of the greatest return of volume, the rotation must coincide with the year in which the mean annual increment culminates.

The following example will illustrate this :—

The curront annual increment of spruco in the above example culminates at 30 years, the mean annual increment at 70 years. In the course of 210 years there would be seven rotations of 30 years, or threo rotations of 70 years. The former would yield in each rotation 173 × 30 = 5,190 cubic feet, and in seven rotations = 5,190 × 7 = 36,330 cubic feet. The latter would yield in one rotation 237 × 70 = 16,590 cubic feet, and in three rotations = 16,590 × 3 = 49,770 cubic feet, or 13,440 cubic feot more.

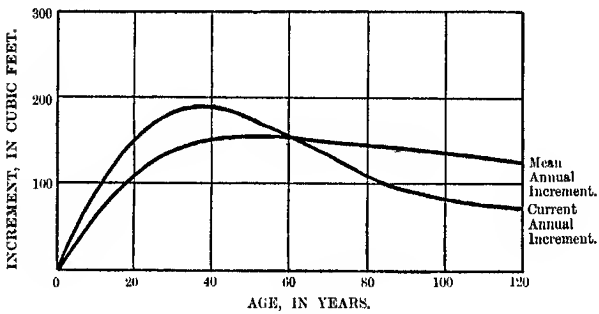


Fig. 42.

Tho maximum for final yield only occurs earlier than that for final *plus* intermediate yield; the difference may amount to 10 and even 20 years.

2. Volume Increment Per Cent.

So far the increment has been expressed in actual volume. In addition, it is useful to ascertain the proportion which exists between the total volume of a tree or wood at a certain age and the increment laid on during the year before or the following year. In order to express this proportion independently of the actual volume, it is usual to give it in per cents., and to call it the *increment per cent.;* by this is, therefore, understood the current annual increment which is laid on by every 100 units of volume.

If a capital C yields annually the interest R, then the per cent. p, with which the capital has worked, is found by the equation—

$$\frac{R}{C} = \frac{p}{100}$$

and

$$p = \frac{R}{C} \times 100.$$

In the same way, it may be said, if $v =$ the volume of a tree or wood, $i =$ the increment added during one year's growth, and $p_v =$ the per cent. with which the wood capital v has worked, then the following equation holds good:—

$$\frac{i}{v} = \frac{p_v}{100},$$

and

$$p_v = \frac{i}{v} \times 100,$$

or, if $i = V - v$,

$$p_v = \frac{V - v}{v} \times 100.$$

The matter can be considered from a different point of view, as follows:—

$$\frac{p_v}{100} = \frac{i}{v} = \frac{V - v}{v},$$

or

$$\cdot 0p_v = \frac{V}{v} - 1,$$

or

$$1 \cdot 0p_v = \frac{V}{v}$$

and

$$V = v \times 1 \cdot 0p_v.$$

This means that the volume at the end of one year, V, may be considered as having been produced by v working for one year with p_v per cent.

The increment per cent. is used, sometimes to calculate from the present volume the increment which is likely to be laid on in the immediate future, but chiefly for the purpose of testing the activity of the capital invested in forestry.

The increment per cent. p_v is naturally very large during

the early youth of a tree or wood; but as the volume increases year by year, that is to say the denominator in the above equation, while the annual increment does not increase in anything like the same proportion, and in fact begins to decrease comparatively early, it follows that the increment per cent. becomes smaller year by year. Heavy thinnings can temporarily produce an exception to the above rule, as they may retard the sinking of the increment per cent. by diminishing the producing capital.

As the determination of the increment of a single year is a difficult and inaccurate operation, it is usual to determine it for a number of years, 5, 10, or, generally, n years, and to consider V as the volume produced by placing v for n years at compound interest, working with p_v per cent.:—

$$V = v \times (1{\cdot}0p_v)^n.$$

From this—

$$\left(1{\cdot}0p_v\right)^n = \frac{V}{v}$$

and

$$1{\cdot}0p_v = \sqrt[n]{\frac{V}{v}},$$

and

$$\cdot 0p_v = \sqrt[n]{\frac{V}{v}} - 1$$

$$p_v = 100\left(\sqrt[n]{\frac{V}{v}} - 1\right), = 100 \times \sqrt[n]{\frac{V}{v}} - 100,$$

or

$$100 + p_v = 100 \times \sqrt[n]{\frac{V}{v}}$$

and

$$\log(100 + p_v) = 2 + \frac{\log V - \log v}{n}.$$

In order to avoid the use of logarithms, several formulæ have been evolved which give approximately accurate results. Pressler obtained such a formula by assuming that the increment during the n years is laid on in annually equal quantities, and by comparing the increment with the volume

which is present in the middle of the period of n years. He thus obtains the proportion—

$$\text{Capital} : \text{annual increment} = 100 : p_v$$

$$\frac{V+v}{2} : \frac{V-v}{n} = 100 : p_v$$

and

$$p_v = \frac{V-v}{V+v} \times \frac{200}{n}.$$

This formula gives p_v somewhat too small; but the difference is so slight that it can be neglected for all practical purposes.

Example:—

Let v in the year 70 = 3,820 cubic feet;
,, V in the year 80 = 4,260 ,,

then—

$$p_v = 100\left(\sqrt[n]{\frac{V}{v}} - 1\right) = 100\left(\sqrt[10]{\frac{4,260}{3,820}} - 1\right) = 1{\cdot}096 \text{ per cent.}$$

or

$$p_v = \frac{V-v}{V+v} \times \frac{200}{n} = \frac{4,260 - 3,820}{4,260 + 3,820} \times \frac{200}{10} = 1{\cdot}089 \text{ per cent.}$$

If any thinnings have been made during the n years, their amount must be added to V, before the increment per cent. is calculated. Supposing that in the above case 327 cubic feet were cut between the years 60 and 70, then—

$$p_v = 100\left(\sqrt[10]{\frac{4,260 + 327}{3,820}} - 1\right) = 1{\cdot}847 \text{ per cent.};$$

or

$$p_v = \frac{4,260 + 327 - 3,820}{4,260 + 327 + 3,820} \times \frac{200}{10} = 1,825 \text{ per cent.}$$

It remains to add that the formulæ for the increment per cent. can be applied to height, diameter, or basal area increment, as well as to volume increment.

The results of investigations made on the Continent have led to the preparation of yield tables for a number of species, an abstract of which will be found in Appendix III. An examination of these tables will confirm the conclusions so far drawn. The tables for oak refer only to the lowlands of Germany; the returns of oak on hilly ground are somewhat lower than those given in the tables.

SECTION II.—QUALITY INCREMENT.

By quality increment is understood the increase in the value per unit of volume. It is produced, in the first place, by larger pieces of timber frequently fetching higher prices per unit of measurement, and secondly by a reduction of the cost of harvesting per unit of measurement. Quality increment is independent of any alteration in the general price of forest produce.

If, in the course of n years, the net value of the unit of volume rises from q to Q, then the quality increment is $= Q - q$, and the corresponding per cent. is obtained by the formula—

$$Q = q \times \left(1{\cdot}0p_q\right)^n;$$

and

$$p_q = 100\left(\sqrt[n]{\frac{Q}{q}} - 1\right);$$

or

$$\log\,(100 + p_q) = 2 + \frac{\log Q - \log q}{n}.$$

An approximately correct value for p_q is obtained by the formula—

$$p_q = \frac{Q - q}{Q + q} \times \frac{200}{n}.$$

The quality increment may be rising, falling, or its movements may be more or less irregular; hence, it must be ascertained in each case.

Woods grown for *firewood* show only little or no quality increment after middle age; except, perhaps, in so far as the percentage of stem- to branch-wood increases. The latest investigations seem to indicate even that wood taken from middle-aged trees has a higher heating power than wood taken from older trees, although the latter may be perfectly sound.

Matters are different in the case of *timber forests;* here the quality increment rises, in the majority of cases at any

rate, beyond middle age and frequently to an advanced age, because:

(1.) Trees of large dimensions are, up to a certain limit, more valuable per unit of volume than those of small dimensions, though exceptions to this rule occur frequently.

(2.) The percentage of timber to firewood increases, at any rate up to a certain age.

The quality increment per cent. sinks, on the whole, with advancing age, though more or less irregularly; it can become nil and even negative if the timber commences to decay.

Example.—A Scotch pine wood 60 years old contains—

Timber = 3,300 cubic feet, worth 4*d.* per cubic foot.
Firewood = 760 ,, ,, ,, 1*d.* ,, ,,

Hence, mean quality—

$$q = \frac{3{,}300 \times 4 + 760 \times 1}{4{,}060} = 3{\cdot}44 \text{ pence.}$$

The same wood in the year 70 has—

Timber = 3,820 cubic feet, worth 5*d.* a cubic foot.
Firewood = 710 ,, ,, ,, 1*d.* ,, ,,

Hence—

$$Q = \frac{3{,}820 \times 5 + 710 \times 1}{4{,}530} = 4{\cdot}37 \text{ pence.}$$

And

$$4{\cdot}37 = 3{\cdot}44 \times 1{\cdot}0p_q{}^{10}$$

$$p_q = 100\left(\sqrt[10]{\frac{4{\cdot}37}{3{\cdot}44}} - 1\right)$$

$$\log(100 + p_q) = 2 + \frac{\log 4{\cdot}37 - \log 3{\cdot}44}{10}.$$

And

$$p_q = 2{\cdot}42 \text{ per cent.}$$

Approximate value—

$$p_q = \frac{4{\cdot}37 - 3{\cdot}44}{4{\cdot}37 + 3{\cdot}44} \times \frac{200}{10} = 2{\cdot}38 \text{ per cent.}$$

What has been said above can also be applied to the intermediate returns. Indeed, the quality increment of that part of

a wood which yields the thinnings can be very considerable, especially while the wood is still young. Here, a few years' extra growth may cause a great rise in the quality per unit of measurement. On the other hand, if thinnings are kept over too long, they interfere with the proper development of the major part of the wood; hence, extremes in this respect must be avoided.

SECTION III.—PRICE INCREMENT.

By price increment is understood the increment caused by a change in the price of forest produce generally, independent of the accompanying quality increment. It can be positive, nil, or negative.

Example.—A hitherto inaccessible forest is brought into communication with a large town by the construction of a railway; the increase in the prices of the produce of the forest represents the price increment, which in this case is positive.

Or, owing to an increased import of forest produce, the price of the home production falls generally; this represents a negative price increment.

The price increment depends partly on the forester, and partly on external causes over which he has little or no control. Of the former class of causes are, for instance, the construction of good roads, development of industries which consume forest produce, improvement in the general management leading to a higher net value per unit of measurement.

It is out of the question to construct a law showing the changes in price. In some cases, such changes affect all classes of produce, in others only certain kinds. Under any circumstances, it is almost impossible to foresee them, except in special definite cases. At the same time, the price increment is of considerable importance, as it affects the financial ripeness of woods, and in this way influences the lines upon which the management of the forest should proceed.

The price increment is calculated in the same way as the quality increment. If s represents the value of the unit of

measurement at the present time, and S the corresponding value after n years, the price increment is $= S - s$, and

$$S = s \times 1{\cdot}0p_s^{\,n}$$

$$p_s = 100\left(\sqrt[n]{\frac{S}{s}} - 1\right),$$

$$\log\,(100 + p_s) = 2 + \frac{\log S - \log s}{n}.$$

Again, the approximate value—

$$p_s = \frac{S - s}{S + s} \times \frac{200}{n}.$$

SECTION IV.—ADDITION OF THE SEVERAL INCREMENT PER CENTS, LEADING TO THE FOREST PER CENT.

Instead of calculating with the volume-, quality-, and price-increments separately, it is more convenient to combine them. The combined increment per cent. is determined in the following manner:—

A forest, which has at present a value $= F_m$, increases during the next year—

In volume by p_v per cent.
,, quality ,, p_q ,, ,,
,, price ,, p_s ,, ,,

Its value at the end of one year may be expressed by the formula—

$$F_{m+1} = F_m \times 1{\cdot}0p_v \times 1{\cdot}0p_q \times 1{\cdot}0p_s.$$

For convenience sake—

$$1{\cdot}0p_v \times 1{\cdot}0p_q \times 1{\cdot}0p_s \text{ is placed } = 1{\cdot}0p_f,$$

where p_f represents the combined effect of p_v, p_q, and p_s; hence,

$$F_{m+1} = F_m \times 1{\cdot}0p_f.$$

This formula could be used to determine the financial activity of a wood at any time, if it were possible to determine accurately the value increment of the wood for a single year. This, however, is a very uncertain operation; hence, the difference in the value of the crop produced during a series

of years must be ascertained, a difference due to the combined effect of volume-, quality-, and price-increment during, say, n years.

In that case, the value of the forest after n years is—

$$F'_{m+n} = F'_m \times 1{\cdot}0p_f^n.$$

Out of this,

$$1{\cdot}0p_f^n = \frac{F'_{m+n}}{F'_m}$$

$$1{\cdot}0p_f = \sqrt[n]{\frac{F'_{m+n}}{F'_m}}$$

$$\cdot 0p_f = \sqrt[n]{\frac{F'_{m+n}}{F'_m}} - 1,$$

and

$$^{curr.}\mathbf{p_f} = 100\left(\sqrt[n]{\frac{\mathbf{F_{m+n}}}{\mathbf{F_m}}} - 1\right)$$

$$\log\left(100 + p_f\right) = 2 + \frac{\log F'_{m+n} - \log F'_m}{n}$$

This formula was introduced by Pressler, who called the $^{curr.}p_f$ thus obtained the *indicating per cent.* (Weiserprocent.).

The indicating per cent. (or current forest per cent.) indicates the per cent. with which the capital, represented by a wood, works at the various periods of the wood's life; in other words, it indicates at any time, whether a wood is financially ripe or not. As long as the indicating per cent. is larger than the general per cent. p, at which money can be invested otherwise with equal security, or at which money can be obtained for investment in forestry, the wood is financially not ripe; when the indicating per cent. has become smaller than p, the financial ripeness of the wood is past; *the wood is financially ripe at the time, when the indicating per cent. is equal to p.*

It remains to substitute the proper values for F'_m and F'_{m+n}. The capital value of the forest at the present time m is represented by the value of the growing stock and soil, correctly $= {}^mG_c + S$. As the formula is, as a rule, only used in the case of woods which are at or near maturity, the

utilisation value may be substituted for the cost value of the growing stock, so that

$$F_m = Y_m + S.$$

This is the capital which it is proposed to let work for another n years. During that period, it increases to the value of the forest in the year $m + n$, from which amount must be deducted the annual costs during n years with compound interest, so that—

$$F_{m+n} = Y_{m+n} + S - E\,(1{\cdot}0p^n - 1)$$

and

$$^{curr.}p_f = 100\left(\sqrt[n]{\frac{Y_{m+n} + S - E\,(1{\cdot}0p^n - 1)}{Y_m + S}} - 1\right).^*$$

If, between the years m and $m + n$, a thinning has been made, say in the year x, its value, with compound interest to the year $m + n$, must be added to F_{m+n}, so that the formula becomes:—

$$^{curr.}p = 100\left(\sqrt[n]{\frac{Y_{m+n} + T_x \times 1{\cdot}0p^{m+n-x} + S - E(1{\cdot}0p^n - 1)}{Y_m + S}} - 1\right).^*$$

In either case, the value S of the soil can be taken as the cost value or as the expectation value.

If n is placed $= 1$, the above formula reduces to—

$$^{curr}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{Y_m + S},$$

agreeing with that given at page 163 for the current annual forest per cent.

Example.—Taking the data in the table at page 120, and putting $p = 2\frac{1}{2}$ per cent., $S = 404$ shillings, $e = 3$ shillings, the following values of p_f are obtained:—

For the period 70—80 years:—

$$\log\,(100 + p_f) = 2 + \frac{\log\,(2{,}860 + 144 + 404 - 34) - \log\,(2{,}187 + 404)}{10},$$

$$^{70-80}p_f = 2{\cdot}68.$$

* This formula differs from that given by Pressler and Judeich for the reasons indicated in the footnote at page 157.

For the period 80—90 years:—

$$\log(100 + p_f) = 2 + \frac{\log(3{,}558 + 149 + 404 - 34) - \log(2{,}860 + 404)}{10},$$

$$^{80-90}p_f = 2{\cdot}25.$$

The current annual forest per cents. given in the table at page 195 have been calculated in this way, and they show that the financial ripeness occurred during the period 70 to 90, or, more precisely, in the year 79.

CHAPTER II.

THE ROTATION.

By rotation is understood that period of years which elapses between the formation of a wood and the time when it is finally cut over and regenerated.

The end of this period, that is to say, the age of the wood when cut over is called the "final age." If it coincides with that which is considered the one best suited to the system of management, it is called the *normal* final age; if a wood has, for one reason or another, to be cut over at a different age, the latter is called an *abnormal* final age.

The determination of the rotation is one of the most important measures in forest management. It depends entirely on the various objects of management; hence, the rotation differs with every change of conditions. In economic forestry the following deserve to be distinguished:—

(1.) The financial rotation.
(2.) The rotation of the highest income.
(3.) The rotation of the greatest volume production.
(4.) The technical rotation.
(5.) The physical rotation.

Any one of these may be indicated by the objects of management, and it is necessary to explain them in some detail.

1. The Financial Rotation.

a. Calculation of the Financial Rotation.

By the financial rotation is understood that, under which a forest yields, if calculated with a given per cent. and

compound interest, the highest net return. The financial rotation is, therefore, identical with that which—

(*a.*) Gives the maximum soil rental as expressed by the formula—

$$\text{Soil rental} = S_e \times \cdot 0p$$
$$= \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{\dfrac{1{\cdot}0p^r - 1}{\cdot 0p}} - e.$$

(See page 148.)

(*b.*) Or yields the highest profit:—

$$P = S_e - S_c, \text{ respectively} = F_r - F_c.$$

(See pages 151 and 153.)

(*c.*) Or yields the maximum mean annual forest per cent.:—

$${}^{mean}p_f = \frac{S_e}{S_c} \times p, \text{ respectively} = \frac{F_r}{F_c} \times p.$$

(See pages 158 and 159.)

Of these, the first formula is the most convenient, and the procedure is as follows:—In the forest (for which the financial rotation shall be determined), a number of typical woods are examined and as many data as possible collected. These can be augmented by data taken from suitable yield tables, if such are available. Then the soil rental is calculated for various rotations, and that for which the rental becomes a maximum is the financial rotation.

Example:—In order to explain the method, the appended table has been calculated from the money yield table for the Scotch pine given at page 120. In calculating that table, it has been assumed that the cost of formation comes to 60 shillings, the annually recurring net costs to 3 shillings, and that the general per cent. $p = 2\frac{1}{2}$ per cent. It has also been assumed that the thinnings during the several periods of ten years are made at the end of each period; for instance, the thinnings during the period of 40—50 are assumed to be made in the year 50.

The table on pp. 194, 195 shows that the financial rotation falls between the years 70 and 90. In order to ascertain the exact year, the rentals given in column *k* of the table have been plotted (see Fig. 43). It will be seen that the financial rotation falls into the year 79, when the rental reaches its maximum.

In column l the current annual forest per cent. is given, calculated according to the formula on page 189. These data show that the forest, or indicating per cent. passes from above $2\frac{1}{2}$ per cent. to below $2\frac{1}{2}$ per cent. between the years 75 and 85. By plotting the per cents. (see Fig. 44), it is found that the exact time falls into the year 79, that is to say, the year when the annual soil rental culminates. Hence, the wood was financially ripe in the year 79.

b. Notes on the Financial Rotation.

Owing to the uncertainty of the data upon which the calculation is based, the financial rotation can be determined only approximately; moreover, it changes with every change of conditions. Under these circumstances, it can only serve as a general guide.

Of the several items which appear in the formula for the soil rental, the rate of interest is the most important. A low rate gives a high financial rotation, and *vice versâ*. An alteration of 1 per cent. in the general per cent. p may cause the financial rotation to rise or fall by 10 to 20 years.

As has been explained on a previous occasion, the general per cent. applicable to forest finance may, for Britain, at present be placed at $2\frac{1}{2}$ per cent.

Of the receipts, the final yield is by far the most important item. Its present value can easily be ascertained, but forecasts for the future are of a risky nature. If, in the future, the proportion between the prices of the different classes of produce remains about the same, then a change in the financial rotation does not necessarily follow; but great changes can be produced in the reverse case; that is to say, if, for instance, timber of small dimensions rises in price, while that of large dimensions falls, or *vice versâ*. Such changes are difficult to foresee. The experience of the last decades seemed to show that timber of large dimensions is not unlikely to rise in price. During the last few years, however, middle-sized timber has, comparatively, risen in price; still, on the whole, the selected rotation should be rather above than below the financial rotation.

· FINANCIAL YIELD TABLE I

Data in Column *b* and *c* tal

a	*b*	*c*	*d*	*e*	*f*
Length of Rotation, Years.	Net Value of Yields, in Shillings.		Sum of Intermediate Yields with Compound Interest to date, Shillings, $p=2\cdot5$.	Total Yield to Date ($b+d$). Shillings.	Cost of Form tion = 60s., w Compoun Interest to Date. Shillings
	Final.	Intermediate.			
30	195	5	5	200	126
40	590	44	50	640	161
50	985	87	152	1,137	206
60	1,563	118	312	1,875	264
70	2,187	134	533	2,720	338
80	2,860	144	827	3,687	433
90	3,558	149	1,207	4,765	554
100	4,273	165	1,710	5,983	709
110	5,017	167	2,356	7,373	907
120	5,792	135	3,015	8,807	1,161

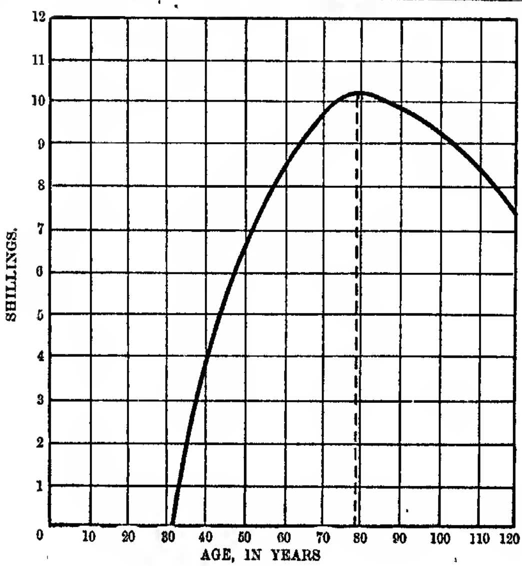


Fig. 43.

ONE ACRE OF SCOTCH PINE WOOD.

from the Table at page 120.

g	h	i	k	l	m
Total Yield less Cost of Formation $(e-f)$. Shillings.	Value of $\frac{1 \cdot 025r-1}{\cdot 025}$.	Soil. Gross Rental $\frac{g}{h}$. Shillings.	Soil Net Rental $i-$Annual Expenses. Shillings.	Current Forest or Indicating Per Cent. during every 10 years.	Length of Rotation. Years.
74	43·904	1·69	−1·31		30
				5·30	
479	67·404	7·11	+4·11		40
				3·89	
931	97·484	9·55	6·55		50
				3·97	
1,611	135·992	11·85	8·85		60
				3·19	
2,382	185·284	12·32	9·32		70
				2·68	
3,254	248·383	13·10	10·10		80
				2·25	
4,211	329·154	12·79	9·79		90
				1·95	
5,274	432·549	12·19	9·19		100
				1·73	
6,466	564·902	11·45	8·45		110
				1·51	
7,646	734·326	10·41	7·41		120

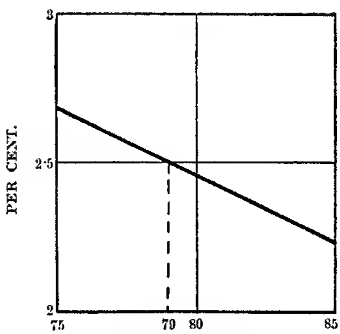


Fig. 44.

The intermediate returns exercise a considerable influence upon the actual amount of the rental, but a comparatively small effect upon its culminating point; in other words, early thinnings reduce the financial rotation only to a limited extent, and if they are made so heavy that they reduce the value of the final return, they may have even the opposite effect. Of late years it has been argued that the thinnings, especially in coniferous woods, should be made much heavier than has been customary on the Continent for almost a century, thus favourably affecting the financial results. Indeed, it has been maintained that the amount taken out in thinnings should be equal to the final yield. The author cannot accept that theory, because such a method of treatment would lead to the production of an inferior quality of timber, which would more than neutralise the advantages of heavy thinnings at an early period of the life of the wood. Those who advocate the theory of early and heavy thinnings have not seen the evil effects of such a treatment brought about in the United Kingdom in past years.

Of the costs, the annual expenses do not affect the financial rotation, unless they alter in amount with the rotation. The cost of formation affects the rental to a considerable extent, but its effect upon the financial rotation is small.

Taking all effects together, it may be said that the financial rotation is *low*—

(1.) in the case of localities where only firewood is saleable, that is to say, where an increase in quality per unit of volume ceases at a comparatively early age; in other words, the financial rotation would approximately coincide with that of the greatest volume production;

(2.) if trees of small dimensions can be sold as timber, for instance, in mining districts, in hop-growing countries, etc.

The financial rotation is *high*—

(1.) in localities with an unfavourable soil or climate, such

as high exposed situations, where the trees take a longer time to reach marketable dimensions;

(2.) in thinly populated districts, where prices generally rule low for small dimensions, while large timber can be exported to other better paying markets.

c. *Correction of the Calculated Financial Rotation.*

The length of the financial rotation as obtained by a first calculation is subject to correction, because it is based upon certain rates obtainable for the various classes of produce, whereas a change in the actual rotation may alter those rates. If, for instance, the calculated financial rotation is lower than that actually existing and the former is introduced, more small and less large timber will be produced; also the proportion between timber and firewood will be altered. This may produce a fall in the average price of produce. The reverse effect would be produced if the calculated financial rotation were higher than the one actually existing. In either case, it must be taken into consideration that a change in the rotation is accompanied by a change in the growing stock, and that either more or less material is brought into the market, at any rate for a number of years.

It follows that the first calculation is generally subject to some correction, in accordance with the alteration of prices which may be produced by a change in the rotation.

d. *Introduction of the Financial Rotation.*

Although every deviation from the financial rotation is accompanied by a financial loss, yet it is very desirable that, if it has been decided to introduce that rotation, this should be done with great caution, because, in the first place, it can only be determined approximately, and, secondly, its introduction is accompanied by a change in the existing growing stock. If a true, or assumed, surplus of growing stock has been disposed of, it would take much time to re-establish it should further

experience indicate a higher rotation than that originally calculated. Hence, it is desirable always to keep somewhat above the theoretical financial rotation.

If a change of rotation has been decided on, it can be carried out at once, provided the forest is of small extent and the demand for produce sufficiently large to absorb the extra supply of produce thrown upon the market, without causing any appreciable change in prices. If the forest is, however, of some extent and the demand for produce uncertain, it is always desirable to make the change gradually, so as either to spread the extra supply of produce over a number of years, or to accumulate the extra growing stock gradually, thus disturbing the market as little as possible.

2. Rotation of the Highest Income.

By this is understood the rotation which yields the highest income, calculated without interest and irrespective of the time when the items of income occur. The net income is thus calculated according to the arithmetical mean of incomes reduced by the costs. All items of income and costs during one rotation are added up and the sum of the latter deducted from the former; the difference, divided by the number of years in the rotation, represents the mean annual net income.

Hence, the rotation in this sense is that under which the expression—

$$\text{Annual income} = \frac{Y_r + T_a + T_b + \ldots + T_q - c - r \times e}{r}$$

becomes a maximum.

This rotation falls, as a rule, a considerable number of years beyond the financial rotation.

Example.—Taking the data contained in the table at page 120, the net annual income amounts to—

For a rotation of *Shillings.*

$$70 \text{ years} = \frac{2{,}187 + 5 + 44 + 87 + 118 + 134 - 60 - 70 \times 3}{70} = 33$$

$$80 \text{ years} = \frac{2{,}860 + 5 + 44 + 87 + 118 + 134 + 144 - 60 - 80 \times 3}{80} = 39$$

Shillings.

$$90 \text{ years} = \frac{3{,}558 + 5 + 44 + 87 + 118 + 134 + 144 + 149 - 60 - 90 \times 3}{90} = 43$$

$$100 \text{ years} = \frac{4{,}273 + 5 + 44 + 87 + 118 + 134 + 144 + 149 + 165 - 60 - 100 \times 3}{100} = 48$$

$$110 \text{ years} = \frac{5{,}017 + 5 + 44 + 87 + 118 + 134 + 144 + 149 + 165 + 167 - 60 - 110 \times 3}{110} = 51$$

$$120 \text{ years} = \frac{5{,}792 + 5 + 44 + 87 + 118 + 134 + 144 + 149 + 165 + 167 + 135 - 60 - 120 \times 3}{120} = 54$$

$$130 \text{ years} = \frac{6{,}591 + 5 + 44 + 87 + 118 + 134 + 144 + 149 + 165 + 167 + 135 + 133 - 60 - 130 \times 3}{130} = 57$$

$$140 \text{ years} = \frac{7{,}410 + 5 + 44 + 87 + 118 + 134 + 144 + 149 + 165 + 167 + 135 + 133 + 135 - 60 - 140 \times 3}{140} = 60$$

It will be observed that the annual income still rises under a rotation of 140 years, and will continue to do so, until the volume- and quality-increment become so much reduced and the expenses increased that they will cause a reduction in the average income. At the same time, a rotation of 140 years would involve a financial loss, because interest on the invested capital has been altogether omitted. This can easily be seen by a reference to column *k* of the financial yield table at page 194. The net soil rental under a rotation of 120 years comes to 7·41 shillings, and under one of 80 years to 10·10 shillings. This is due to the fact that the income under a rotation of 120 years represents the interest on a much larger capital invested in the forest than is the case under a rotation of 80 years.

3. Rotation of the Greatest Production of Volume.

This is the rotation under which a forest yields the greatest quantity of material per unit of area; it coincides

with the year in which the mean annual volume increment culminates.

$$
\begin{aligned}
&\text{Let volume of final yield} = V_r \\
&\text{Volume of thinning in the year } a = v_a \\
&\quad ,, \qquad ,, \qquad ,, \qquad b = v_b \\
&\qquad \vdots \\
&\quad ,, \qquad ,, \qquad ,, \qquad q = v_q,
\end{aligned}
$$

then the rotation of the greatest production is that under which the value $\frac{V_r + v_a + v_b + \ldots v_q}{r}$ becomes a maximum.

The calculation can be made for timber and firewood, or for timber only.

Example.—Taking, for instance, the data for total yield in the table for Scotch pine medium quality, at page 363, for timber only, the rotation of the greatest production would fall about into the year 80, which is approximately the financial rotation. For timber and firewood, the rotation would fall into the year 60, which is considerably below the financial rotation; in this case, a financial loss would be incurred.

4. The Technical Rotation.

By this is understood the rotation under which a forest yields the most suitable material for a certain fixed purpose; for instance, for construction generally, shipbuilding, railway sleepers, telegraph or hop poles, mining props, tanning bark fuel, etc.

As the objects of management and the purposes for which the material is required differ very much, the technical rotation may fall into an age, either before, after, or into the age of the financial rotation. The loss occasioned by following it depends on the difference between the technical and financial rotations.

5. The Physical Rotation.

By the physical rotation is generally understood that age which is most favourable for the natural regeneration of a

species, taking into consideration the conditions of the locality and the silvicultural system. It cannot be lower, in the case of high forest, than the age when the trees have commenced to bear good seed in sufficient quantity, nor as high as the age when the production of good seed has ceased; the best period being that about the end of the principal height growth.

In the case of coppice woods, the age must be below that at which the trees cease to produce good healthy shoots when cut over.

Sometimes a second physical rotation is mentioned as that which coincides with the natural lease of life of the trees. It is of interest only in the case of protection forests, parks, etc.

6. Choice of Rotation.

The choice of rotation, or the age at which a wood is to be cut over, is, as already stated, one of the most important questions in forest management. Many and varied are the arguments which have been advanced in favour of the one or other rotation.

One party maintains that the financial aspect should decide the choice of rotation, since forests represent capital which should yield the highest possible interest. Another party brings the general usefulness more into the foreground, and maintains that other considerations are more important to the general community than purely financial results, especially in the case of State forests.

In the author's view, the "objects of management" should determine the rotation. These frequently demand deviations from the financial rotation. For instance, to begin with an extreme case, for protection forests generally a very high rotation is indicated; where a nation considers it necessary to produce timber fit for naval construction, a rotation which lies far beyond the financial rotation is called for; where hop-poles are wanted, a very low rotation would be advisable; in cases where land is scarce and yet a certain quantity of wood is wanted for existing industries, the rotation of the highest

production of produce is indicated; if a proprietor wishes to invest capital, so as to obtain the highest annual income, irrespective of the rate of interest, he would choose the rotation under which that income culminates, etc.

There may be good reasons in all these cases for adopting the one or other rotation. At the same time, the proprietor should know what financial sacrifice he brings for the realisation of his special object. Hence, the general procedure in fixing the rotation may be described as follows:—

In the first place, the financial rotation should be determined, as it alone gives a true expression of the economic value of the management; then, it should be ascertained in how far the objects of management demand a departure from the financial rotation; lastly, the financial loss involved in such a departure should be determined, so that the proprietor may have a clear conception of the payment which he is called upon to make in order to realise his special object.

It need hardly be pointed out that the above procedure suits all possible cases which may come under consideration. It should, however, never be overlooked that very short rotations may injuriously affect the future returns of the locality, owing to the frequent exposure of the soil to deteriorating atmospheric influences. Hence, purely financial considerations, in the case of all but really fertile localities, should be adopted only after a full inquiry into the effects likely to be produced on the future yield capacity of the locality.

CHAPTER III.

THE NORMAL AGE CLASSES.

When, under the system of working for a sustained annual yield, the rotation has been fixed, it is necessary that, year after year, or period after period, the required mature woods are forthcoming, so that the calculated annual yield may be obtainod. This involvos tho establishmont of a normal series of age gradation. By that term is, therefore, understood a series of age gradations so arranged that at all times, when cuttings are to be made, mature woods of tho normal age are available and so situated that no obstacles to their cutting exist. This means that each age class must be of the proper extent, and that tho several age classes must be properly grouped, or distributed, over the forest.

If a forest is to be managed according to the systom of a sustained annual yield and the clear cutting system, it must contain a series of age gradations equal to the number of years in the rotation; the oldest age gradation must, immediately before cutting, have the age of the rotation, the youngest must be one year old, with a difference of one year in the age of every succeeding two gradations. (See Fig. 50 on page 218.)

If the annual returns are to be equal in volume and the quality of the locality is the same throughout, then all age gradations must be of the same extent; if different qualities occur, the areas of the coupes must be in inverse proportion to the quality of the locality. A series of age gradations so arranged is called *a normal working section*. This subject will again be dealt with further on. For the present it is assumed that the quality of locality is the same throughout.

Frequently, a number of age gradations are thrown together into an age class. The following questions thus arise :—

(1.) What is the area to be cut annually under the different methods of treatment?

(2.) What is the size, or extent, of the age classes? and

(3.) How should the age classes be distributed over the forest?

1. The Annual Coupe, or the Area to be Cut Annually.

This differs according to the method of treatment. (For a description of the latter, see page 92 of Volume II., fourth edition.)

a. Coppice and Coppice with Standards.

The annual coupe is determined by dividing the total area of the forest, or working section, by the number of years in the rotation under which the coppice is worked.

Let total area $= A$, and the rotation of the coppice $= r$, then the annual cutting area $c = \frac{A}{r}$. This holds good for the coppice with standards system, because the annual cutting area is governed by the coppice only.

b. Clear Cutting in High Forest.

Here again:

$$c = \frac{A}{r},$$

if each clearing is at once re-stocked. It frequently happens, however, that the cleared coupes lie fallow for one or more, say s, years; in that case:

$$c = \frac{A}{r + s},$$

so that the forest consists, immediately before cutting, of a series of age gradations from 1 to r years old and s blanks, or altogether $r + s$ coupes.

c. The Shelter-wood Compartment System.

Under this system, the regeneration of each coupe extends over a number of years, say m; hence, it is necessary to throw m annual coupes together into a regeneration coupe, the crop on which, by gradual cuttings, is led over in the course of m years into a young wood. The size of the regeneration coupe is, therefore, $= \frac{A}{r} \times m$.

In this case, the first of the successive cuttings towards regeneration may be made—

Either in the year r, so that the trees removed at the end of the regeneration period would be $r + m$ years old, and the mean age $r + \frac{m}{2}$ years; in other words, the procedure would lead to a raising of the rotation from r to $r + \frac{m}{2}$ years;

Or, the first cutting may be made in the year $r - \frac{m}{2}$ and the last in the year $r + \frac{m}{2}$, so that the mean final age comes to r years.

In the present chapter the latter is assumed.

d. The Selection System.

Strictly speaking, the annual coupe is equal to the total area of the forest. For convenience sake, however, the cuttings of each year are restricted to a portion of the area, so that it takes a number of years to go round the forest, and before cuttings are again made on the same area. If that number is l, then—

$$\text{Annual cutting area} = \frac{A}{l}.$$

Example.—In the beech forests of Buckinghamshire, which are worked under the selection system, it is usual to go round once in seven years; in that case the annual cutting area

would be equal to $\frac{A}{7}$. In other cases, as in the Indian sál and teak forests, l is longer, generally from 15 to 40 years.

2. Size of the Age Classes.

In forests of some extent, which are worked under a high rotation and especially those regenerated naturally, it is, as a rule, impracticable to separate the annual cutting areas, so that a regular series of age gradations, differing by one year in age throughout, exists. In these cases, it is necessary to be satisfied with larger groups, that is to say, to join a number of age gradations into an "age class." The normal size of

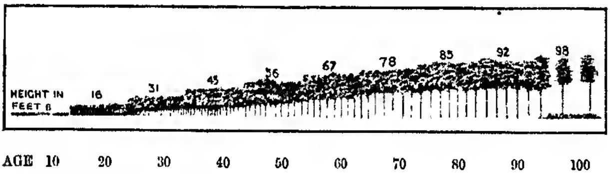


Fig. 45.—A normal Series of Age Classes, treated under the Uniform System.

such an age class depends on the area of the annual coupe and the number of such coupes thrown together.

If a class contains n gradations, its area would be $= n \times c$.

The number of age classes $= \frac{r}{n}$ is variable.

Another way is to fix the number of age classes; in that case n is variable, but this procedure is not to be recommended, as it is likely to lead to confusion.

It is usual to take for n a round number, say 10, 20, or even 30; in coppice woods, n is usually taken as $= 5$. The age classes are numbered. It is best to call the youngest I., the next youngest II., and so on; for instance, if $n = 20$—

First age class I., contains all woods up to 20 years old.
Second „ II., „ „ from 21 to 40 years old.
Third „ III., „ „ „ 41 to 60 „

And so on.

In this way, the number of the age class indicates directly the limit of ages of the woods contained in it. The reverse method, of calling the oldest age class I., the next oldest II., etc., is less desirable, but unfortunately it has been largely adopted.

The number of years includod in an age class is called a "period," and the area dealt with in the course of a period is called a "periodic coupo" (French: "Affectation").

The aroa of the age classes under the several methods of treatment will be as follows:—

a. Clear Cutting in High Forest.

The area of oach ago class, C, in a normal state, is—

$$C = n \times c = n \times \frac{A}{r}, \text{ or } C = n \times \frac{A}{r + s},$$

according as to whether each clearing is at once re-stocked, or allowed to lio fallow for s years.

Example:—Let area A = 1,000 acres, rotation r = 100 yoars, s = 0 years, n = 20 years.
Then,

$$\text{Annual age gradation} = \frac{A}{r} = \frac{1{,}000}{100} = 10 \text{ acros};$$

and the age classes:

C_1 (1— 20 years old woods)	= $c \times n$	=	10×20	=	200 acres.
C_2 (21— 40 ,, ,,)	= ,,	=	,,	=	200 ,,
C_3 (41— 60 ,, ,,)	= ,,	=	,,	=	200 ,,
C_4 (61— 80 ,, ,,)	= ,,	=	,,	=	200 ,,
C_5 (81—100 ,, ,,)	= ,,	=	,,	=	200 ,,
				A =	1,000 acres.

b. Shelter-wood Compartment System.

As already explained, under this system the old crop is gradually led over into a young wood in the course of a number of years which has been indicated by m. There is

always an area under regeneration which contains a certain number of old trees and young growth, and this may be called *the regeneration class* $= C_v$; it wanders gradually through the whole forest, until, at the beginning of the second rotation, it is found in the original position. As regeneration in some cases takes only a few and in others more years, it is impossible to define its duration accurately, and least of all can m be placed equal to n, the number of years in the period. Under these circumstances, the arrangement of age classes can be indicated only approximately, somewhat in the following manner:—

Cuttings in the oldest age class commence when the crop is $r - \frac{m}{2}$ years old, and the last cuttings occur when the crop is $r + \frac{m}{2}$ years old; then the annual cutting area, as before, $= \frac{A}{r}$, and the area of the regeneration class $= \frac{A}{r} \times m$. If $m < n$, then the regeneration class contains areas as yet blank, young trees from 1 to m years old, and the remaining old trees up to $r + \frac{m}{2}$ years old. It forms part of the youngest age class, C_1. Now, it may happen that $m = n$; in that case the youngest age class does not exist by itself, but is identical with C_v. Again, it may occur, that $m > n$, in which case C_v contains not only the youngest age class, but also a portion, if not the whole, of the second age class. Hence, the size of the several age classes may be expressed as in the following example, assuming five age classes:—

(1.) $m < n$.

$$C_v = \frac{A}{r} \times m \qquad C_1 = \frac{A}{r}(n - m)$$

$$C_2 = \frac{A}{r} \times n \qquad C_3 = \frac{A}{r} \times n$$

$$C_4 = \frac{A}{r} \times n \qquad C_5 = \frac{A}{r} \times n$$

Example:— $m = 15$; $n = 20$; $r = 100$; $A = 1{,}000$ acres.

Then

$$C_v = 10 \times 15 = 150 \text{ acres.}$$
$$C_1 = 10 \times 5 = 50 \text{ ,,}$$
$$C_2 = 10 \times 20 = 200 \text{ ,,}$$
$$C_3 = 10 \times 20 = 200 \text{ ,,}$$
$$C_4 = 10 \times 20 = 200 \text{ ,,}$$
$$C_5 = 10 \times 20 = 200 \text{ ,,}$$

Total = 1,000 acres.

(2.) $m = n$.

Example:— $m = n = 20.$

$$C_v = \frac{A}{r} \times m = 200 \text{ acres.}$$
$$C_1 = \frac{A}{r}(n - m) = 0 \text{ ,,}$$
$$C_2 = \frac{A}{r} \times n = 200 \text{ ,,}$$
$$C_3 = \frac{A}{r} \times n = 200 \text{ ,,}$$
$$C_4 = \frac{A}{r} \times n = 200 \text{ ,,}$$
$$C_5 = \frac{A}{r} \times n = 200 \text{ ,,}$$

Total = 1,000 acres.

(3.) $m > n$, but $m < 2 \times n$.

Example:— $m = 30.$

$$C_v = \frac{A}{r} \times m = 300 \text{ acres.}$$
$$C_1 = \quad = 0 \text{ ,,}$$
$$C_2 = \frac{A}{r}(2 \times n - m) = 100 \text{ ,,}$$
$$C_3 = \frac{A}{r} \times n = 200 \text{ ,,}$$
$$C_4 = \frac{A}{r} \times n = 200 \text{ ,,}$$
$$C_5 = \frac{A}{r} \times n = 200 \text{ ,,}$$

Total = 1,000 acres.

And so on.

It is obvious that, for the shelter-wood system with natural regeneration, the above allotment is only of an ideal character,

because the duration of regeneration is so uncertain. The regeneration class, the oldest and youngest classes are subject to modifications amongst themselves, so that they cannot easily be separated the one from the other; hence, they are best thrown together. The important point in that case is that the middle-aged classes are of the proper size. The allotment may then be represented as follows:—

Example:—

$$C_2 = \frac{A}{r} \times n \qquad = 200 \text{ acres.}$$

$$C_3 = \frac{A}{r} \times n \qquad = 200 \quad ,,$$

$$C_4 = \frac{A}{r} \times n \qquad = 200 \quad ,,$$

$$C_5 + C_v + C_1 = \frac{A}{r}(r - 3 \times n) = 400 \quad ,,$$

$$\text{Total} = 1{,}000 \text{ acres.}$$

Or again, if regeneration extends over a still longer period:—

$$C_3 = \frac{A}{r} \times n \qquad = 200 \text{ acres.}$$

$$C_4 = \frac{A}{r} \times n \qquad = 200 \quad ,,$$

$$C_5 + C_v + C_1 + C_2 = \frac{A}{r}(r - 2 \times n) = 600 \quad ,,$$

$$\text{Total} = 1{,}000 \text{ acres.}$$

c. *Coppice Woods.*

As the rotation of coppice woods is short, it is usually possible to mark the annual coupes on the ground, so that grouping in age classes is not necessary. If the latter should, nevertheless, be considered desirable, generally not more than five gradations are thrown together, so that C_1 comprises the 1- to 5-years-old gradations, C_2 those from 6 to 10 years, etc.

Example:—

$$\text{Area} = 200 \text{ acres.}$$
$$r = 20 \text{ years.}$$
$$n = 5 \quad ,,$$

The arrangement of age classes would be normal, if—

$$C_1 = \frac{200}{20} \times 5 = 50 \text{ acres.}$$

$$C_2 = \frac{200}{20} \times 5 = 50 \;\;,,$$

$$C_3 = \frac{200}{20} \times 5 = 50 \;\;,,$$

$$C_4 = \frac{200}{20} \times 5 = 50 \;\;,,$$

Total = 200 acres.

d. Coppice with Standards.

Here each coupe contains coppice (underwood) and standards (overwood). As far as the underwood is concerned, the arrangement is exactly the same as in the case of simple coppice; tho annual ago gradation is $= \frac{A}{r}$, and the age class, if any, $= \frac{A}{r} \times n$.

The distribution of the overwood, in its normal condition, is somewhat peculiar, which may usefully be explained here, though it is only of a *theoretical* value.

In the first place, it should be remembered that cuttings in both the under- and overwood on the same area must be mado at tho same timo, or rather those in the overwood must be made immediately after the underwood has been cut over, and before the new coppice shoots appear; hence, the rotation R of the overwood must be a multiple of the rotation r of the underwood, say $R = r \times t$.

In each annual coupe, when cutting comes round to it, a certain portion of the underwood (chiefly seedling trees) is left standing to form the youngest age gradation of the overwood. That portion should occupy an area $= \frac{A}{R}$, assuming that each age gradation of the overwood occupies the same extent of ground. The area occupied by each age class of overwood comes to $= \frac{A}{R} \times r = \frac{A}{t}$.

Assuming now that the youngest overwood class, 1 to r years old, though still forming part of the underwood, is already counted as belonging to the overwood, thon there are t overwood classes. The latter are not separated according to area, as in the case of clear cutting or coppice, but t gradations are standing mixed on each annual coupe, so that each of the latter contains $\frac{1}{r}$th part of each overwood class.

Immediately before cutting, the arrangement would be as follows:—

Underwood, Age in Years.	1	2	3	......	$r-1$	r
Overwood Age Class C_1 age	1	2	3	.....	$r-1$	r
" " " C_2 "	$r+1$	$r+2$	$r+3$	......	$2r-1$	$2\times r$
" " " C_3 "	$2r+1$	$2r+2$	$2r+3$	...	$3r-1$	$3\times r$
⋮						
" " " C_r "	$(t-1)\ r+1$	$(t-1)\ r+2$	$(t-1)\ r+3$	......	$t\times r-1$	$t\times r$

It will be seen that a normal coppice with standards forest must havo an overwood which consists of $t \times r = R$ age gradations ranging from 1 year up to R years old.

Example.—A forest of 200 acres, worked under a rotation of 20 years for the underweed and 100 years for the overwood, has $\frac{100}{20} = 5$ everwood classes. On the 10 acres which are about to be cut will be found—

Underwood = 20 years old
Overwood = 20, 40, 60, 80 and 100 years old.

The next oldest coupo contains—

Underwood = 19 yoars old
Overwood = 19, 39, 59, 79 and 99 years old.

The youngost coupe contains—

Underwood = 1 year old
Ovorwood = 1, 21, 41, 61 and 81 years old.

The figures, 46, 47, 48 and 49 on pages 214, 215 illustrate the distribution of the several age gradations over the area.

The area occupied by each overwood class can be determined only by assuming that each gradation occupies an equal extent of ground; hence, the youngest gradation will have most trees and the oldest least. Imagining now that the age classes of the overwood were not intermixed, but that the trees of each class were brought together on separate areas, then the overwood, apart from the coppice, would form an open high forest. Of these woods, the youngest would contain the standards from 1 to r years, the next those from $r + 1$ to $2\,r$ years, and so on. By degrees, the youngest class passes through all the intermediate stages, until it becomes the oldest and is cut over in the course of r years. At each annual cutting, therefore, an equal area must be cut over, on which the new, that is the youngest, gradation is started, either naturally or artificially.

The annual coupe is $c = \frac{A}{r}$ and $A = c \times r$.

The number of overwood classes is $= \frac{R}{r} = t$, hence—

$$R = t \times r.$$

Area of each age class on each annual coupe $= \frac{A}{R} = \frac{A}{t \times r} = \frac{c}{t}$,

or,

$$\frac{200}{100} = \frac{200}{5 \times 20} = \frac{10}{5} = 2 \text{ acres.}$$

As the whole forest consists of r coupes, each overwood class, consisting of r gradations, contains, in a normal forest, $\frac{c}{t} \times r = \frac{A}{t}$ units of area, or $\frac{200}{5} = 40$ acres. This shows that, theoretically, the proportion of the age classes is the same as in high forest, although the distribution is different.

Example.—Data as before.

$A = 200$; $R = 100$; $r = 20$, number of overwood classes $t = 5$.

Normal annual cutting area $c = \frac{A}{r} = \frac{200}{20} = 10$ acres.

On each coupe each age gradation of overwood occupies . . $\frac{c}{t} = \frac{10}{5} = 2$ acres.

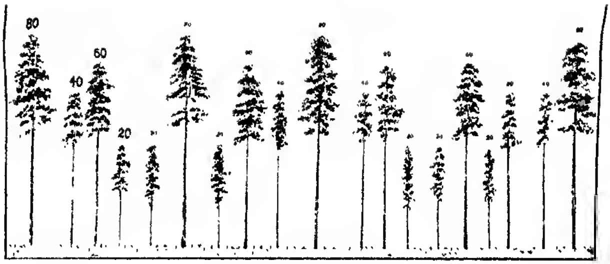


Fig. 46.—Coppice with Standards.
Rotation of underwood = 20 years. Rotation of overwood = 100 years.
Coppice just cut.

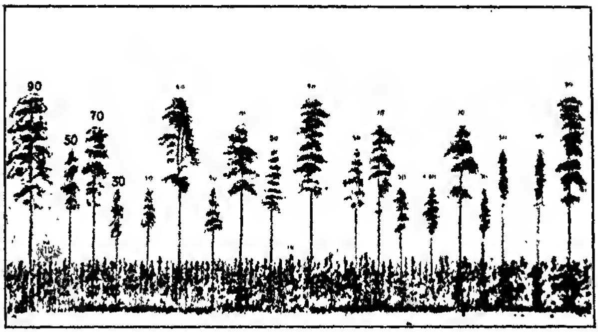


Fig. 47.—Coppice with Standards.
Rotation of underwood = 20 years. Rotation of overwood = 100 years.
Age of coppice, 10 years.

Fig. 48.—Coppice with Standards.
Rotation of underwood = 20 years. Rotation of overwood = 100 years.
Age of coppice, 20 years.

DISTRIBUTION OF THE OVERWOOD IN COPPICE WITH STANDARDS.

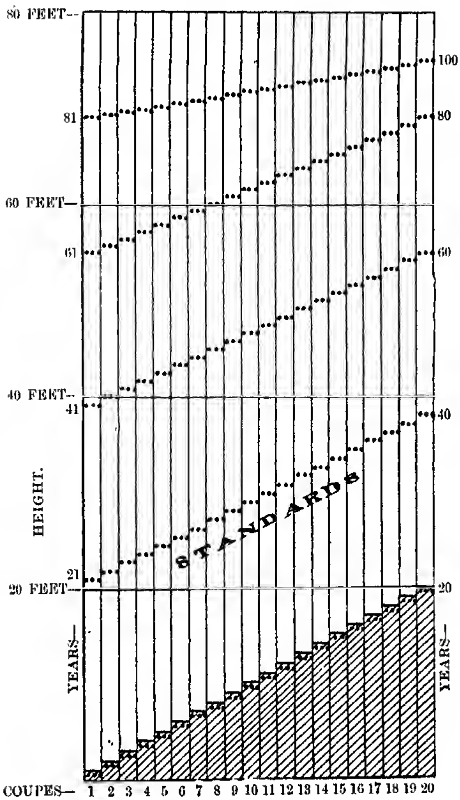


Fig. 49.—(Conventional.)

The area and distribution of the several age classes is as follows :—

Coupe No. 20, *oldest :*

Underwood	= 10 acres	= 20 years old.	
Overwood on	2 ,,	= 20 ,,	,,
,, ,,	2 ,,	= 40 ,,	,,
,, ,,	2 ,,	= 60 ,,	,,
,, ,,	2 ,,	= 80 ,,	,,
,, ,,	2 ,,	= 100 ,,	,,

Coupe No. 1, *youngest :*

Underwood	= 10 acres	= 1 year old.	
Overwood on	2 ,,	= 1 ,,	,,
,, ,,	2 ,,	= 21 years	,,
,, ,,	2 ,,	= 41 ,,	,,
,, ,,	2 ,,	= 61 ,,	,,
,, ,,	2 ,,	= 81 ,,	,,

The normal state of the age classes in the case of coppice with standards is of a still more ideal character than in the case of the shelter-wood compartment system ; it can only serve as a mathematical guide for the treatment of such woods, as it gives some idea of the relative number of trees which should be found in each class or gradation. Each of these should occupy about the same area ; hence, the youngest class must contain a large number of trees which is gradually reduced to a comparatively small number in the oldest age class. The actual proportion in these numbers depends on the species and the quality of the locality.

e. The Selection Forest.

If the annual cuttings extend over the whole area, then all age classes are, theoretically speaking, represented in all parts of the forest ; if, on the other hand, the cuttings pass over the forest in the course of a number of years, say, l, then the age classes will gradually become located similar to the distribution of the overwood in a coppice with standards forest. The number of age classes will, theoretically, be equal to $\frac{r}{l}$.

Example.—Let A = 1,000 acres; r = 100; l = 20; then each annual cutting area $= \frac{A}{l} = \frac{1,000}{20} = 50$ acres, and the distribution would approximately be as follows:—

Coupe No. 1 (*youngest*).	*Coupe No.* 2.
1 year old trees = 10 acres	2 year old trees = 10 acres
21 ,, ,, ,, = 10 ,,	22 ,, ,, ,, = 10 ,,
41 ,, ,, ,, = 10 ,,	42 ,, ,, ,, = 10 ,,
61 ,, ,, ,, = 10 ,,	62 ,, ,, ,, = 10 ,,
81 ,, ,, ,, = 10 ,,	82 ,, ,, ,, = 10 ,,
Total = 50 acres	Total = 50 acres

Coupe No. 19.	*Coupe No.* 20 (*oldest*).
19 years old trees = 10 acres	20 years old trees = 10 acres
39 ,, ,, ,, = 10 ,,	40 ,, ,, ,, = 10 ,,
59 ,, ,, ,, = 10 ,,	60 ,, ,, ,, = 10 ,,
79 ,, ,, ,, = 10 ,,	80 ,, ,, ,, = 10 ,,
99 ,, ,, ,, = 10 ,,	100 ,, ,, ,, = 10 ,,
Total = 50 acres	Total = 50 acres

Each year the 100 years old trees in the oldest coupe would be cut, and they should cover an area equal to one-fifth of the coupe, equal to 10 acres, thus cutting once the whole area of the forest in 100 years. It is needless to add that such regularity is never reached in practical forest management.

3. Distribution of the Age Classes over the Forest.

By a normal distribution of the age classes is understood that which admits of a proper succession of cuttings, so that each wood is cut at the proper age, and that the other woods are protected against external dangers, in so far as this can be done by careful management.

It has already been explained that every deviation from the normal age interferes with the full realisation of the objects of management; hence, the age classes should be so distributed that no such deviations are called for. Of special importance, in this respect, are threatening dangers, such as damage by strong winds, dry air currents, danger from frost, fire, insects, etc., and sometimes considerations for a successful regeneration.

Strong winds or gales are a most important consideration. Their prevailing direction must be ascertained, and cuttings must proceed against it. Assuming that the strong winds generally blow from the west, the youngest age class should, at the commencement, be situated at that side and the oldest on the east, so that the cuttings proceed gradually from east to west. (See diagram, Fig. 50, upper part.) In this case, the younger age classes gradually break the force of the wind, while the youngest (in the diagram) will gradually grow up

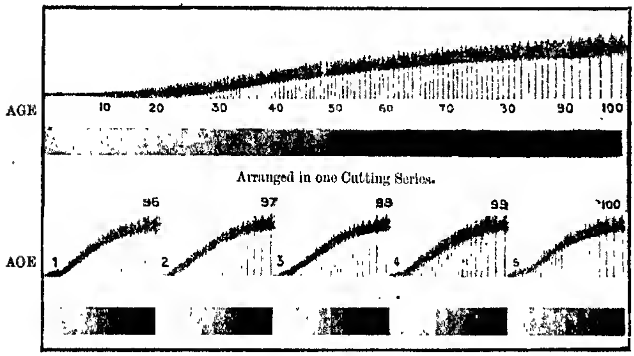


Fig. 50.—A normal Series of Age Gradations, treated under the Clear Cutting System.

exposed to the strong wind; its edge trees will develop strong root systems, and the wood will then be able to resist the force of the wind when it grows up to become the oldest age gradation.

In determining the prevailing wind direction, it must not be overlooked that it is frequently changed in hilly and mountainous tracts according to the direction of the valleys and hill ranges.

Dry winds may frequently blow from a direction differing from that of strong winds; in that case, the forester must decide which is the more important consideration of the two, and determine the cutting direction accordingly. Frequently,

the seeds of trees fall under the effect of a dry wind, so that the cleared areas which are to be naturally regenerated must be situated to the leeward of the seed-bearing trees, as, for instance, under the strip system with regeneration by seed fallen from trees standing on the adjoining area.

Large clearings in one place are generally objectionable, because the soil is liable to dry up, and damage by frost is more likely to occur; hence, in extensive forests the area to be cut annually may have to be divided into a number of small coupes situated in different parts of the forest.

Insects and fire are likely to be most injurious when several cuttings made in consecutive years adjoin each other, because the former wander from one coupe to the next, while fire spreads more rapidly in young woods, than if they are interrupted by older woods.

These circumstances demand in many cases, and especially where clear cutting is practised in coniferous woods, that a second cutting should not be made in any locality, until the former coupe has been successfully re-stocked. This leads to the splitting up of a working section, or a series of age gradations, into several sub-divisions which are called *cutting series*. Supposing, in a forest worked under a rotation of 100 years, it was considered necessary not to cut in the locality adjoining a previous cutting except after a lapse of 5 years, the series of 100 age gradations would be divided into five cutting series, of which each would comprise 20 coupes. (See Fig. 50, lower part.)

Cutting Series	A	would	comprise	the coupes	now	100, 95 . . 10, 5,	years	old.
,, ,,	B	,,	,,	,,	,,	99, 94 . . 9, 4,	,,	,,
,, ,,	C	,,	,,	,,	,,	98, 93 . . 8, 3,	,,	,,
,, ,,	D	,,	,,	,,	,,	97, 92 . . 7, 2,	,,	,,
,, ,,	E	,,	,,	,,	,,	96, 91 . . 6, 1,	,,	,,

As a general rule, a careful distribution of the age classes over the area of the forest is of special importance in the case of species which are easily thrown by wind, liable to attacks by insects, to danger from fire and frost, and also those which are difficult to regenerate naturally. In all these cases, a

distribution must be aimed at which allows the cutting of each wood when mature, without thereby endangering, on the one hand the adjoining woods, and on the other tho successful regeneration of the cleared area.

The above considerations must specially guide the forester in the case of forests workod under tho systems of clear-cutting, and also of the sholter-wood compartment system. They are of less importance in coppice, coppico with standards, and selection forests; but even here the cutting direction should be carefully determined.

At the same time, the forester should not go to extremes, as thero is something to be said on both sides.

Reasons for adjoining the annual coupes are :—

(1.) Best security against damage by storms.

(2.) Reduction to a minimum of damage by overhanging trees.

(3.) Production of a larger percentage of high class timber.

(4.) Reduction of the cost of transport of forest produce.

Reasons against adjoining the annual coupes are :—

(1.) Increase of danger through fire and insects.

(2.) Reduced protection of young growth against raw or dry winds.

The subject will again be referred to in Part IV., when dealing with the division and allotment of aroas.

CHAPTER IV.

THE NORMAL GROWING STOCK.

IT has been stated at page 171 that by the normal growing tock is understood that present in a forest which has a normal roportion of age classes and a normal increment. This being o, the forester need only see that the age classes and incre-nent are normal, and the normal growing stock will be present s a natural consequence.

It happens, however, that, as far as quantity is concerned, he normal growing stock may be present if neither the normal age classes nor increment havo boon established ; for nstance, if the deficit in one age class is made good by a urplus in another. If, in such a case, an annually equal uantity of wood were cut, it would lead to a deviation from he normal final age and consequently to loss. Indeed, the normal growing stock according to quantity might be present, f the whole forest consisted of only one uniform age class of about half the normal final age. In that case, no ripe wood t all would be found in the forest, and cuttings would have o be suspended for a considerable number of years.

Under these circumstances, the normal growing stock measured by a certain number of cubic feet is of subordinate importance in determining the yield of a forest, and yet it is useful to look at its determination for tho following two reasons :—

(1.) Because the yield, taken out of a forest in the course of a rotation, consists partly of the growing stock which was present at the beginning of the rotation, and partly of increment added to that growing stock during the rotation.

(2.) Because several methods base the calculation of the yield upon the difference between the normal and real growing stock.

The amount of the normal growing stock depends on the length of the rotation; the higher the latter, the greater the former for one and the same area.

In calculating the normal growing stock, only the principal part of the woods which gives the final yield is taken into account, because, as previously explained, the determination of a sustained yield is, in the first place, based upon the final yield.

The normal growing stock can be looked at from the volumetric or the financial point of view.

1. Calculation of the Normal Growing Stock as regards its Volume.

a. Clear Cutting in High Forest.

It has already been explained on page 140 that, under the system of clear cutting, the normal growing stock consists of a series of age gradations ranging from 0 years old to $r-1$ years old, with a difference of one year between the ages of every two succeeding age gradations; this occurs in a temperate climate in spring, before the annual increment has been laid on.

(1.) *Calculation from Yield Tables.*—If a yield table is available for a forest which gives the produce standing in it from year to year, the normal growing stock is equal to the sum of all the growing stocks given in that table from the year 0 to the year $r-1$; that sum would represent the normal growing stock of r units of area for spring.

If the yield table, and this is generally the case, gives the volumes only from period to period, say for every n years, then the approximate amount of the normal growing stock can be calculated (according to Pressler) by assuming that the volumes rise within each period of n years according to an

arithmetical series, that is to say, by adding the same number of cubic feet each year.

Let the volume in the year	0	$= 0$ cubic feet,			
,, ,, ,,	n	$= a$	,,	,,	
,, ,, ,,	$2 \times n$	$= b$	,,	,,	
,, ,, ,,	$3 \times n$	$= c$	,,	,,	
,, ,, ,,	$4 \times n$	$= d$	,,	,,	

then—

Total volume from the year 0 to the year, n, $-a = (0+a) \times \frac{n+1}{2} - a$

,, ,, n ,, $2 \times n$, $- b = (a+b) \times \frac{n+1}{2} - b$

,, ,, $2 \times n$,, $3 \times n$, $- c = (b + c) \times \frac{n+1}{2} - c$

,, ,, $3 \times n$,, $4 \times n$ $= (c + d) \times \frac{n+1}{2}$.

Total volume from 0 to $4 \times n$ years $=$

$$\frac{n+1}{2}\left(0 + 2a + 2b + 2c + d\right) - \left(a + b + c\right)$$

$$= \left(n + 1\right)\left(a + b + c + \frac{d}{2}\right) - \left(a + b + c\right)$$

$$\textbf{Normal Gr. Stock, } \mathbf{G_n = n\left(a + b + c + \frac{d}{2}\right) + \frac{d}{2}}.$$

This is the calculation for *autumn*.

If the oldest age gradation is cut away during winter, the normal growing stock in *spring* must be d cubic feet less than that in autumn, that is to say :—

$$\mathbf{G_n \textbf{ in Spring} = n\left(a + b + c + \frac{d}{2}\right) - \frac{d}{2}}.$$

In spring, the growing stock consists only of a series of gradations running from 0 to (r—1) years old. In the course of the summer (the growing season) the series is brought up again to one running from 1 to r years old. If, therefore, the calculation is made for the middle of summer, it may be assumed that one-half of the annual increment has been laid

on ; in other words, that the growing stock is then equal to the arithmetical mean of those in spring and autumn :—

$$\text{Summer's middle } G_n = n\left(a + b + c + \frac{d}{2}\right).$$

Example. - A forest of 100 acres, to which the data given in the table at page 120 apply, worked under a rotation of 100 years, has the following normal growing stock :—

$$\begin{aligned}
\text{In autumn } {}^{100}G_{normal} &= 10\,(100 + 600 + 1{,}170 + 2{,}830 + 3{,}940 + 4{,}690 \\
&\quad + 5{,}250 + 5{,}720 + 6{,}100 + 3{,}205) + 3{,}205 \\
&= 10 \times 33{,}605 + 3{,}205 = 339{,}225 \text{ cubic feet.} \\
\text{In spring } {}^{100}G_{normal} &= 10 \times 33{,}605 - 3{,}205 = 332{,}845 \text{ cubic feet.} \\
\text{In summer } {}^{100}G_{normal} &= 10 \times 33{,}605 \qquad = 336{,}050 \text{ cubic feet.}
\end{aligned}$$

The same forest, if worked under a rotation of 80 years, would have, for summer, the following growing stock :—

$$^{80}G_n = \left[10\,(100 + 600 + 1{,}170 + 2{,}830 + 3{,}940 + 4{,}690 + 5{,}250 + 2{,}860) - 2{,}860\right]\frac{100}{80} = 264{,}425$$ cubic feet, which is considerably less than if the area is worked under a rotation of 100 years. If, therefore, the rotation were reduced from 100 to 80 years, 332,845 – 264,425 = 68,420 cubic feet of wood would have to be removed in addition to the normal increment.

(2.) *Calculation with the Mean Annual Increment.*—A shorter, but less accurate, method of calculating the normal growing stock is based upon the assumption that the normal final yield is produced in annually equal instalments throughout the rotation; in other words, that the growing stock of the several age gradations forms an arithmetical series. Indicating one year's increment by i, the growing stock in successive age gradations would be in the

$$\begin{array}{ll}
\text{Year} & = 1, 2, \quad 3 \quad . \quad . \ r - 1, \ r. \\
\text{Growing Stock} & = i, 2 \times i, 3 \times i \ . \quad . \ (r - 1)\, i, r\, i.
\end{array}$$

$$\text{and their sum } G_n = (i + r\,i)\frac{r}{2} = \frac{r\,i}{2} + r\,i \times \frac{r}{2}.$$

As $r \times i$ represents the growing stock of the oldest age gradation, and is also equal to the total increment, I, laid

on by all age gradation during one year, the above formula may be written thus:—

$$G_n = \frac{I \times r}{2} + \frac{I}{2}.$$

This is the growing stock calculated for autumn. For spring, it is $G_n = \frac{I \times r}{2} - \frac{I}{2}$, and for the middle of summer

$$G_n = \frac{I \times r}{2}.$$

The growing stock calculated according to this formula is larger than that calculated from yield tables for short, and smaller for high rotations. The subjoined table shows this.

Length of Rotation. Years.	Normal Growing Stock, for the Middle of Summer.		
	Calculated from a Yield Table. Cubic feet.	Calculated from the Mean Annual Increment. Cubic feet.	Excess of Calculation from Mean Annual Increment. Cubic feet.
30	12,850	17,550	+ 4,700
40	32,850	56,600	+ 23,750
50	66,700	98,500	+ 31,800
60	109,850	140,700	+ 30,850
70	159,550	183,750	+ 24,200
80	214,400	228,800	+ 14,400
90	273,500	274,500	+ 1,000
100	336,050	320,500	− 15,550
110	401,550	367,950	− 33,600
120	469,750	417,000	− 52,750

b. Shelter-wood Compartment System.

The normal growing stock is the same as for the clear cutting system, provided the regeneration cuttings are so arranged that one-half are made before the year r, and the other half after it. Strictly speaking, this is only correct if the timber in the regeneration class is removed in annually equal quantities, and if regeneration takes place in the middle of the period. In reality, this occurs only rarely, but the deviations compensate each other in the long run; anyhow, a more accurate determination is practically impossible.

c. Coppice and Coppice with Standards.

The calculation for simple coppice is the same as in the case of clear cutting in high forest.

For coppice with standards forest, the calculation must be made separately for under- and overwood, and the results added together. The former is of small account, as the presence of the overwood reduces the quantity of the underwood considerably.

The calculation of the normal growing stock of overwood is a complicated and uncertain operation and at the best only of *theoretical* value. It must be based upon the number of trees and the average volume per tree in each age class somewhat in the following manner:—

If the normal number of trees in each of the r, $2r$, $3r$ . . . old age gradation is known, and also the volume of the average tree in each of these gradations, then it can be assumed that the trees increase in volume within each class according to an arithmetical series; this makes it possible to interpolate the volume of the trees $r + 1$, $2r + 1$ . . . years old. In that case, the normal growing stock of the first age class would be expressed by—

$$\frac{r}{2}\left(V_{r+1} + V_{2r}\right),$$

where V_{r+1} represents the volume of all trees $r + 1$ years old, and V_{2r} that of all trees $2r$ years old. In the same way, the next age class would be represented by—

$$\frac{r}{2}\left(V_{2r+1} + V_{3r}\right),$$

and so on. Adding all positions together, the normal growing stock of overwood comes to—

$$G_n = \frac{r}{2}\left(V_{r+1} + V_{2r} + V_{2r+1} + V_{3r} + \ldots + V_{(n-1)r+1} + V_{nr}\right).$$

This amount does not comprise the youngest age class of all which still forms part of the underwood. Its volume is, however, very small, and may be neglected.

Example :—

Area of a coppice with standards forest = 100 acres.
Rotation of underwood = 20 years.
,, ,, overwood = 100 ,,

Number of overwood classes $= \frac{100}{20} = 5$.

Area of each coupe = 5 acres.

Age of Gradation.	Number of Trees in Gradation.	Mean Volume per Tree. Cubic feet.	Total Volume of Gradation. Cubic feet.
21	200	·2	40·
40	200	2·	400·
41	130	2·65	344·
60	130	15·	1950·
61	80	15·75	1260·
80	80	30·	2400·
81	40	31·	1240·
100	40	50·	2000·

$$G_n = \frac{20}{2}\left(40 + 400 + 344\cdot + 1,950 + 1,260 + 2,400 + 1,240 + 2,000\right).$$

$G_n = 10 \times 9,634\cdot = 96,340$ cubic feet, or per acre $= 963$ cubic feet. In practice, such calculations are rarely made, but they give a clear idea of the proportion which ought to exist between the several age classes.

d. The Selection Forest.

The growing stock of a normal selection forest may be placed equal to that of a forest under the clear cutting system, as all the age gradations are represented in a similar way, though differently arranged over the area; hence, it can be ascertained by summing up the quantities given in a yield table. At the same time, the calculation is likely to be less accurate, since the younger age gradations stand under the shade of the older trees, and it is difficult to say, in how far the loss of increment of the former is covered by an increased increment of the latter.

2. Calculation of the Financial Value of the Normal Growing Stock.

The various methods of calculating the financial value of the normal growing stock have been explained at pages 140 to 143.

It has there been stated that the same results are obtained, whether the value is calculated as the cost value, expectation value, or the capitalised rental of the growing stock, provided that the expectation value of the soil is introduced into the account. In each of these cases, the value is expressed, for r units of area worked under a rotation of r years, by the formula:—

$$G_n = \frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{\cdot 0p} - r \times S_e;$$

in words, the normal growing stock is equal to the capitalised annual net rental minus the expectation value of the soil. As it is impossible to keep a forest always in the normal state, it follows that the financial normal growing stock has only a theoretical value, which assists in the comprehension of the working of the capital invested in forestry, but is of little importance in determining the yield of forests.

CHAPTER V.

THE NORMAL YIELD.

By the normal yield is understood that which a normal forest can permanently give. The yield may be annual, or intermittent. Instead of determining the yield for each year, or certain intermittent years, it can be ascertained for a number of years, in which case it is called the periodic yield.

The yield is composed of the final and intermediate returns. The regulation of the yield deals principally with the former, for reasons which have been explained at page 169.

The yield of major produce is further sub-divided according to the different classes of wood, such as timber, cord-wood, fagots, root-wood, etc. In order to bring them into the account, all the different classes of produce are reduced to one common standard, that is "the solid cubic foot."

The yield can be determined by area and volume, or by its financial value.

1. The Yield Determined by Area and Volume.

a. Clear Cutting in High Forest.

The Normal Final Yield is equal to the volume which stands on the oldest age gradation.

The normal cutting area is $c = \frac{A}{r}$ or $= \frac{A}{r + s}$, according as to whether the cleared area is at once re-stocked or allowed to lie fallow for s years (see page 204). The volume standing on c must be equal to the volume of the oldest age gradation in a normal series of age gradations, if it is to give the normal yield.

The periodic normal coupe is $= \frac{A}{r} \times n$, or $= \frac{A}{r + s} \times n$.

Example :—

Area of forest . . = 1,000 acres
Rotation . . . = 100 years

Annual cutting area $= \frac{1{,}000}{100} = 10$ acres,

if the area is at once re-stocked.

The annual yield of final returns, according to the table at page 120, amounts to 6,410 + 397 = 6,807 cubic feet per acre ; for ten acres = 6,807 × 10 = 68,070 solid cubic feet. (Final yield during every period of 20 years = 68,070 × 20 = 1,361,400 cubic feet.)

Intermediate Yields.—These consist of all the thinnings which are made.

Taking the same tablo, the following thinnings would bo made in each year :—

In the coupe 30 years old	= 10 acres,	each giving	57 c′ =	570
„ „ 40 „ „	= 10 „	„	„ 353 c′ =	3,530
„ „ 50 „ „	= 10 „	„	„ 524 c′ =	5,240
„ „ 60 „ „	= 10 „	„	„ 565 c′ =	5,650
„ „ 70 „ „	= 10 „	„	„ 536 c′ =	5,360
„ „ 80 „ „	= 10 „	„	„ 493 c′ =	4,930
„ „ 90 „ „	= 10 „	„	„ 446 c′ =	4,460
			Total =	29,740

Total Normal Annual Yield = 68,070 + 29,740 = 97,810 cubic feet.

The total normal annual yield must be equal to the total annual increment. To cut less than that amount leads to a higher rotation, and *vice versâ*.

b. Shelter-wood Compartment System.

The calculation of the yield is the same as under the system of clear cutting, as long as the rotation r is maintained. If regeneration is commenced later than in the year $r - \frac{m}{2}$, the rotation is increased, and the calculation must be made accordingly. Supposing the first cutting is made in the year r, and the last in the year $r + m$, then the rotation $= r + \frac{m}{2}$, and the mean annual cutting area $= \frac{A}{r + \frac{m}{2}}$.

Example :—

Let m = 20; then rotation = 110 years. Annual cutting area = $\frac{1,000}{110}$ = 9·09 acres. Volume standing on an acre at the age of 110 years = 6,690 + 334 = 7,024; hence, annual yield = 7,024 × 9·09 = 63,848 cubic feet.

The intermediate yields would amount to 30,642 cubic feet, and :—

Total annual yield = 94,490.

The raising of the rotation has led to a reduction of the volume yield, because the mean annual increment began to decrease before the year 100.

c. Selection Forest.

If all trees which are cut in one year were brought together on a portion of the area, the latter should be = $\frac{A}{r}$: hence, the yield is practically the same as in the case of clear cutting.

Another way of looking at the matter is to determine the area on which cuttings are made in each year; this has been placed above (page 217), = $\frac{A}{l}$. Everything which has to be cut on this area forms the normal annual yield.

Example :—

Area of a selection forest	=	1,000 acres
Rotation . . .	=	100 years
l . .	=	20 years.

Then :

$$\text{Annual cutting area} = \frac{A}{l} = \frac{1,000}{20} = 50 \text{ acres.}$$

On these 50 acres, the following material is cut :—

(1.) All trees which have reached the age of 100 years.

(2.) A certain proportion of trees in the younger age classes, so as to reduce their number gradually to that which should reach maturity at the age of 100 years.

These cuttings should give, theoretically speaking, the same yield as if the area were treated under the clear-cutting system.

d. Coppice and Coppice with Standards.

The normal yield of coppice woods is calculated in the same way as for clear cutting in high forest. In this case, th

annual cutting area is $= \frac{A}{r}$, and the volumetric yield is composed of the material standing on that area, plus thinnings in the younger age gradations.

In coppice with standards, the annual cutting area is the same as in simple coppice. The normal annual yield consists of:—

(1.) The underwood on the oldest age gradation, less those trees which are left to grow into standards.

(2.) The contents of the oldest, R years old, age gradation of the overwood.

(3.) The thinnings amongst the younger age gradations of overwood standing on the annual coupe, and occasionally in the younger underwood gradations.

Example:—

Taking the data given at page 227, the yield in overwood is as follows:—

40 trees (mature)	100 years old, each	= 50 c′	= 2,000
40 ,,	80 ,, ,, ,,	= 30 c′	= 1,200
50 ,,	60 ,, ,, ,,	= 15 c′	= 750
70 ,,	40 ,, ,, ,,	= 2 c′	= 140
		Total	= 4,090 c′,

to which the volume of the underwood has to be added.

The normal annual yield of overwood must also be equal to the annual increment laid on by all the overwood during one year, or

$$I = 2{\cdot}0 \times 200 + (15 - 2) \times 130 + (30 - 15) \times 80 + (50 - 30) \times 40$$
$$I = 4{,}090 \text{ cubic feet.}$$

As already indicated, this calculation is only of *theoretical* value.

2. The Financial Value of the Normal Yield.

The financial value of the normal yield is that which secures interest on all capital invested in a forest at the rate of the general per cent. p, at which money can be obtained for forestry, or at which money taken out of the forest can be invested with equal security as in forestry. The financial yield is realised, as long as a financial equilibrium on the above lines exists in the forest, that is to say, when the forest per

cent. is equal to the general per cent. p. This occurs under a rotation equal to that for which the expectation value of the soil reaches its maximum.

Example :—

Taking the data in the table at page 120, and a rotation of 80 years—

		Shillings.
Soil expectation value for 80 units of area = 80 × 404 .	. =	32,320
Financial value of normal growing stock (See page 141).	. =	91,360
	Total =	123,680

Financial Normal Y = 123,680 × ·025 = 3,092 shillings; or 38·65 shillings for each acre of forest.

CHAPTER VI.

RELATIONS BETWEEN INCREMENT, GROWING STOCK, AND YIELD.

Between the increment, growing stock, and yield of a normal forest relations exist which are of great importance in determining the yield. In order to bring them out clearly, the system of clear cutting in high forest will be used as an illustration.

1. Allotment of Increment during a Rotation.

Every normal series of age gradations contains, at the commencement of the rotation, the normal growing stock. Every year the oldest age gradation is cut over which gives the normal final annual yield, and this is replaced during the following growing season by the laying on of the normal increment. The latter is added partly to the old growing stock, and removed with it during the first rotation; but partly it accumulates on the cleared areas, forming a new growing stock which is carried over into the second rotation. The question then is, how the total increment of one rotation is divided between the two.

Making the calculation for spring, the youngest age gradation is 0 years old, and the oldest r—1 years. The former grows for r growing seasons, and is cut over during the last winter of the first rotation, so that all its increment is removed during the first rotation; hence, all goes to the old growing stock and nothing to the new stock. The gradation now one year old grows for r—1 years during the first rotation, when it is cut over. All the increment laid on during these years goes to the old growing stock; but that laid on during the last year is

not cut, but goes over to the second rotation. The gradation now two years old, grows for r—2 years during the first rotation, and the increment laid on during these years goes to the old growing stock; but the increment of the last 2 years of the first rotation is carried over to the second rotation, and so on. If the calculation is carried through on these lines, the following division of the total increment laid on during each rotation will be obtained:—

At the commencement of the first rotation the normal growing stock is assumed to be present; it amounts, for an area of 100 acres and a rotation of 100 years, to 332,845 cubic feet (see page 224). The total increment laid on during one rotation amounts to 978,100 cubic feet (see page 230). As the normal growing stock must again be present at the end of the first rotation, it follows that the total increment during each rotation is divided as follows:—

Cut away with the old growing stock	=	645,255 c′
Carried over into the second rotation	=	332,845 c′
Total increment	=	978,100 c′

Of the quantity cut, 645,255 cubic feet, 297,400 cubic feet represent intermediate cuttings and 347,855 cubic feet final yields.

2. Relation between Normal Yield and Normal Growing Stock.

If the normal yield (Y_n) is divided by the normal growing stock (G_n) and the quotient multiplied by 100, the result is called the "utilisation per cent."

$$\text{Utilisation per cent.} = \frac{Y_n}{G_n} \times 100.$$

It gives the units of yield for every 100 units of growing stock, just as the increment per cent. gives the units of increment for every 100 units of growing stock. As the increment

of a whole series of age gradations is equal to the yield of the same, it follows that—

Utilisation per cent. = increment per cent. of the whole series of age gradations.

The utilisation per cent. must fall with the increase of the rotation, just as the increment per cent. has been shown to fall.

PART II.

FOREST VALUATION.

FOREST VALUATION.

Forest Valuation deals with the determination of the value of forest soil, the growing stock, the forest as a whole, and the rental derivable from the soil or the forest as a whole.

These values must be determined in all cases of sale, or for the purpose of assessing forest property, or when it is proposed to divide a property. Soil and growing stock also form the capital invested in forestry, and their value must be ascertained for the purpose of gauging the financial success of the forest industry. The latter subject is generally dealt with in a separate part called, "Forest Statics." In the present instance it is proposed to compress that part into one chapter, and to add it to "Forest Valuation."

In order to deal with the matter here contemplated, it is necessary to explain the various methods according to which the value of property may be ascertained, to determine the rate of interest applicable to the forest industry, to give certain formulæ for calculating with compound interest, and to explain the methods of estimating receipts and expenses. All these matters will be dealt with in a preliminary chapter, and the subjects here under consideration will be arranged as follows:—

Chapter I.—Preliminary Matters.
,, II.—Valuation of Forest Soil.
,, III.—Valuation of the Growing Stock.
,, IV.—Valuation of Whole Woods or Forests.
,, V.—Valuation of the Rental.
,, VI.—Methods of Calculating the Financial Results of the Forest Industry and of determining the Most Profitable Treatment of Forests.

CHAPTER I.

PRELIMINARY MATTERS.

SECTION I.—VALUE OF PROPERTY GENERALLY.

PROPERTY means an object which serves for the satisfaction of a requirement. The degree of utility of a property indicates its value.

The value of property may present itself in various ways:—A piece of property may possess value, because it can be used for a certain purpose (as articles of food), or for the production of another kind or class of property (as a set of carpenters' tools, or raw materials).

Again, the value of a proporty may be general or special; the former is that which a property has in the open market; the latter is that which it has for a particular person (as a piece of land situated in the middle of another estate), or under special circumstances (as a loaf of bread in a famine-stricken district).

By the price of a property is understood the amount of another class of property which is offered for it in exchange; the ordinary means of exchange is money.

The value of property can be ascertained in various ways, of which four must be mentioned:—

(1.) *The expectation value*, by which is understood the present net value of all yields, which a property may be able to give; it is determined by discounting to the present time all incomes derivable from the property, and deducting from them the present value of all expenses necessary for the realisation of the incomes.

(2.) *The cost value*, or the total outlay on the acquisition or production, of a property.

(3.) *The sale value*, or the price which can be realised by the sale of a property; if the sale is open to general competition, the sale value becomes the *market value* which depends on supply and demand.

(4.) *The rental value*, by which is understood the capital corresponding to the rental which the property is capable of yielding. This value is ascertained by capitalising the rental according to the formula :—

$$\text{Capital} = \frac{\text{Rental} \times 100}{\text{rate per cent}} = \frac{R \times 100}{p}.$$

It is commonly expressed as "a certain number of years' purchase."

In determining the value of a forest property, or indeed any other property, all calculations must be made with compound interest, as all money, whether principal or interest, is capable of again yielding interest.

SECTION II.—CHOICE OF RATE OF INTEREST

By rate of interest is understood the proportion between the yearly interest (I) and the capital (C) which has yielded it, as represented by the formula :—

$$\text{Rate of interest} = \frac{I}{C}.$$

By rate per cent., or shortly per cent., is understood the yearly interest yielded by a capital of 100; hence the per cent. :—

$$p = \frac{I}{C} \times 100.$$

The rate of interest applicable to an industry depends on many things, of which the most important are :—

(*a.*) The degree of security of the investment and the safety with which it yields a return. In a general way, it may be said that the rate of interest should be inversely proportional to the safety of the investment.

(*b*.) The supply and demand of capital, which change from time to time and with the locality.

(*c*.) The general credit of the country in which the industry is carried on, in other words, the interest yielded by Government securities (called Consols in Britain).

It follows that the general rate of interest applicable to the forest industry cannot be a fixed quantity, but that it changes with locality, time, and a variety of other circumstances.

The question then arises, what rate of interest is applicable to the forest industry under a given set of conditions? In attempting to answer this question the following points must be taken into consideration:—

(1.) The safety of capital invested in forests. The soil offers (apart from changes in prices) almost absolute security. The growing stock is subject to damage by men, insects, fungi, wind, snow, rime, and, above all, by fire. The degree of danger from fire differs much according to species, method of treatment, length of rotation, climate, etc.; in temperate climates the damage keeps within narrow limits.

(2.) The price of forest produce is, on the whole, subject to less sudden fluctuations than the value of money.

(3.) Investment in forest property possesses the great convenience of yielding a steady income.

(4.) Forests cannot easily be let on lease, as inroads on the growing stock are difficult to control; for the same reason, forests, beyond the value of the land, are not well suited as security for loans.

(5.) Compared with the cultivation of field crops, it must be noted that:—

(*a*.) A forest once placed under systematic management yields annually equal returns, or nearly so, whilst those of fields differ much according to the seasons.

(*b*.) Forests require much less labour.

(*c*.) Temporary high prices can be fully utilised by cutting more than the normal yield for a time, or *vice versâ*.

(*d*.) As a rule, successful forest management requires larger estates than the cultivation of field crops.

Bearing these matters in mind, attempts may be made to determine the rate of interest for the forest industry in one of the following ways:—

(1.) Determination based upon the rental and capital value of the forest.

If the value F of a forest, so managed as to yield an annual net return R, is known from a sale which has taken place, the per cent. would be:—

$$p = \frac{R}{F} \times 100.$$

The conditions for the applicability of the mothod are:—

(*a*.) That the annual rental of the forest is accurately known.

(*b*.) That the forest is, at any rate approximately, in such a condition that it can yield a steady annual return.

(*c*.) That the price realised for tho forest was the result of genuine competition.

There is great difficulty in complying with all these conditions, so that frequently this method is not of much practical value, unless the forest is in a normal condition.

(2.) Determination based upon the rate of interest obtainable in agriculture.

As indicated above, differences exist between the two methods of using the soil which are difficult to estimate. At the same time, forestry and agriculture resemble each other in many respects, so that the agricultural rate of interest can frequently be applied to the forest industry.

(3.) Determination according to the rate of interest yielded by Government securities.

The rate of interest of Consols may be too high in some cases and too low in others, according to the general credit of the State and the nature of the forests. On the whole,

however, this is a safe way of determining the rate of interest for the forest industry in all well regulated States.

The rate of interest generally adopted in the calculations contained in the following chapters is that of British Consols, that is to say, $2\frac{1}{2}$ per cent.

SECTION III.—FORMULÆ OF COMPOUND INTEREST.*

The following formulæ, based upon compound interest, are required in forest valuation :—

1. Amount, or the Future Value to which a Capital accumulates.

A capital C_o put out at p per cent. compound interest accumulates in the course of n years to the value—

$$C_n = C_o \times 1{\cdot}0p^n \quad . \quad . \quad . \quad \text{(I.)}$$

or $$\log. C_n = \log. C_o + n \times \log. 1{\cdot}0p,$$

where $1{\cdot}0p = \dfrac{100 + p}{100}$ and $1{\cdot}0p^n = (1{\cdot}0p)^n = \left(\dfrac{100 + p}{100}\right)^n.$

2. Discount, or Determination of Present Value.

The present value C_o of a capital C_n to be realised n years henco is—

$$C_o = \frac{C_n}{1{\cdot}0p^n} \quad . \quad . \quad . \quad . \quad \text{(II.)}$$

$$\log. C_o = \log. C_n - n \times \log. 1{\cdot}0p.$$

3. Summation of Rentals.

a. Future Value.

A rental R becomes due for the first time after m years, and is payable altogether n times at intervals of m years; its value at the end of $m \times n$ years is—

$$C_{mn} = \frac{R\,(1{\cdot}0p^{mn} - 1)}{1{\cdot}0p^m - 1} \quad . \quad . \quad . \quad \text{(III.)}$$

* For the benefit of those who are not versed in calculations with compound interest, the details showing how the formulæ have been obtained are given in Appendix VI.

A rental R is due at the end of each year, altogether n times; its value at the end of n years is—

$$C_n = \frac{R\,(1{\cdot}0p^n - 1)}{{\cdot}0p} \quad . \quad . \quad . \quad (\text{IV.})$$

b. Present Value.

A rental R becomes due for the first time after m years, and is payable altogether n times after intervals of m years; its present value is—

$$C_o = \frac{R\,(1{\cdot}0p^{mn} - 1)}{1{\cdot}0p^{mn}\,(1{\cdot}0p^m - 1)} \quad . \quad . \quad . \quad (\text{V.})$$

A rental R is due at the end of each year, altogether n times; its present value is—

$$C_o = \frac{R\,(1{\cdot}0p^n - 1)}{1{\cdot}0p^n \times {\cdot}0p} \quad . \quad . \quad . \quad (\text{VI.})$$

A rental R is due at the end of each year, for ever; its present value is—

$$C_o = \frac{R}{{\cdot}0p} \quad . \quad . \quad . \quad . \quad (\text{VII.})$$

A rental R is due after n years, and again every n years, for ever; its present value is—

$$C_o = \frac{R}{1{\cdot}0p^n - 1} \quad . \quad . \quad . \quad (\text{VIII.})$$

A rental R is due after m years, and again every n years, for ever; its present value is—

$$C_o = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1} \quad . \quad . \quad . \quad (\text{IX.})$$

A rental R is due now, and again after every n years, for ever; its present value is—

$$C_o = \frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1} \quad . \quad . \quad . \quad (\text{X.})$$

4. Conversion of an Intermittent into an Annual Rental.

A rental R is due after n years, and again every n years for ever; it is equal to an annual rental of—

$$r = \frac{R}{1{\cdot}0p^n - 1} \times {\cdot}0p \quad . \quad \text{(XI.)}$$

A rental R is due after m years, and then every n years for ever; its value is equal to an annual rental of—

$$r = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1} \times {\cdot}0p \quad . \quad . \quad \text{(XII.)}$$

A rental R is due now, and again every n years, for ever; its value is equal to an annual rental of—

$$r = \frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1} \times {\cdot}0p \quad . \quad . \quad \text{(XIII.)}$$

All the above calculations can be made with logarithms, or by means of tables which give future or present values for various rates of interest. The tables in Appendix II., page 338, which give the values for Formulæ I., II., VI., and VIII., suffice for all ordinary forest calculations; they will be utilised in the subsequent chapters.

SECTION IV.—ESTIMATE OF RECEIPTS AND EXPENSES.

1. Receipts.

The receipts of forests are derived from a variety of produce which are generally divided into two classes: Major or principal produce, and minor produce.

a. Major Produce.

This comprises all yields of wood, that is to say, timber and firewood. Bark is generally included; if it is severed from the timber before disposal, it is sometimes classed as minor produce.

Major produce is subdivided into that obtained from final cuttings (final yield) and that from thinnings, etc. (intermediate yields).

The value of major produce is ascertained by means of *money yield tables* which are calculated from *volume yield tables*.

The preparation of volume yield tables and the selection of the proper table have been explained in Forest Mensuration. If tables are available, the age, volume, or height, of each wood are ascertained, and these data are used in the selection of the proper yield table. If the data do not agree with any table, the nearest is selected, and its quantities are modified in proportion to the volumes, or heights, as the case may be, of the wood in question and those given in the yield table.

When the produce yield table has been selected, it is converted into a money yield table, for which purpose the local prices of the several classes of timber and firewood must be ascertained. In doing this, it must be remembered that the average value of material per unit of measurement generally rises with the rotation up to a certain age.

These tables refer to fully stocked or normal woods; hence, the quantities must be modified before use, in the same degree as a particular wood differs from the normal condition. In some cases, a further reduction is made as a kind of insurance against future events. Some authors give 10 per cent. as a proper reduction.

b. Minor Produce.

This comprises all yields which do not consist of timber and firewood; their amounts and values must be locally ascertained.

2. Expenses.

The expenses comprise the cost of administration, protection, formation, harvesting, construction of roads, slides, houses, taxes, etc. The amounts must be locally ascertained

3. Examples of Money Yield Tables.

Subjoined follow two money yield tables. The first is for Scotch pine and the second for beech, it being assumed that the quality of the locality is about the same in both cases.

For simplicity's sake, it has been assumed that the thinnings are made at the end of every 10 years; hence, column *h* gives the values of the wood before the thinnings are made, and column *f* the values which remain after the thinnings have been executed.

MONEY YIELD TABLE FOR ONE ACRE OF SCOTCH PINE WOOD.

Calculated with English prices; brushwood under 3 inches diameter omitted.

Age.	Yield in Cubic Feet.		Net Value of Returns.				
			Per Cubic Foot, Pence.		Total, Shillings.		
	Final.	Inter-mediate.	Final.	Inter-mediate	Final.	Inter-mediate.	Total, Final and Inter-mediate.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
20	600	—	1·5	—	75	—	75
30	1,170	57	2·	1·	195	5	200
40	2,830	353	2·5	1·5	590	44	634
50	3,940	524	3·	2·	985	87	1,072
60	4,690	565	4·	2·5	1,563	118	1,681
70	5,250	536	5·	3·	2,187	134	2,321
80	5,720	493	6·	3·5	2,860	144	3,004
90	6,100	446	7·	4·	3,558	149	3,707
100	6,410	397	8·	5·	4,273	165	4,438
110	6,690	334	9·	6·	5,017	167	5,184
120	6,950	232	10·	7·	5,792	135	5,927
130	7,190	200	11·	8·	6,591	133	6,724
140	7,410	180	12·	9·	7,410	135	7,545

Example.—In the year 60 the volume amounts to 4,690 + 565 = 5,255 cubic feet, of which 565 are taken out in the thinning, leaving 4,690 cubic feet to go on with. These increase during the next 10 years to 5,250 + 536 = 5,786 cubic feet. Similarly, of the total value in the year 60 = 1,681 shillings, 118 are taken out in the thinning, while a value of 1,563 shillings remains. This increases during the next 10 years to 2,187 + 134 = 2,321 shillings, and so on.

The Scotch pine wood is supposed to be grown on fairly good soil, so that the returns stand about halfway between *best* and *middling* quality according to the yield tables in Appendix III., pages 360—363. Such returns can, on an

average, be expected from Scotch pine woods grown in England. The values per cubic foot are those prevailing in the south and centre of England.

MONEY YIELD TABLE FOR ONE ACRE OF BEECH WOOD.

Calculated with English prices; brushwood under 3 inches diameter omitted.

Age.	Yield in Cubic Feet.		Net Value of Returns.				
			Per Cubic Foot, Pence.		Total, Shillings.		
	Final.	Intermediate.	Final.	Intermediate.	Final.	Intermediate.	Total, Final and Intermediate.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
30	86	—	2·	—	14	—	14
40	943	—	3·	- -	236	—	236
50	2001	86	4·5	3·	750	21	771
60	2915	272	6·	4·	1457	91	1548
70	3716	329	7·	4·5	2167	123	2290
80	4430	357	8·	5·	2953	149	3102
90	5045	400	9·	6·	3784	200	3984
100	5573	414	10·	7·	4641	241	4885
110	6045	400	11·	8·	5541	267	5808
120	6460	400	12·	9·	6460	300	6760
130	6817	386	13·	10·	7385	322	7707
140	7117	372	14·	11·	8303	341	8644

CHAPTER II.

VALUATION OF FOREST SOIL.

The soil can be utilised in two ways:—

Either by being used direct, as for mining, quarrying, construction of dwellings, etc.;

Or by letting it produce other goods, as field or forest crops.

In each of these cases the soil may have a different value. Forest valuation ascertains the value of the soil under the supposition that it is used for the production of forest crops; for this purpose it determines the expectation, cost, and sale values.

SECTION I.—THE EXPECTATION VALUE OF FOREST SOIL.

1. Method of Calculation.

In conformity with the definition given at page 112, by the expectation value of forest soil is understood the sum of the present values of all returns expected from the soil in the course of time, *less* the present value of all expenses which must be incurred to obtain those returns.

a. Present Value of Returns.

(1.) *Final Yields.*—Let the rotation contain r years, and let the value of each final yield, less cost of harvesting, $= Y_r$, to be realised every r years, then the present value of all final yields for ever amounts, according to formula VIII. (page 117), to:—

$$\text{Present value of all final yields} = \frac{Y_r}{1{\cdot}0p^r - 1}.$$

(2.) *Intermediate Cuttings, or Thinnings.*—Let T_a, T_b, T_c . . . T_q, represent the value of thinnings, less cost of

harvesting, occurring in the years a, b, c . . . q. Each of these thinnings occurs during every rotation, that is to say, every r years. Thus, the first thinning occurs after a years, again after $a + r$ years, again after $a + 2 \times r$ years, and so on; hence, the present value of all these thinnings, according to formula IX.:—

$$= \frac{T_a \times 1{\cdot}0p^{r-a}}{1{\cdot}0p^r - 1} + \frac{T_b \times 1{\cdot}0p^{r-b}}{1{\cdot}0p^r - 1} + \ldots + \frac{T_q \times 1{\cdot}0p^{r-q}}{1{\cdot}0p^r - 1}$$

$$= \frac{T_a \times 1{\cdot}0p^{r-a} + T_b \times 1{\cdot}0p^{r-b} + \ldots + T_q \times 1{\cdot}0p^{r-q}}{1{\cdot}0p^r - 1}.$$

(3.) *Minor Produce.*—These are dealt with in the same way as thinnings, if they occur at regular intervals. Let M_s, M_t . . . M_z be the values of minor produce occurring in the years s, t . . . z, and again every r years, then their present value is ÷

$$= \frac{M_s \times 1{\cdot}0p^{r-s} + M_t \times 1{\cdot}0p^{r-t} + \ldots + M_z \times 1{\cdot}0p^{r-z}}{1{\cdot}0p_r - 1}.$$

If items of minor produce, M, occur regularly every year, as grazing fees, rent from shooting, etc., and of equal value year after year, their present value amounts, according to formula VII., to $\frac{M}{{\cdot}0p}$.

b. Present Value of Expenses.

(1.) *Cost of Formation.*—Assuming that c represents the cost of formation, whether artificial or natural, which has to be incurred at the commencement of each rotation, then the present value of all such expenses comes, according to formula X., to $= \frac{c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1}$.

If the first cost differs from that on future occasions, and if the latter is represented by c', then the total present value is $= c + \frac{c'}{1{\cdot}0p^r - 1}$.

(2.) *Annually Recurring Expenses.*—Let e represent the

value of all annually recurring expenses, as cost of administration, taxes, etc., then the total present value of all such expenses comes to $\frac{e}{\cdot 0p}=E$, being the capitalised value of the annual expenses.

(3.) *Periodically Occurring Expenses.*—These are dealt with as has been shown in the case of thinnings or items of minor produce.

(4.) *Expenses of Harvesting and of Collection of Revenue.*—These are always deducted from the receipts, and they do not appear in the account.

c. *Formula for the Expectation Value of Forest Soil.*

This formula would be represented by an addition of all the items enumerated under *a* less those under *b*. In order to shorten the formula, without destroying its usefulness, it may be assumed—

(1.) That $T_a, T_b \ldots T_q$ represent thinnings as well as items of regularly occurring minor produce;

(2.) That the expenses are reduced to the cost of formation and the annually recurring expenses;

(3.) That the cost of formation at the commencement of each rotation comes to the same amount;

(4.) That *e* represents the difference between the annually recurring expenses and the annually recurring receipts such as grazing fees, rent of shooting, etc. If the latter are higher than the former, then *e*, respectively *E*, would be positive.

The expectation value S_e of the soil is then represented by the formula—

$$S_e = \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} - E.$$

Example.—An acre of land is to be cultivated at once with Scotch pine, and to be worked under a rotation of 80 years. It is expected to yield

the returns given in the money yield table at page 120. The expenses are expected to be as follows:—

Cost of formation every 80 years = 60 shillings.

Annual expenses for administration, taxes, etc., less annually recurring incomes = 3 shillings.

Interest = $2\frac{1}{2}$ per cent.

The expectation value of the soil will, in that case, amount to ÷

$$S_e = \frac{3{,}004 + 5 \times 1{\cdot}025^{50} + 44 \times 1{\cdot}025^{40} + 87 \times 1{\cdot}025^{30} + 118 \times 1{\cdot}025^{20} + 134 \times 1{\cdot}025^{10} - 60 \times 1{\cdot}025^{80}}{1{\cdot}025^{80} - 1} - \frac{3}{{\cdot}025}.$$

By using the tables in Appendix II., page 338, this value becomes—

$$S_e = (3{,}004 + 5 \times 3{\cdot}4371 + 44 \times 2{\cdot}6851 + 87 \times 2{\cdot}0976 + 118 \times 1{\cdot}6386 + 134 \times 1{\cdot}2801 - 60 \times 7{\cdot}2096) \times {\cdot}1610 - 120$$

$$S_e = 3{,}254{\cdot}1333 \times {\cdot}1610 - 120 = 403{\cdot}9155 \text{ shillings},$$

or,

$$S_e = 404 \text{ shillings} = £20\ 4s.\ 0d.$$

This shows that, for land yielding the above returns and involving the above expenses, the proprietor may pay £20 4*s*. an acre, if he is satisfied with $2\frac{1}{2}$ per cent. compound interest on his investment. It will be shown later on that he must obtain the land for less than £20 4*s*., if he desires to get more than $2\frac{1}{2}$ per cent. on his investment.

Assuming now that the same acre of land is planted with beech, that it yields the returns given in the money yield table at page 121, and that all other conditions remain the same, the expectation value of the soil will be ÷

$$S_e = \frac{3{,}102 + 21 \times 1{\cdot}025^{30} + 91 \times 1{\cdot}025^{20} + 123 \times 1{\cdot}025^{10} - 60 \times 1{\cdot}025^{80}}{1{\cdot}025^{80} - 1} - 120$$

$$S_e = 366 \text{ shillings} = £18\ 6s.\ 0d.$$

It follows that, under the above conditions, it pays to plant beech on such land if the latter can be purchased for £18 6*s*. an acre or under, which is less than in the case of Scotch pine. The highest expectation value of the soil in the case of beech is obtained under a rotation of 90 years, when S_e = £18 15*s*., which is somewhat lower than that obtained in the case of Scotch pine.

2. Matters which affect the Expectation Value of the Soil.

As indicated by the formula, the expectation value of the soil depends on—

(*a*.) The absolute amount of receipts and expenses.

(*b*.) The length of the rotation.

(*c*.) The time when the intermediate returns are realised.

(*d.*) The time when the costs of production have to be incurred.

(*e.*) The rate of interest with which the calculation is made.

Each of these matters must be further considered.

a. The absolute Amounts of Receipts and Expenses.

These are influenced, not only by careful management, but also by the choice of species. If, for one and the same piece of land, the calculation is made for different species, the expectation values thus obtained are likely to differ considerably; hence, the importance of selecting the most suitable species in each case.

b. The Length of Rotation.

As the growing stock has, in the majority of cases, very little value during the first part of the life of a wood, so that the yield would not even cover the cost of harvesting, it follows that the expectation value of the soil may, under a very short rotation, be negative. With advancing age, the value of the growing stock increases, so that the expectation value of the soil becomes positive; it goes on augmenting until it reaches a maximum, after which time it falls again. A second maximum may occur owing to a sudden heavy increase in the value per unit of measurement of the growing stock. Under an excessively high rotation, the expectation value of the soil would again become negative.

Example.—Making the calculation for successive rotations for a Scotch pine wood with the same conditions as before, the following amounts of the expectation value are obtained:—

S_e	for a rotation of	30	years	.	=	− 52	shillings.
S_e	,, ,,	40	,,	.	=	+ 164	,,
S_e	,, ,,	50	,,	.	=	+ 262	,,
S_e	,, ,,	60	,,	.	=	+ 353	,,
S_e	,, ,,	70	,,	.	=	+ 394	,,
S_e	,, ,,	80	,,	.	=	+ 404	,,
S_e	,, ,,	90	,,	.	=	+ 392	,,
S_e	,, ,,	100	,,	.	=	+ 368	,,
S_e	,, ,,	110	,,	.	=	+ 338	,,
S_e	,, ,,	120	,,	.	=	+ 304	,,

The above data show that, financially, the most favourable rotation must be that under which the expectation value reaches its maximum; in the above example a rotation of about 80 years. The more the rotation differs from 80 years, the less favourable are the financial results.

c. The Time when the Intermediate Returns are Realised.

The earlier the intormediate returns occur, other matters being the same, the higher will be the expectation value.

Example.—The present value of a thinning worth 86 shillings made in the year 60, and again every 100 years, is =

$$\frac{86 \times 1{\cdot}025^{40}}{1{\cdot}025^{100} - 1} = 21 \text{ shillings.}$$

If the same thinning were made in the year 30, and again every 100 years, its present value would be =

$$\frac{86 \times 1{\cdot}025^{70}}{1{\cdot}025^{100} - 1} = 45 \text{ shillings.}$$

It follows that strong thinnings made at an early ago can considerably increase tho expectation value of the soil. At the same time, it must not be overlooked that the small material obtained from early thinnings is not always saleable at remunerative rates; moreover, such thinnings may disastrously affect the later returns, especially the final yield, so that the advantage gained in the first instance may be more than counterbalanced by loss later on.

Early receipts from minor produce affect the expectation value in the same way as those from thinnings; hence, they are of great financial importance, as long as they do not unduly affect later returns.

d. The Time when the Costs of Production have to be Incurred.

This affects the expectation value in a manner the reverse of that produced by early thinnings and incomes from minor produce. Expenses, incurred during the early part of the rotation, affect the expectation value unfavourably; hence, it

is important to keep the cost of formation as low as possible. Thus, it may happen that tho expectation value is higher under the system of natural regeneration than under the clear cutting system with planting, provided that natural regeneration is easy; if it involves loss of time, it may be more profitable to regenerate artificially, so as to prevent the lengthening of the rotation.

e. The Rate of Interest with which the Calculation is made.

A high rate of interest gives a low expectation value of the soil, and *vice versâ*. The value is, however, not in inverse proportion to the rate of interest; the former rises more rapidly than the latter falls.

Again, under a low rate of interest the expectation value culminates later than under a high rate.

Example.—Taking the same data as above, and calculating the expectation value of the soil with $2\frac{1}{2}$, and again with 3, and 4 per cent., the following values are obtained :—

Rotation, Years.	Calculation made with		
	$2\frac{1}{2}$%	3%	4%
	s.	*s.*	*s.*
50	262		43
60	353	214	**61**
70	394	**230**	56
80	**404**	225	43
90	392	185	
100	368		

This example shows :—

(1.) By raising the per cent. from $2\frac{1}{2}$ to 3, the maximum expectation value falls from 404 to 230 shillings, and calculated with 4 per cent. to 61 shillings.

(2.) By making the calculation with 4 per cent. the expectation value becomes very small, and under a higher per cent. it becomes negative.

(3.) The expectation value of the soil culminates:—

Calculated with 2½ per cent. at about 80 years.

„	„	3	„	„	„	70	„
„	„	4	„	„	„	60	„

(4.) If the proprietor is satisfied with 2½ per cent. on his investment, he can afford to pay £20 4*s*. per acre for the land; if he expects 3 per cent., only £11 10*s*.; if 4 per cent. only £3 1*s*.

3. Merits of the Method of the Expectation Value.

The expectation value indicates the true economic value of the soil for forest culture, because it is based upon the productive power of the land when used for the rearing of forest crops. It gives the value which corresponds to the net returns calculated with the adopted rate of interest. On the other hand, the method gives correct results only under the following conditions:—

(*a*.) That all items of expected incomes and expenses are accurately known. In order to comply with this condition, accurate and suitable yield tables are required. Moreover, future prices of produce and expenses must be estimated on the basis of those at present prevailing a matter which introduces much uncertainty into the calculation.

(*b*.) That the calculation is made with a suitable rate of interest. It has been shown above that this is a matter beset by considerable difficulty.

(*c*.) That the rotation, corresponding to the maximum expectation value of the soil, can be adopted and retained without thereby lowering the price of forest produce; in other words, that the market can readily absorb any extra cuttings which may be necessary, in order to introduce a rotation lower than that hitherto followed.

Generally speaking, the expectation value of forest soil is not a fixed quantity; it changes, not only in the ways indicated

above, but also with alterations in the price of forest produce, consequent on changes in the areas set aside for the production of forest crops.

SECTION II.—THE COST VALUE OF FOREST SOIL.

By the cost value of the soil is understood the sum of all expenses incurred in acquiring the land and rendering it fit for forest culture. These expenses consist of:—

(1.) The price paid for the land.

(2.) The sum expended in rendering it fit for cultivation, such as drainage, or irrigation, levelling, fixation of soil, etc.

(3.) The interest accumulated on the outlay mentioned under (1.) and (2.) up to the date when the first forest crop is started.

Example.—An acre of land has been purchased for £10; a sum of £5 has been expended at once in breaking through an impermeable substratum in strips 6 feet apart; a further sum of £2 has been spent after the lapse of 3 years in breaking up the intermediate strips; the land was allowed to lie fallow for another 2 years. The cost value of the land at the end of 5 years, when it is planted, amounts, calculating with $2\frac{1}{2}$ per cent., to

$$\mathbf{S}_c = (10 + 5) \times 1{\cdot}025^5 + 2 \times 1{\cdot}025^2 = £19\ 1s.\ 5d.$$

The cost value of the land may be accepted as the true value:—

(1.) If the expectation value of the soil cannot be ascertained with any degree of accuracy.

(2.) If the owner accepts the price, which represents his own outlay on the land.

The cost value of the soil may be equal to, smaller, or larger than the expectation value.

SECTION III.—THE SALE VALUE OF FOREST SOIL.

By the sale, or market, value of forest soil is understood the value which it realises in the open market. It represents the true economic value only, if it agrees with the expectation value.

In most localities, a sale price has established itself, but this represents in the majority of cases the value which the land has for other purposes, such as agriculturo. It may differ considerably from the value which the land has if used for the production of forest crops, the sale value being generally higher than the forest value in the case of good lands and lower in the case of inferior lands, because the former yield a higher rental under field crops, and the latter undor forest crops.

The salo value of forest soil may be taken as expressing the truo value:—

(1.) If forost soil is to be disposod of voluntarily for other uses.

(2.) In the case of forced salcs, when the local valuo has to be ascertained, rather than the forest value.

CHAPTER III.

VALUATION OF THE GROWING STOCK.

The value of the growing stock can, as in the case of the soil, be determined as the expectation, cost, or sale value.

The valuation may refer to—

(1.) One particular wood.

(2.) A whole series of age gradations, representing the normal growing stock of a working section or of a whole forest.

SECTION I.—VALUE OF THE GROWING STOCK OF A SINGLE WOOD.

1. The Expectation Value.

The expectation value of the growing stock of a wood now m years old is equal to the present values of all incomes which may be expected from the wood, *less* the present value of all expenses which must be incurred between the year m and the time when the wood is finally cut over.

a. Method of Calculation.

Starting from the same data as those given in the case of the valuation of forest soil, the *receipts* consist of—

(1.) Final yield in the year $r = Y_r$; present value $= \frac{Y_r}{1{\cdot}0p^{r-m}}$.

(2.) Intermediate yields, such as thinnings, to be realised in the years n, o, p, and q; their present value amounts to—

$$\text{Thinning in the year } n = \frac{T_n}{1{\cdot}0p^{n-m}} = \frac{T_n \times 1{\cdot}0p^{r-n}}{1{\cdot}0p^{r-m}}.$$

$$\text{,, \quad ,, \quad ,, \quad } o \ \ldots = \frac{T_o \times 1{\cdot}0p^{r-o}}{1{\cdot}0p^{r-m}}.$$

Thinning in the year p . . $= \dfrac{T_p \times 1{\cdot}0p^{r-p}}{1{\cdot}0p^{r-m}}$.

" " " q . . $= \dfrac{T_q \times 1{\cdot}0p^{r-q}}{1{\cdot}0p^{r-m}}$.

Expenses :

(1.) Annual expenses to be incurred from the year m to the year r; their present value amounts to (according to Formula VI.)—

$$\frac{e}{1{\cdot}0p}+\frac{e}{1{\cdot}0p^2}+\ldots+\frac{e}{1{\cdot}0p^{r-m}}=\frac{e\,(1{\cdot}0p^{r-m}-1)}{1{\cdot}0p^{r-m}\times{\cdot}0p}=\frac{E\,(1{\cdot}0p^{r-m}-1)}{1{\cdot}0p^{r-m}}.$$

(2.) Rent of soil to be paid from the year m to the year r; its annual amount may be denoted by $S \times {\cdot}0p$; the total present value of the rent of the soil during $r-m$ years amounts to—

$$\frac{S\times{\cdot}0p}{1{\cdot}0p}+\frac{S\times{\cdot}0p}{1{\cdot}0p^2}+\ldots+\frac{S\times{\cdot}0p}{1{\cdot}0p^{r-m}}=\frac{S\times{\cdot}0p\,(1{\cdot}0p^{r-m}-1)}{1{\cdot}0p^{r-m}\times{\cdot}0p}$$
$$=\frac{S\,(1{\cdot}0p^{r-m}-1)}{1{\cdot}0p^{r-m}}.$$

The *formula* for the expectation value of a wood stands, therefore, as follows:—

$$^{m}G_{e}=\frac{Y_r+T_n\times1{\cdot}0p^{r-n}+\ldots+T_q\times1{\cdot}0p^{r-q}-(S+E)(1{\cdot}0p^{r-m}-1)}{1{\cdot}0p^{r-m}}$$

Example.—A fully-stocked Scotch pine wood, worked under a rotation of 80 years, is feloniously burned when 45 years old; what compensation per acre should be paid to the owner, if the expected returns are those indicated in the table at page 120?

Rate of interest = $2\frac{1}{2}$ per cent. Value of soil = 404 shillings.
Annual expenses = 3 shillings, to be incurred at the end of each year:—

$$^{45}G_e=\frac{3{,}004+87\times1{\cdot}025^{30}+118\times1{\cdot}025^{20}+134\times1{\cdot}025^{10}-(404+120)(1{\cdot}025^{35}-1)}{1{\cdot}025^{35}}.$$

$$^{45}G_e=1{,}193 \text{ shillings} = £59\ 13s.\ 0d.$$

b. Notes on the Expectation Value of the Growing Stock.

The expectation value of the growing stock depends on the following matters:—

i. The Absolute Amount of Receipts and Expenses.

Regarding the value of the soil to be introduced into the calculation, it should be noted that the maximum expectation

value should be chosen (in the above example 404 shillings), if the soil is again to be used for forest purposes; if the soil can be more profitably used for agriculture or other purposes, the correspondingly increased value must be introduced into the account.

ii. The Length of the Rotation.

In the case of fully-stocked normal woods, the highest expectation value is obtained for that rotation, for which the expectation value of the soil culminates, provided that value is introduced into the account. (In the above example, rotation = 80 years and S_e = 404 shillings.) If a larger value of soil is introduced, then the maximum expectation value of the growing stock is obtained for a lower rotation, than that for which the expectation value of the soil culminates, and *vice versâ*.

In the case of insufficiently stocked abnormal woods, the rotation, for which the highest expectation value of the growing stock is obtained, can only be determined by experimental calculations based upon the data of each special case.

Example.—Taking the same data as before,

For a rotation of	70 years	${}^{45}G_e$ =	1,169 shillings.
,, ,,	80 ,,	,, =	1,193 ,,
,, ,,	90 ,,	,, =	1,160 ,,
,, ,,	100 ,,	,, =	1,092 ,,

The maximum value is obtained for a rotation of 80 years, that is to say, the rotation under which the expectation value of the soil culminates (see page 126).

Assuming now that the wood has been injured by wind at a previous period, so that no thinnings can be made before the year 90, and that the growing stock would realise—

At an age of 70 years	= 1,800	shillings,
,, ,, 80 ,, ...	= 2,700	,,
,, ,, 90 ,, ...	= 3,707	,,

then the expectation value of the wood under these rotations would be as follows:—

For a rotation of 70 years—

$$ {}^{45}G_e = \frac{1,800 - 524\ (1\cdot025^{25} - 1)}{1\cdot025^{25}} = 729 \text{ shillings.} $$

For a rotation of 80 years—

$$ {}^{45}G_e = \frac{2,700 - 524\,(1{\cdot}025^{35} - 1)}{1{\cdot}025^{35}} = 835 \text{ shillings.} $$

For a rotation of 90 years—

$$ {}^{45}G_e = \frac{3,707 - 524\,(1{\cdot}025^{45} - 1)}{1{\cdot}025^{45}} = 868 \text{ shillings.} $$

For a rotation of 100 years—

$$ {}^{45}G_e = \frac{4,438 + 149 \times 1{\cdot}025^{10} - 524\,(1{\cdot}025^{55} - 1)}{1{\cdot}025^{55}} = 801 \text{ shillings.} $$

It will be observed that the maximum expectation value of the growing stock is in this case obtained under a rotation of 90 years.

iii. The Age of the Wood.

The expectation value of the growing stock rises (given a fixed rotation) with the age of the wood, but not in exact proportion. If the thinnings are made after regular intervals, say every 10 years, it generally happens that, immediately before making thinnings, the expectation value is slightly higher than immediately after the thinning has been made. For instance, if a thinning is made in the year 60, the expectation value of the growing stock for the year 59 will probably be higher than for the year 61.

Immediately after the area has been stocked in the beginning of the rotation, and if the maximum expectation value of the soil has been introduced in the account, the expectation value of the growing stock is equal to the cost of formation, and at the end of the rotation it is equal to the value of the final yield.

iv. The Rate of Interest with which the Calculation is made.

A high rate of interest gives a low expectation value of the growing stock, and *vice versâ*, similar to what has been shown for the expectation value of the soil.

2. Cost Value of the Growing Stock of a Single Wood.

The cost value of the growing stock of a wood now m years old is equal to the present value of all costs of production, less

the present value of all returns which the wood has yielded before the year m.

a. Method of Calculation.

Costs of Production.—(1.) The present value of the rent of the soil during m years comes to—

$$S \times 1{\cdot}0p^m - S = S\,(1{\cdot}0p^m - 1).$$

(2.) The present value of the annual expenses during m years (Formula IV.) $= e \times 1{\cdot}0p^{m-1} + e \times 1{\cdot}0p^{m-2} + \ldots + e,$

$$= \frac{e}{{\cdot}0p}\,(1{\cdot}0p^m - 1) = E\,(1{\cdot}0p^m - 1).$$

(3.) Present value of cost of formation $= c \times 1{\cdot}0p^m$.

Receipts.—These consist of all previous thinnings and items of other incomes realised before the year m; they may be represented by $T_a, T_b \ldots T_l$. Their present value is—

$$= T_a \times 1{\cdot}0p^{m-a} + T_b \times 1{\cdot}0p^{m-b} + \ldots + T_l \times 1{\cdot}0p^{m-l}.$$

Should any items of income have occurred in annually equal amounts, such as shooting, grazing, etc., they can be summed up according to Formula IV., or, better still, be deducted from the annually recurring expenses.

The general formula for the cost value is, therefore—

$$^{m}G_c = (S + E)\,(1{\cdot}0p^m - 1) + c \times 1{\cdot}0p^m$$
$$- (T_a \times 1{\cdot}0p^{m-a} + T_b \times 1{\cdot}0p^{m-b} + \ldots + T_l \times 1{\cdot}0p^{m-l}).$$

Here, $^{m}G_c$ still includes the thinning of the year m. If the calculation is made for what remains as final yield to go on with, then the thinning in the year m $(= T_m)$ must also be deducted.

Example.—Taking the same data as before, the cost value comes to—

$$^{45}G_c = (404 + 120)\,(1{\cdot}025^{45} - 1) - (5 \times 1{\cdot}025^{15} + 44 \times 1{\cdot}025^{5}).$$
$^{45}G_c = 1{,}193$ shillings, as in the case of the expectation value.

b. Notes on the Cost Value.

The cost value of the growing stock depends on—

(1.) *The absolute amount of the receipts and expenses up to the year* m.

(2.) *The Age of the Wood.*—The value rises with the age, but temporary exceptions occur, as immediately after a thinning the cost value may be smaller than immediately before it.

At the commencement of the rotation, the cost value is equal to the cost of formation.

At the end of the rotation, the cost value is equal to the value of the final yield, provided that—

(*a*.) the calculation has been made with the expectation value of the soil (404 shillings in the above case);

(*b*.) the receipts and expenses were of the normal amounts, and

(*c*.) the wood is fully stocked, or normal, at the end of the rotation.

Proof.—Let $m = r$, then,

$$^{r}G_c = (S + E)\,(1{\cdot}0p^r - 1) + c \times 1{\cdot}0p^r - (T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q}).$$

By introducing the expectation value of the soil, the above equation becomes:—

$$^{r}G_c = \left(\frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} - E + E\right)$$
$$\times (1{\cdot}0p^r - 1) + c \times 1{\cdot}0p^r - (T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q})$$

which, after reduction, leads to—

$$^{r}G_c = Y_r.$$

(3.) *The Rate of Interest.*—If the calculation is made with the maximum expectation value of the soil and the receipts and costs corresponding to it, then a higher rate of interest yields a lower cost value and *vice versâ*.

If the above assumptions do not hold good, it depends on the value of the soil and the amounts of receipts and costs which are introduced into the calculation, whether a higher rate of interest gives a greater or smaller cost value of the growing stock.

3. Sale Value, or Utilisation Value, of the Growing Stock of a Single Wood.

By the sale value of the growing stock of a wood is understood that price which it would realise if offered for sale in the open market.

The growing stock may be sold under one of two conditions:—

(*a.*) The wood is allowed to grow on for a number of years. In this case, the purchaser would have to rent the soil for a number of years, and to meet certain other expenses. Hence, the sale value should be equal to the expectation value.

(*b.*) The wood is to be cut down at once. In this case the sale value would be the price which the cut material realises in the open market. It is ascertained by determining the volume of the growing stock and multiplying it by the net mean rate per unit of measurement.

The sale value of very young woods undor condition *b* is generally negative, until the receipts obtained by the sale of the produce cover the cost of harvesting; from that period it becomes positive, rising at first slowly, then more rapidly, reaching its maximum value far beyond the period at which the mean annual increment culminates, in fact not until the annual increase in the value per unit of measurement is no longer sufficient to cover the falling off caused by thinning or decay. This period occurs, generally speaking, earlier in the case of light-demanding species, than in the case of shade-bearing species which maintain a full stocking for a longer space of time.

4. Relation existing between the Expectation and Cost Values of the Growing Stock of a Normal Wood.

Looking at the formulæ for the expectation and cost values, it will be observed that the rental of the soil and annual

expenses appear in the negative form in the one and in the positive form in the other; again, the thinnings appear positive in the first and negative in the second. It follows that any change in these items affects the two values in opposite directions; what raises the one value reduces the other, and *vice versâ*. Nevertheless, the one value can become equal to the other. This is the case, other items remaining the same in both instances, if the calculation is made in either case with the maximum expectation value of the soil.

Proof.—Let ${}^{m}G_e = {}^{m}G_c$, immediately after the thinning in the year m has been made, then:—

$$\frac{Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} - (S+E)(1{\cdot}0p^{r-m} - 1)}{1{\cdot}0p^{r-m}}$$
$$= (S + E)(1{\cdot}0p^{m} - 1) + c \times 1{\cdot}0p^{m} - (T_a \times 1{\cdot}0p^{m-a} + \ldots + T_l \times 1{\cdot}0p^{m-l} + T_m).$$

After making the necessary reduction, it will be seen that this equation can hold good only, if—

$$S = \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^{r}}{1{\cdot}0p^{r} - 1} - E;$$

in other words, if the expectation value of the soil is introduced for S. Even then, the above holds good only in the case of normally stocked woods. If a wood has been too thinly stocked from early youth, so that both thinnings and final yield are below the normal amounts, tho cost value will be found to be greater than the expectation value.

5. Relation existing between the Expectation and Cost Values of the Growing Stock of a Normal Wood on the one hand, and the Utilisation Value on the other.

The utilisation value of the growing stock is equal to the expectation or cost value at the end of the rotation, provided the maximum expectation value of the soil is introduced into the account, and the rotation is that for which the expectation value of the soil culminates. An equality can also occur at

a previous stage according to the values introduced into the account.

Generally speaking, the utilisation value of young woods is smaller than the expectation or cost value. On approaching the end of the rotation, the difference is small, and it is then the safest plan to value the woods according to their utilisation value, as the calculation of the expectation and cost values is based upon more or less uncertain data.

SECTION II.—VALUE OF THE GROWING STOCK OF A NORMAL SERIES OF AGE GRADATIONS. (NORMAL GROWING STOCK.)

If a forest is so managed that it yields annually an equal return, it must contain a regular series of woods of equal yield capacity ranging in age from 1 year up to r years, with one year's difference between every two successive gradations. Whether these age gradations are found on separate areas or are mixed with each other makes no difference. In either case, every year the oldest age gradation will be cut over, giving a yield $= Y_r$, and every year there will be thinnings in the gradations which have reached the ages of $a, b \ldots q$ years; at the same time, every year formation will cost c shillings, while supervision will cost $r \times e$ shillings. It is of interest to the forester to ascertain the value of the growing stock in such a forest, the same being known as "the growing stock of a normal series of age gradations."

The annual net return of a normal series of age gradations, or a working section, forms the rental of the soil and normal growing stock of that working section. Like the interest yielded by any ordinary capital, that rental is produced within the year, so that the growing stock at the end of the year represents the capital plus one year's rental. Hence, the capital alone is present immediately after the year's rental has been removed. At that moment the oldest age gradation is $(r-1)$ years old, the next $(r-2)$, etc., and the youngest (which has just been cleared) is 0 years old.

Where cuttings are made in winter, the normal growing

stock is present in spring, before the trees have commenced to lay on the new annual increment.

The simplest way of calculating the value of the normal growing stock is to capitalise the annual net income and to deduct from the amount the value of the soil.

The annual net income is represented by the expression—

$$Y_r + T_a + \ldots + T_q - (c + r \times e);$$

hence, the value of tho growing stock is equal to—

$$\text{Normal } G = \frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{\cdot 0p} - r \times S_e,$$

or for the unit of area

$$\frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{r \times \cdot 0p} - S_e.$$

Example.—Taking the same data as before and a rotation of 80 years, the normal growing stock standing on 80 acres is—

$${}^{80}G_{normal} = \frac{3{,}004 + 5 + 44 + 87 + 118 + 134 - (60 + 80 \times 3)}{\cdot 025} - 80 \times 404$$

$${}^{80}G_{normal} = 91{,}360 \text{ shillings},$$

and the average value of one age gradation

$$= \frac{91{,}360}{80} = 1{,}142.$$

Tho value of the normal growing stock can also be obtained by adding up either the expectation, or cost, values of all the different age gradations running from 0 years up to r—1 years. If the maximum soil expectation value is introduced into these values, then the result is exactly the same as if the annual net rental is capitalised and the maximum soil expectation value deducted. If a different value for the soil is introduced, the result will not be the same.

Calculations for the Expectation Value.—Assuming, in the first instance, that any one intermediate return in the year q is obtained, the values of the various age gradations will be as follows (see formula on page 133):—

$$^{r-1}G_e = \frac{Y_r - (S+E)(1{\cdot}0p^1 - 1)}{1{\cdot}0p^1}$$

$$^{r-2}G_e = \frac{Y_r - (S+E)(1{\cdot}0p^2 - 1)}{1{\cdot}0p^2}$$

$$\vdots$$

$$^{q}G_e = \frac{Y_r - (S+E)(1{\cdot}0p^{r-q} - 1)}{1{\cdot}0p^{r-q}}$$

$$^{q-1}G_e = \frac{Y_r + T_q \times 1{\cdot}0p^{r-q} - (S+E)(1{\cdot}0p^{r-(q-1)} - 1)}{1{\cdot}0p^{r-(q-1)}}$$

$$\vdots$$

$$^{o}G_e = \frac{Y_r + T_q \times 1{\cdot}0p^{r-q} - (S+E)(1{\cdot}0p^{r} - 1)}{1{\cdot}0p^{r}}$$

By adding up these quantities and introducing the other intermediate returns in the years a, $b \ldots p$, the following formula is obtained:—

$$\textbf{Norm. } G_e = \frac{(Y_r + S + E)(1{\cdot}0p^r - 1) + T_a \times 1{\cdot}0p^{r-a}(1{\cdot}0p^a - 1) + \ldots + T_q \times 1{\cdot}0p^{r-q}(1{\cdot}0p^q - 1)}{1{\cdot}0p^r \times {\cdot}0p} - r(S+E)$$

By introducing the soil expectation value, and substituting its value in the first part of the formula, the latter reduces to:—

$$G_e = \frac{Y_r + T_a + \ldots + T_q - c}{{\cdot}0p} - r(S_e + E),$$

or, as $E = \frac{c}{{\cdot}0p}$:—

$$\text{Normal } G_e = \frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{{\cdot}0p} - r \times S_e,$$

as before.

Calculation for the Cost Value.—(See formula on page 136).

$$^{o}G_e = (S+E)(1{\cdot}0p^o - 1) + c \times 1{\cdot}0p^o.$$

$$^{1}G_e = (S+E)(1{\cdot}0p^1 - 1) + c \times 1{\cdot}0p^1.$$

$$\vdots$$

$$^{a}G_e = (S+E)(1{\cdot}0p^a - 1) + c \times 1{\cdot}0p^a - T_a.$$

$$^{a+1}G_e = (S+E)(1{\cdot}0p^{a+1} - 1) + c \times 1{\cdot}0p^{a+1} - T_a \times 1{\cdot}0p.$$

$$\vdots$$

$$^{r-1}G_e = (S+E)(1{\cdot}0p^{r-1} - 1) + c \times 1{\cdot}0p^{r-1} - T_a \times 1{\cdot}0p^{r-a-1}.$$

The sum of these expressions, adding the other thinnings, comes to:—

$$\textbf{Normal } G_c = \frac{(S+E+c)(1{\cdot}0p^r-1)-\left[T_a(1{\cdot}0p^{r-a}-1)+\ldots+T_q(1{\cdot}0p^{r-q}-1)\right]}{{\cdot}0p} - r(S+E).$$

By introducing the soil expectation value, this goes over into:—

$$\text{Normal } G_c = \frac{Y_r + T_a + \ldots + T_q - (c + r \times e)}{{\cdot}0p} - r \times S_e,$$

as before.

CHAPTER IV.

VALUATION OF WHOLE WOODS, OR FORESTS.

THE value of a whole wood, or a forest, is equal to the 'alue of the soil plus the value of the growing stock; hence, t can be ascertained by adding together these two values. At he same time, that value can also be determined direct out of he returns and expenses, or from sales of forests, or it can be alculated out of the rental which the forest yields.

1. EXPECTATION VALUE OF A FOREST.

As this is based upon future returns and costs, which differ ınder different methods of treatment and species, two cases nust be distinguished:—

ı. Calculation under the Supposition that the same Species and System of Management are retained after the present Crop has been Harvested.

(1.) The value of the forest is equal to the expectation value ɔf the soil plus the expectation value of the growing stock. Γhe greatest expectation value of the forest is obtained for that rotation, under which the expectation value of the soil culminates:—

$$ {}^mF_e = {}^mG_e + {}^rS_e $$

Let the age of the existing growing stock $= m$, then the expectation value of the forest is—

$$ {}^mF_e = \frac{Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} - ({}^rS_e + E)\,(1{\cdot}0p^{r-m} - 1)}{1{\cdot}0p^{r-m}} + {}^rS_e $$

$$ = \frac{Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} - E\,(1{\cdot}0p^{r-m} - 1) + {}^rS_e}{1{\cdot}0p^{r-m}} $$

By introducing the value of ${}^rS_e$—

$$ {}^rS_e = \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T'_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} - E, $$

the above formula becomes

$$ {}^mF_e = \frac{1{\cdot}0p^m\left(Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} + \frac{T_a}{1{\cdot}0p^a} + \ldots + \frac{T_m}{1{\cdot}0p_m} - c\right)}{1{\cdot}0p^r - 1} - E. $$

If $m = 0$, and cultivation has not yet taken place, the formula reduces to $F_e = {}^rS_e$.

(2.) The forest expectation value can also be calculated direct out of the expected receipts and expenses, in which case—

$$ {}^mF_e = \frac{Y_r}{1{\cdot}0p^{r-m}} + \frac{Y_r}{1{\cdot}0p^{2r-m}} + \ldots + \frac{T_n}{1{\cdot}0p^{n-m}} + \frac{T_n}{1{\cdot}0p^{r+n-m}} $$
$$ + \frac{T_n}{1{\cdot}0p^{2r+n-m}} + \ldots + \frac{T'_a}{1{\cdot}0p^{r-(m-a)}} + \frac{T'_a}{1{\cdot}0p^{2r-(m-a)}} $$
$$ + \ldots - \left(\frac{c}{1{\cdot}0p^{r-m}} + \frac{c}{1{\cdot}0p^{2r-m}} + \ldots\right) - E; $$

$$ {}^mF_e = \frac{Y_r \times 1{\cdot}0p^m}{1{\cdot}0p^r - 1} + \frac{T_n \times 1{\cdot}0p^{r+m-n}}{1{\cdot}0p^r - 1} + \ldots + \frac{T'_a \times 1{\cdot}0p^{m-a}}{1{\cdot}0p^r - 1} $$
$$ + \ldots - \frac{c \times 1{\cdot}0p^m}{1{\cdot}0p^r - 1} - E; $$

$$ {}^mF_e = \frac{1{\cdot}0p^m\left(Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + \frac{T'_a}{1{\cdot}0p^a} + \ldots - c\right)}{1{\cdot}0p^r - 1} - E, $$

as before.

Example.—Determine the expectation value of a forest now 45 years old, if that forest yields the returns given in the money yield table for the Scotch pine at page 120; rotation = 80 years; cost of formation = 60 shillings; per cent. = $2\frac{1}{2}$; and annual costs per acre = 3 shillings.

$$ {}^{45}F_e = \frac{1{\cdot}025^{45}\left(3{,}004 + 87 \times 1{\cdot}025^{30} + 118 \times 1{\cdot}025^{20} + 134 \times 1{\cdot}025^{10} + \frac{5}{1{\cdot}025^{80}} + \frac{44}{1{\cdot}025^{40}} - 60\right)}{1{\cdot}025^{80} - 1} - 120, $$

$$ {}^{45}F_e = 1{,}597 \text{ shillings} = £79\ 17s.\ 0d. $$

It was found before—

At page 125.	${}^{60}S_e$	= 404 shillings.		
,, 133.	${}^{46}G_e$	= 1,193	,,	
Total . .		= 1,597	,,	= £79 17s. 0d.

as above.

In the case of the present growing stock being abnormal, the corresponding values must be introduced into the account (see page 134).

b. Calculation under the Supposition that, after the cutting over of the present Crop, another Species or another Method of using the Soil is introduced.

In this case, the value S' of the soil, corresponding to the new conditions, must be introduced into the account; then the rotation r' must be determined, under which the expectation value of the present growing stock reaches its maximum.

The value of the forest is then represented by the formula—

$$ {}^{m}F'_e = \frac{Y_{r'} + T_{n'} \times 1{\cdot}0p^{r'-n} + \ldots - E\,(1{\cdot}0p^{r'-m} - 1) + S'r}{1{\cdot}0p^{r'-m}}. $$

2. Cost Value of a Forest.

a. The Cost value of a Forest is equal to the Cost Value of the Soil plus that of the Growing Stock.

(1.) For any soil value—

$$ {}^{m}F_c = S + (S+E)\,(1{\cdot}0p^{m}-1) + c \times 1{\cdot}0p^{m} - [T_a \times 1{\cdot}0p^{m-a} + \ldots] $$

$$ {}^{m}F_c = (S + E + c)\,1{\cdot}0p^{m} - [T_a \times 1{\cdot}0p^{m-a} + \ldots + E]. $$

(2.) By introducing the expectation value of the soil, the above becomes, for normal woods—

$$ {}^{m}F_c = \left(\frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots - c \times 1{\cdot}0p^{r}}{1{\cdot}0p^{r}-1} - E + E + c\right) 1{\cdot}0p^{m} $$

$$ -[T_a \times 1{\cdot}0p^{m-a} + \ldots + E] $$

$$ = \frac{1{\cdot}0p^{m}\left(Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + \dfrac{T_a}{1{\cdot}0p^{a}} + \ldots - c\right)}{1{\cdot}0p^{r}-1} - E. $$

This, it will be observed, is equal to the expectation value of the forest.

b. The Cost Value can be calculated direct out of the Expenses Incurred.

The method is similar to that followed in calculating the cost value of the growing stock, but the value of the soil is added instead of the rental only; hence—

$$
{}^{m}F_{c} = S \times 1{\cdot}0p^{m} + E(1{\cdot}0p^{m} - 1) + c \times 1{\cdot}0p^{m} - (T_{a} \times 1{\cdot}0p^{m-a} + \ldots)
$$
$$
= (S + E + c)\,1{\cdot}0p^{m} - [T_{a} \times 1{\cdot}0p^{m-a} + \ldots + E],
$$

as before.

3. Sale Value of a Forest.

By the sale value of a forest is understood the value at which it could be disposed of in the open market. It can be estimated according to prices realised for forests of a similar description; but as it is difficult to estimate existing differences, the method is of subordinate importance.

4. Rental Value of a Forest.

By the rental value of a forest is understood the capitalised rental, which it is capable of yielding. If the annually equal rental is $= R$, the rental value would be—

$$
\text{Rental value} = \frac{R}{{\cdot}0p}.
$$

This method is only applicable in the case of a forest which can be so managed that it yields an annually (or periodically) equal rental. The rental is represented by—

$$
Y_{r} + T_{a} + T_{b} + \ldots + T_{q} - (c + r \times e),
$$

and the rental value of the forest is—

$$
F = \frac{Y_{r} + T_{a} + T_{b} + \ldots + T - (c + r \times e)}{{\cdot}0p}.
$$

CHAPTER V.

DETERMINATION OF THE RENTAL OF FORESTS.

In order to convert any item of income, whether it occurs once or after stated intervals, into an annual rental, it is necessary to ascertain the capital value of the income and then to multiply it by $\cdot 0p$. For instance, the rental which corresponds to the thinning in the year a, and its recurrence every r years, is equal to—

$$\frac{T_a \times 1{\cdot}0p^{r-a}}{1{\cdot}0p^r - 1} \times \cdot 0p.$$

The annual payment corresponding to the periodic cost of cultivation is expressed by—

$$\frac{c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} \times \cdot 0p.$$

1. Rental of the Soil.

By the rental of the soil is understood the annual net return of the soil. It is represented by the difference between the rentals of incomes and the annual payment of expenses of a wood, hence :—

Soil rental R

$$= \frac{Y_r}{1{\cdot}0p^r - 1} \times \cdot 0p + \frac{T_a \times 1{\cdot}0p^{r-a}}{1{\cdot}0p^r - 1} \times \cdot 0p + \ldots + \frac{T'_q \times 1{\cdot}0p^{r-q}}{1{\cdot}0p^r - 1} \times \cdot 0p$$

$$- \left[\frac{c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} \times \cdot 0p + e\right]$$

$$= \left[\frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p_r}{1{\cdot}0p^r - 1} - \frac{e}{\cdot 0p}\right] \times \cdot 0p.$$

This rental, it will be observed, is the rental of the soil expectation value. Soil rental $= {}^rS_e \times \cdot 0p$.

2. Rental of the Forest.

The rental of the forest is equal to the net return yielded by the forest (soil plus growing stock). In the case of the annual working, the forest rental is equal to—

$$R_f = Y_r + T_a + \ldots + T_q - (c + r \times e).$$

CHAPTER VI.

THE FINANCIAL RESULTS OF FORESTRY.

THE subjects which will be treated in this chapter belong to that part of scientific forestry which is called, on the continent, "Forest Statics"; that is to say, the science which weighs and considers the comparative merits of the different methods of treatment to which forests may be subjected. The financial results, or the rent-yielding power, of an undertaking are expressed by the proportion existing between the yield and the capital which produces it. Hence, when several methods of treatment lead to the realisation of the otherwise desired object, it should always be ascertained which of them does so in the most profitable manner: in other words, which of them gives the highest rate of interest on the invested capital.

In the present instance, only the most necessary matters will be given, namely, the methods of calculating the financial results of forestry and their application to a few of the more important questions.

SECTION I.—THE METHODS OF CALCULATING THE FINANCIAL RESULTS OF FORESTRY.

The financial results of forestry can be determined in one of two ways:—

(1.) by ascertaining the "profit," that is to say, the surplus of receipts over costs of production, allowing compound interest at a certain rate on both;

(2.) by ascertaining the rate of interest yielded by the capital invested in forestry, here called the "forest per cent."

Each of these two methods must be explained in detail.

1. Determination of the Profit of Forestry.

a. Calculation for the Intermittent Working.

In the case of a single wood of approximately uniform age, the returns and costs do not occur at the same time, but at intervals of various duration; hence, they must be calculated for one and the same time. In the first place, that time shall be the commencement of the rotation when the area is about to be planted, or sown, for the production of a forest crop.

i. Calculation for a Blank Area.

Let as before—

Y_r be the final yield occurring in the year r and again in $2 \times r$, $3 \times r$, and so on for ever;

$T_a, T_b \ldots T_q$, the thinnings occurring in the years $a \ldots q$, and again in $a + r \ldots q + r$ years, again in $a + 2 \times r \ldots q + 2 \times r$ years, and so on;

S_c the cost value of soil;

E the capitalised annual expenses $= \frac{e}{\cdot 0p}$;

c the cost of formation expended now, and again every r years;

C the capitalised cost of formation $= \frac{c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1}$;

p the per cent. at which money can be made available for investment in forestry, and at which money taken out of the forest can be invested with equal security.

Then the present value of all returns is—

$$= \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q}}{1{\cdot}0p^r - 1}$$

while the present value of all costs comes to—

$$S_c + E + C.$$

Hence, the profit of forestry is expressed by the formula—

$$\textbf{Profit} = \mathbf{P} = \frac{\mathbf{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q}}}{\mathbf{1{\cdot}0p^r - 1}} - (\mathbf{S_c + E + C}).$$

This formula can also be written as follows:—

$$P=\left(\frac{Y_r+T_a\times 1{\cdot}0p^{r-a}+\ldots+T_q\times 1{\cdot}0p^{r-q}-c\times 1{\cdot}0p^r}{1{\cdot}0p^r-1}-E\right)-S_c.$$

It will be observed that the part in brackets represents tho expectation value of the soil; hence, the above formula reduces to—

$$\mathbf{P} = \mathbf{S_e} - \mathbf{S_c}.$$

In words: the profit in the case of a blank area is equal to the difference between the expectation and cost values of the soil. From this fact the following conclusions can be drawn:—

(1.) A profit is realised, if the soil had been acquired at a lower rate than that indicated by the expectation value of forest soil.

(2.) A profit is also realised, if, although $S_e = S_c$ at the outset, the soil expectation value is afterwards increased, either by higher returns or by smaller costs, or both; in other words, by improved and more economic management.

(3.) The greatest profit is obtained by tho adoption of the rotation, species, and method of treatment, which give the highest expectation value of the soil.

(4.) If the cost value of the soil is equal to the expectation value, tho profit is nil, and the capital invested in the forest yields exactly p per cent. If the cost value is greater than the expectation value, the forest industry involves a financial loss; in that case it is more profitable to take the capital out of the forest and invest it otherwise, as long as in this way p per cent. can be obtained with equal security.

ii. Calculation of the Profit for a Wood now m years old.

It is assumed that the returns and costs are of the normal amount.

The receipts consist of—

(*a.*) All items of income realised between the formation of the wood and the year m, with accumulated interest to the year m :—

$$T_a \times 1{\cdot}0p^{m-a} + T_b \times 1{\cdot}0p^{m-b} + \ldots T_m.$$

(*b.*) All items of income to be realised between the year m and the end of the first rotation r, discounted to the year m :—

$$\frac{T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} + Y_r}{1{\cdot}0p^{r-m}}.$$

(*c.*) All items of income to be realised during subsequent rotations, amounting to—

$$\frac{T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} + Y_r}{(1{\cdot}0p^r - 1) \times 1{\cdot}0p^{r-m}}.$$

The costs are—

(*a.*) Past costs with compound interest to the year m :—

$$S_c \times 1{\cdot}0p^m + E\,(1{\cdot}0p^m - 1) + c \times 1{\cdot}0p^m.$$

(*b.*) Costs to be incurred from the year m to the end of the first rotation :—

$$\frac{E\,(1{\cdot}0p^{r-m} - 1)}{1{\cdot}0p^{r-m}}.$$

(*c.*) Costs to be incurred during future rotations :—

$$\frac{E}{1{\cdot}0p^{r-m}} + \frac{c \times 1{\cdot}0p^r}{(1{\cdot}0p^r - 1) \times 1{\cdot}0p^{r-m}}.$$

By deducting all costs from the receipts, the profit is obtained and represented by the following formula :—

Profit $P =$

$$\frac{Y_r + T_n \times 1{\cdot}0p^{r-n} + \ldots + T_q \times 1{\cdot}0p^{r-q} - E\,(1{\cdot}0p^{r-m} - 1)}{1{\cdot}0p^{r-m}} \quad \text{(I.}$$

$$\frac{1}{\cdot 0p^{r-m}}\left(\frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} - E\right) \quad \text{(II.}$$

$$(S_c + E)\,(1{\cdot}0p^m - 1) + c \times 1{\cdot}0p^m - T_a \times 1{\cdot}0p^{m-a} - \ldots - T_m] - S_c \quad \text{(III.}$$

In this formula, part (II.) represents the expectation value of the soil discounted for $r - m$ years, and the bracketed part of expression (III.) represents the cost value of the m years old growing stock; hence, the above formula can be written as follows :—

$$P = \frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} + {}^rS_e - E\,(1{\cdot}0p^{r-m} - 1)}{1{\cdot}0p^{r-m}}$$
$$- (S_c + {}^mG_c).$$

Here again, the positive part represents the expectation value of the forest (see page 144), and the negative part the cost value of the forest; hence—

$$\text{Profit } \mathbf{P} = {}^m\mathbf{F}_e - {}^m\mathbf{F}_c = (\mathbf{S}_e - \mathbf{S}_c) \times 1{\cdot}0\mathbf{p}^m.$$

It will be seen that this formula agrees with that given for a blank, because for the year 0 the expectation value of the forest is equal to the expectation value of the soil, and the same holds good for the cost values.

The conclusions drawn on page 151, headings (1.) to (4.), with regard to the formula $P = S_e - S_c$, also hold good with regard to the formula $P = {}^mF_e - {}^mF_c$.

b. Calculation for the Annual Working.

In this case, the returns and costs occur regularly every year. If they are of annually equal amounts, the returns are $= Y_r + T_a + \ldots\ldots + T_q$.

The costs consist every year of—

(1.) Interest on the value of the soil . $= r \times S_c \times {\cdot}0p$;

(2.) Interest on the value of the normal growing stock . . . . $= {}^nG_c \times {\cdot}0p$;

(3.) The annually recurring cost of administration, protection, taxes, etc. $= r \times e$;

(4.) The cost of formation . . . $= c$;

hence, the annual profit—

$$P = Y_r + T_a + \ldots + T_q - c - r \times e - [r \times S_c + {}^nG_c] \times {\cdot}0p;$$

or,

$$\frac{\text{Annual } P}{{\cdot}0p} = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{{\cdot}0p} - (r \times S_c + {}^nG_c).$$

Now:—

$$\frac{\text{Annual } P}{\cdot 0p} = \text{capitalised value of annual profit} = \text{total profit};$$

$$\frac{Y_r + T_a + \ldots + T_q - c - r \times e}{\cdot 0p} = \text{the expectation value}$$

of the forest under the annual working $= F_e$;

$$r \times S_c + {}^{n}G_c = \text{cost value of forest} = F_c;$$

hence, total or capital value of profit of the whole forest:—

$$\textbf{Total P} = \mathbf{F_e} - \mathbf{F_c}.$$

In words, the profit is equal to the difference between the expectation and cost values of a forest. For the rest, what has been said as regards the intermittent working holds also good in respect of the annual working, and *vice versâ*, because a series of age gradations may be considered as so many separate woods of various ages; the profit calculated for the series as a whole must be equal to that obtained by adding together the profits derived from the several age gradations, each calculated for itself.

Examples.—Given the data as before, that is to say, the returns to be those of the yield table on p. 120,

$p = 2\frac{1}{2}$ per cent.; $e = 3$ shillings; $c = 60$ shillings; $r = 80$ years.

(1.) Calculate the profit per blank acre, if the soil was bought for 300*s.*—

From previous examples it is known that $S_e = 404s.$; hence—

$$\text{Profit} = 404 - 300 = 104s.$$

(2.) Calculate the profit of a wood 45 years old under the same conditions.

It was found at page 145, that ${}^{45}F_e = 1{,}597s.$, while

$${}^{45}F_c = (300 + 120 + 60) \times 1{\cdot}025^{45} - [5 \times 1{\cdot}025^{15} + 44 \times 1{\cdot}025^{5} + 120]$$

${}^{45}F_c = 1{,}281$ shillings; hence—

$$P = {}^{45}F_e - {}^{45}F_c = 1{,}597 - 1{,}281 = 316 \text{ shillings}.$$

But also $P = (S_e - S_c)\, 1{\cdot}0p^{45} = 104 \times 1{\cdot}025^{45} = 316$ shillings.

(3.) Calculate the profit of a normal series of age gradations, that is to say, for the strictly annual working.

$$\text{Total Profit} = {}^{normal}F_e - {}^{normal}F_c.$$

Now, ${}^{normal}F_e = S_e + {}^{normal}G_e = 80 \times 404 + 91{,}360$ (see page 141).

$${}^{normal}F_e = 123{,}680 \text{ shillings},$$

while

$$^{normal}F_c = S_c + {}^{normal}G^c ;$$

$$S_c = 80 \times 300 = 24{,}000 \text{ shillings.}$$

$$^{normal}G_c = \Big[(300 + 120 + 60)(1{\cdot}025^{60} - 1) - \Big\{ 5 \times (1{\cdot}025^{50} - 1)$$
$$+ 44 (1{\cdot}025^{40} - 1) + 87 (1{\cdot}025^{30} - 1) + 118 (1{\cdot}025^{20} - 1)$$
$$+ 134 (1{\cdot}025^{10} - 1) \Big\} \Big] \div {\cdot}025 - 80 (300 + 120);$$

$$^{normal}G_c = 73{,}836 \text{ shillings,}$$

hence—

$$^{normal}F_c = 24{,}000 + 73{,}836 = 97{,}836 \text{ shillings,}$$

and

$$\text{Profit} = 123{,}680 - 97{,}836 = 25{,}844 \text{ shillings.}$$

2. Determination of the Rate of Interest yielded by the Capital invested in Forestry, called the Forest per cent.

a. Calculation for the Intermittent Working.

Under tho intermittent working, the rate of interest changes from year to year, because the capital as well as the increment change with the advancing age of the wood; hence, it is necessary to distinguish between the current annual and the mean (or average) annual rate of interest.

i. Current Annual Rate of Interest.

As indicated on page 113, the rate of interest is expressed by the fraction

$$\text{Rate of interest} = \frac{I}{C}.$$

where C represents the capital which has produced the amount I in the shapo of interest. The per cent. is then

$$\text{per cent. } p = \frac{I}{C} \times 100.$$

This formula is used in the case of forestry, and it remains only to introduce the proper amounts for I and C. In other words, the current annual rate of interest is equal to the net annual value increment of a wood divided by the cost value of the wood at the commencement of the year in

question. The current annual per cent. is equal to that quotient multiplied by 100.

Let—

Y_m be the utilisation value of a wood at the end of the year m,

Y_{m+1} „ „ „ „ „ „ „ „ $m+1$,

then $Y_{m+1} - Y_m$ represents the increase in value which has been produced during the year $m + 1$. Deducting from this c, the annual costs, the net increase in value is equal to—

$$Y_{m+1} - Y_m - c.$$

The cost value of the wood in the year m (or beginning of the year $m + 1$) is represented by—

(*a*.) The value of the soil with compound interest to the year $m = S_e \times 1{\cdot}0p^m$.

(*b*.) The cost of formation with compound interest to the year $m = c \times 1{\cdot}0p^m$.

(*c*.) The value of the annual costs during m years with compound interest calculated for the end of the year $m = \frac{e\,(1{\cdot}0p^m - 1)}{{\cdot}0p} = E\,(1{\cdot}0p^m - 1)$.

(*d*.) From these amounts must be deducted all returns realised between the formation of the wood and the year $m =$

$$T_a \times 1{\cdot}0p^{m-a} + T_b \times 1{\cdot}0p^{m-b} + \ldots + T_l \times 1{\cdot}0p^{m-l} + T_m.$$

The current forest per cent. ${}^{curr.}p_f$, with which the invested capital has worked during the year $m + 1$, is, therefore, expressed by the formula—

$${}^{curr.}p_f = \frac{(Y_{m+1} - Y_m - e)\,100}{(S_e + E + c) \times 1{\cdot}0p^m - (T_a \times 1{\cdot}0p^{m-a} + T_b \times 1{\cdot}0p^{m-b} + \ldots + T_m + E)}.$$

The denominator in the above formula can also be written thus:—

$$(S_e + E)\,(1{\cdot}0p^m - 1) + c \times 1{\cdot}0p^m - (T_a \times 1{\cdot}0p^{m-a} + \ldots + T_m) + S_e.$$

On reference to page 136, it will be seen that this expression represents the cost value of the growing stock

immediately after the thinning in the year *m* has been made, *plus* the cost value of the soil; hence, the formula for the forest per cent. reduces to—

$$^{\text{curr.}}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100^*}{S_c + {}^mG_c} = \frac{(Y_{m+1} - Y_m - e) \times 100}{{}^mF_c}.$$

By substituting the utilisation value of the growing stock for its cost value, the formula becomes—

$$^{\text{curr.}}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{S_c + Y_m}.$$

The latter formula is used, when *m* is anywhere near the end of the rotation.

Example.—Let $S = 404$ shillings; $e = 3$ shillings; $p = 2\frac{1}{2}$ per cent.; $c = 60$ shillings; other data those on page 120. Then:—

$$^{50-51}p_f = \frac{(1,055 - 985 - 3)\,100}{404 + 985} = 4{\cdot}8 \text{ per cent.}$$

$$^{60-61}p_f = \frac{(1,639 - 1,563 - 3)\,100}{404 + 1,563} = 3{\cdot}7 \quad ,,$$

$$^{70-71}p_f = \frac{(2,269 - 2,187 - 3)\,100}{404 + 2,187} = 3{\cdot}0 \quad ,,$$

$$^{80-81}p_f = \frac{(2,944 - 2,860 - 3)\,100}{404 + 2,860} = 2{\cdot}5 \quad ,,$$

$$^{90-91}p_f = \frac{(3,646 - 3,558 - 3)\,100}{404 + 3,558} = 2{\cdot}1 \quad ,,$$

It will be observed that the forest per cent., from the 50th year onwards, is at first large but falls with the advancing age of the wood. At the year 80—81 it is equal to the general per cent. $p = 2\frac{1}{2}\,\%$; after that time it is smaller than $2\frac{1}{2}\,\%$. Hence, the current annual forest per cent. is utilised to ascertain the financial ripeness of a wood.

ii. Mean Annual Rate of Interest.

The mean (or average) rate of interest is ascertained by converting all net returns into an equal annual rental and

* This formula differs from that usually given in Continental works, which is as follows:—

$$^{\text{curr.}}p_f = \frac{(Y_{m+1} - Y_m)\,100}{S + mG_c + E}.$$

It is easy to show that this formula is correct only for the year in which $^{\text{curr.}}p_f$ = general per cent. p.

dividing it by the producing capital. The quotient multiplied by 100 gives the mean annual forest por cent.

The best time for making the calculation is the commencement of the rotation. At that time, the annual net rental is represented by the expression—

$$\left(\frac{Y_r+T_a\times1{\cdot}0p^{r-a}+\ldots+T_q\times1{\cdot}0p^{r-q}-c\times1{\cdot}0p^r}{1{\cdot}0p^r-1}-E\right)\times{\cdot}0p.$$

The producing capital at the commencement of the rotation is equal to the cost value of the soil $= S_c$. Hence, the mean annual forest per cent. under the intermittent working is—

$$^{mean}p_f = \frac{\left(\frac{Y_r+T_a\times1{\cdot}0p^{r-a}+\ldots+T_q\times1{\cdot}0p^{r-q}-c\times1{\cdot}0p^r}{1{\cdot}0p^r-1}-E\right)\times{\cdot}0p}{S_c}\times100.$$

$$^{mean}p_f = \frac{S_e}{S_c}\times p.$$

$$\text{If } S_e \gtreqless S_c, \text{ then } {}^{mean}p_f \gtreqless p.$$

If the expectation value of the soil is equal to the cost value, then the mean annual forest per cent. is equal to the general per cent p, which proves the correctness of the above formula.

The highest mean annual forest per cent. is obtained under that rotation for which the expectation value of the soil culminates; it is then equal to the current annual forest per cent.

Example.—Given the same conditions as before—

$$S_e = 404s.$$

If the cost value of the soil is also 404*s.*, then

$$^{mean}p_f = \frac{404}{404}\times2{\cdot}5 = 2{\cdot}5 \text{ per cent.},$$

equal to the general per cent. p, with which the value of S_e has been calculated.

If $S_c = 300$ shillings, then

$$^{mean}p_f = \frac{404}{300} \times 2{\cdot}5 = 3{\cdot}4 \text{ per cent.}$$

If $S_c = 500$ shillings, then

$$^{mean}p_f = \frac{404}{500} \times 2{\cdot}5 = 2{\cdot}0 \text{ per cent.}$$

In the latter case, money is lost; hence, the capital had better be taken out of the forest, and invested elsewhere, if at least $2\frac{1}{2}$ per cent. can be obtained with equal security.

b. Calculation for the Annual Working.

Under the annual working, with equal annual increment, yield, and costs, the current annual forest per cent. is equal to the mean annual forest per cent. In this case, the annual net return of r units of area amounts to—

$$Y_r + T_a + \ldots + T_q - c - r + e,$$

and the producing capital to—

$$r \times S_c + {}^{norm.}G_c \text{ of } r \text{ age gradations;}$$

hence—

$$^{mean}p_f = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{r \times S_c + {}^{n}G_c} \times 100.$$

$$^{mean}p_f = \frac{Y_r + T_a + \ldots + T_q - c - r \times e}{F_c} \times 100.$$

By multiplying and dividing the enumerator in this formula by $\cdot 0p$, the following is obtained:—

$$^{mean}p_f = \frac{\left(\dfrac{Y_r + T_a + \ldots + T_q - c - r \times e}{\cdot 0p}\right) \times 100 \times \cdot 0p}{F_c};$$

or, as the part in brackets is equal to the expectation value of the forest $= F_e$:—

$$^{mean}p_f = \frac{F_e}{F_c} \times p.$$

This formula is identical with that obtained above for the profit $= F_e - F_c$:—

$$\text{If } F_e \gtreqless F_c, \text{ then } {}^{mean}p_f \gtreqless p.$$

The highest mean annual forest per cent. under the annual working is obtained for that rotation under which the expectation value of the forest reaches its maximum; in the above example about 80 years.

SECTION II.—THE FINANCIAL TEST APPLIED TO THE METHOD OF TREATMENT.

If the profitableness of the method of treatment is to be tested, or if several methods are to be compared, it must be assumed that in each case those conditions exist which render the method in itself as profitable as possible. In that case it may be said that the most advantageous method of treatment from a financial point of view is that which yields the highest profit, or the highest mean annual forest per cent., provided, in the latter case, that the capital is the same under each method. If the capitals differ, the following cases must be distinguished:

(1.) The method employing the greater capital is the more profitable, if it gives the higher forest per cent.

(2.) The one with the smaller capital is the more profitable, if it yields an equal or a greater amount of interest. If it yields less interest and yet a higher forest per cent., it cannot be decided off-hand, whether it is the more profitable or not, as the total profit depends on two factors, namely the rate of interest and the amount of the invested capital; hence, it is necessary to calculate the actual amount of profit for each case, and to compare the one with the other.

The above-mentioned tests may be applied to all questions connected with forest management. Of these the following are the most prominent:—

1. Choice between Forestry and Agriculture as regards a Piece of Land.

Of these two methods of using the land, that is the more profitable which yields the highest net rental of the soil.

Rental of soil under forest =

$$\left[\frac{Y_r + T_a \times 1{\cdot}0p^{r-a} + \ldots + T_q \times 1{\cdot}0p^{r-q} - c \times 1{\cdot}0p^r}{1{\cdot}0p^r - 1} - E\right] \times {\cdot}0p.$$

Rental of soil under agriculture = net letting value.

Example.—Is it more profitable to use an acre of land for agriculture or forestry under the following conditions?

	£ s. d.	£ s. d.
Annual letting value under agriculture .		0 15 0
Cost of administration = 5 per cent . .	0 0 9	
Rates and taxes	0 2 0	
		0 2 9
Annual net return per acre		£0 12 3

Expenses under forestry:—

Cost of planting with larch	4 10 0	
Fencing	1 5 0	
	£5 15 0	= 115s.
Cost of administration per acre	0 3 0	
Rates and taxes	0 2 0	
		= 5s.
Underplanting with beech at the age of 20 years		= 60s.

Expected returns:—

Thinning at the age of 20 years, value	20s.
,, ,, ,, 30 ,, ,, ...	150s.
,, ,, ,, 40 ,, ,, ...	200s.
,, ,, ,, 55 ,, ,, ...	250s.
Final yield ,, ,, 70 ,, (4,000c′.) . . .	4,000s.

Shooting rent from the 10th to 70th year = 3s. annually.
Interest to be at the rate of 3 per cent.

The expectation value of the soil under forest will be:—

$$S = \frac{\begin{array}{c}4{,}000 + 20 \times 1{\cdot}03^{50} + 150 \times 1{\cdot}03^{40} + 200 \times 1{\cdot}03^{30} + 250 \times 1{\cdot}03^{15} \\ + \frac{3\,(1{\cdot}03^{60} - 1)}{{\cdot}03} - 115 \times 1{\cdot}03^{70} - 60 \times 1{\cdot}03^{50}\end{array}}{1{\cdot}03^{70} - 1} - \frac{5}{{\cdot}03}$$

Using the tables in Appendix II., the following equation is obtained:—

$$S_e = \left\{\begin{array}{l}4{,}000 + 20 \times 4{\cdot}3839 + 150 \times 3{\cdot}262 + 200 \times 2{\cdot}4273 + \\ 250 \times 1{\cdot}558 + 100 \times 4{\cdot}8916 - 115 \times 7{\cdot}9178 - 60 \times 4{\cdot}3839\end{array}\right\} \times {\cdot}1446$$
$$- 166{\cdot}6667$$

$$S_e = 4767{\cdot}517 \times {\cdot}1446 - 166{\cdot}6667$$
$$S_e = 522{\cdot}7163 \text{ shillings.}$$

Net annual rental =

$$S_e \times \cdot 03 = 522 \cdot 7163 \times \cdot 03 = 15 \cdot 6815s. = £0\ 15s.\ 8d.,$$

which compares with £0 12*s*. 3*d*. under agriculture.

2. Choice of Species and Silvicultural System.

In both cases, the choice is determined by the amount of the expectation value of the soil for various species or systems. It depends chiefly on—

(*a*.) The value of the returns and the time when they occur.

(*b*.) The cost of formation and the amount of the annual expenses, if they differ for different species or different silvicultural systems.

(*c*.) The rate of interest.

Example.—It has been shown on page 125 that an acre of land put under Scotch pine, which gives tho return in the tablo on page 120, shows the value of $S_e = 404s$. Tho same acro put undor beech, which yields the returns in tho tablo on page 121, gives the valuo of $S_e = 366s$. Honce, it is more profitable, under the given conditions, to plant Scotch pine than beech.

3. Choice of Method of Formation.

The choice depends on the amount of the expectation value of the soil under the various methods of formation, such as planting, sowing, or natural regeneration.

Example.—An acre of land planted with Scotch pine, and yielding the returns in the table on page 120, shows a soil expectation value of 404*s*. (see page 125). Assuming that the land is naturally regenerated, and that some assistance would have to be given by planting up blanks, or by sowing seed, 5 years after the commencement of the natural regeneration, at a cost of 20 shillings an acre, and that, owing to the slow development of tho plants while young, the rotation would have to be raised from 80 to 90 years, thereby delaying all returns by 10 years. Will it be more profitable to plant, or to regenerate naturally?

Under the system of natural regeneration,

$$S_e = \frac{3{,}004 + 5 \times 1 \cdot 025^{50} + 44 \times 1 \cdot 025^{40} + 87 \times 1 \cdot 025^{30} + 118 \times 1 \cdot 025^{20} + 134 \times 1 \cdot 025^{10} - 20 \times 1 \cdot 025^{85}}{1 \cdot 025^{90} - 1} - \frac{3}{\cdot 025}$$

This gives $S_e = 305$ shillings. It is more profitabls to plant than to regenerate naturally.

4. Choice of Method of Tending, especially in respect of the Time and Strength of Thinnings.

This is best effected by calculating the expectation value of the growing stock for the different methods of thinning, and comparing them.

Example.—Is it more profitable to thin moderately and make for a heavy final crop, or to thin more heavily and be satisfied with a smaller final crop? Let the value of the returns in the two cases be as follows:—

Age.	Light Thinning.		Heavy Thinning.	
30	Thinning value,	50*s.*	Thinning value,	100*s.*
40	" "	100*s.*	" "	150*s.*
50	" "	150*s.*	" "	200*s.*
60	" "	200*s.*	" "	300*s.*
70	" "	250*s.*	" "	500*s.*
80	Final yield .	4,000*s.*	Final yield .	3,000*s.*

Under the system of light thinnings, the value of the returns calculated for the end of the rotation comes, with 3 per cent. interest, to:—

$$= 50 \times 1{\cdot}03^{50} + 100 \times 1{\cdot}03^{40} + 150 \times 1{\cdot}03^{30} + 200 \times 1{\cdot}03^{20} + 250 \times 1{\cdot}03^{10} + 4{,}000 = 5{,}607s.$$

Under the system of heavy thinnings:—

$$= 100 \times 1{\cdot}03^{50} + 150 \times 1{\cdot}03^{40} + 200 \times 1{\cdot}03^{30} + 300 \times 1{\cdot}03^{20} + 500 \times 1{\cdot}03^{10} + 3{,}000 = 5{,}627s.$$

It is slightly more profitable to thin heavily, provided that the final crop fetches the same price per cubic foot in both cases. If the final crop under the system of heavy thinnings brings less per cubic foot than under that of light thinnings, the latter will be the more profitable.

5. Determination of the Financial Ripeness of a Wood.

The financial ripeness of a wood, say m years old, is gauged by means of the current annual forest per cent. As long as—

$$^{curr.}p_f = \frac{(Y_{m+1} - Y_m - e) \times 100}{S_e + Y_m}$$

is greater than the general per cent. p, the wood is not ripe. If $^{curr}p_f = p$, then the wood is just ripe. If $^{curr.}p_f < p$, then the wood is past ripeness, and, if left standing, a financial loss is incurred.

6. Determination of the Financial Rotation.

The financial rotation, or that which gives the best financial results, may be determined in various ways, as:—

(*a.*) The rotation which yields the highest expectation value of the soil, or the highest soil rental.

(*b.*) The rotation which, taking a certain value of the soil, gives the largest profit or the highest mean annual forest per cent.

Subjects 5 and 6 will be more fully dealt with in Part III.

PART I.

FOREST MENSURATION.

FOREST MENSURATION.

Forest Mensuration deals with the determination of the dimensions, volume, age, and increment of single trees and whole woods.

These determinations are required for the calculation of the material standing on a given area, the yield which a wood can give, and the value of single trees, whole woods, and forests. They serve also as the basis for the calculation of the effects of different methods of treatment.

As a rule, the units of measurement employed in Britain and India are the foot, square foot, and cubic foot.

The subject has been divided into the following chapters :—

CHAPTER I.

INSTRUMENTS USED IN FOREST MENSURATION.

Instruments are required to measure the circumference or diameter of logs and trees, the length of logs, the height of trees, and the increment. Such measurements have for their object either to ascertain the various dimensions, or to calculate from them the volume; in the latter case, the measurement of the girth or diameter is used to calculate the area of the cross-cut section, on the assumption that it forms a circle. This is generally called the sectional or basal area.

The instruments may be classified as follows:—

1. Instruments for the Measurement of the Girth.

The girth may be measured with a tape, or with a string and tape.

The tape consists of a band, about half an inch in breadth, so constructed that it alters its length as little as possible when moist. It is divided on one side into feet, inches, and, if necessary, decimals of inches; on the other side, the sectional areas corresponding to the length of girth are sometimes noted. It is useful to have a small hook on one end which can be pressed into the bark when the girth exceeds 5 feet. Long tapes are rolled up in casss which are made of leather, wood, or metal. Of late years, flexible steel tapes have come much into use.

The advantages of the tape are that it is easy to handle and convenient to carry. Measurements with it are, however, subject to various sources of inaccuracy amongst which the following deserve to be mentioned:—

(*a*.) The sections of most trees are not circles.

(*b.*) Owing to the presence of a rough bark, the measured girth is too large.

(*c.*) Irregularities in the tree are difficult to avoid.

(*d.*) The tape is frequently not applied at right angles to the axis of the tree.

In order to avoid some of the disadvantages of tape measurements, a thin string is sometimes used which is then held parallel to a graduated tape or rule. In this way, more

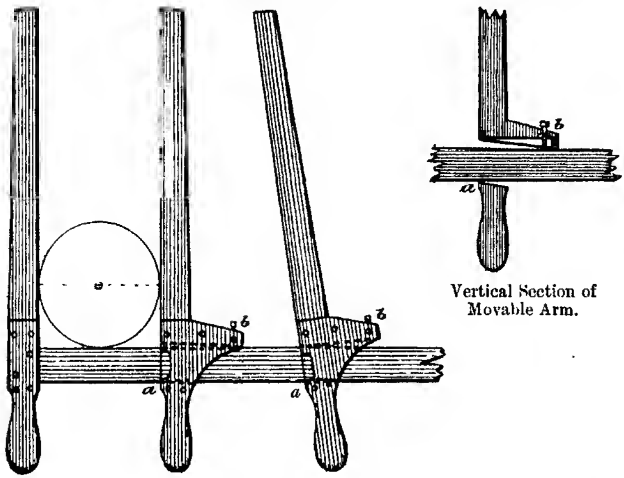


Fig. 1.—Friedrich's Calliper.

accurate results may be obtained, but the procedure takes more time, and is therefore not employed where large numbers of trees have to be measured.

2. Instruments for the Measurement of the Diameter.

The diameter of cross-sections of trees is measured with an ordinary rule or a tape; in all other cases, the calliper is used, or sometimes the tree compass.

a. The Calliper, or Diameter Gauge.

It consists of a graduated rule and two arms. Of the latter, one is fixed at one end at right angles to the rule, so that

its inner plane lies in the starting point of the graduated scale; the other arm moves along the rule, parallel to the fixed arm.

In using the calliper, the tree is brought between the two arms until it touches the rule, then the fixed arm is pressed against the tree on one side and the movable arm shifted until it touches the tree on the other side. The diameter can then be read off on the rule (Fig. 1).

The length of the rule and of the arms depends on the size

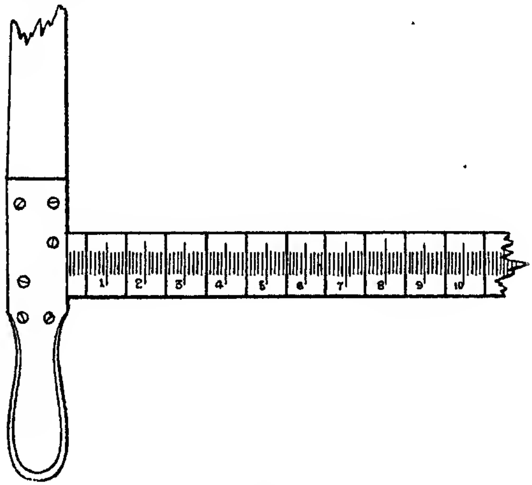


Fig. 2.

of the trees to be measured; each arm should be at least half the length of the rule. Callipers exceeding 4 feet in length are rarely used. The rule is divided into units which depend on the desired degree of accuracy. Ordinarily, they will be inches or two inches; in some cases half inches, and for very accurate measurements decimals of inches.

Where large numbers of trees are to be measured, it is desirable to round off the limits of each unit; for instance, if the rule is divided into intervals of inches, the first division line is placed at $\frac{1}{2}$ inch from zero, the second at $1\frac{1}{2}$, the third

at 2½, and so on (Fig. 2). In this way, all trees measuring from ½ to 1½ inches are recorded as having a diameter of 1 inch, those from 1½ to 2½ inches as 2 inches, and so on.

A good calliper must fulfil the following conditions:—

(1.) It must be sufficiently light so as not to fatigue the labourer, and yet sufficiently strong to resist the wear and tear to which it is likely to be subjected.

(2.) The two arms must be at right angles to the rule, or at least parallel to each other when pressed on to the tree.

(3.) The movable arm must move with sufficient ease along the rule.

Callipers of iron would be too heavy and two cold in winter. The former objection has been removed by making the instrument of aluminium. However, up-to-date callipers are generally made of wood. As wood alters with the degree of humidity, the movable arm is liable to jam at one time, or to move too easily at others. To avoid this drawback, various constructions have been adopted, resulting in a number of callipers of which the following two deserve to be specially mentioned:—

Gustav Heyer's Calliper.—The distinguishing feature of this instrument is that the rule is given, in section, the shape of a trapezium, and that it is pressed up or down in the movable arm by means of a wedge, so as to counteract the swelling or shrinking of the wood. In Fig 3, *a* represents the cross-section of the rule, *b* the wedge, and *c* the section of the movable arm. The wedge is fastened

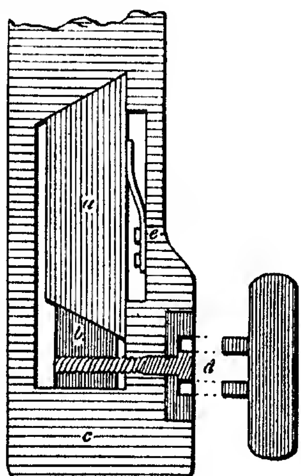


Fig. 3.—Heyer's Calliper.

to a screw which can be moved by a key at *d*. On moving the wedge from left to right, it presses the rule upwards and thus tightens it; on moving the wedge from right to left, it releases the rule, and enables it to move more freely. To force the rule to follow the backward movement of the wedge, a spring is fastened at *e* which pushes it from right to left, so that it is always kept in touch with the wedge.

Friedrich's Calliper.—In this instrument the section of the rule has the shape of a rectangle, while the opening of the movable arm is larger than the section of the rule, and placed slanting towards it. At the same time, it is so shaped that, on being pressed against the tree, it assumes a position which is at right angles to the rule (Fig. 1). In this position the arm rests on the two points, *a* (below) and *b* (above). As these points are liable to wear away, thus causing the arm to assume a position which is no longer at right angles to the rule, Böhmerle has added a spring at *b* which can be moved by a screw, until the true position of the arm is established.

b. Accuracy of Measurements with the Calliper.

To ensure the greatest possible accuracy, the following precautions must be taken :—

(1.) Moss, creepers, etc., found on the tree must be removed before measurement.

(2.) In the case of an abnormal swelling or indenture, the measurement must be taken above or below it, or both and the average taken.

(3.) In the case of eccentric or elliptic trees, two diameters at right angles to each other must be measured and the mean taken.

(4.) The height fixed for the measurement must be strictly adhered to.

(5.) In the case of a tree which is divided into two or more limbs below the fixed height of measurement, each limb must be measured and recorded as a separate tree.

(6.) The calliper must be placed at right angles to the axis of the tree, and the rule must touch the tree.

(7.) The reading must be taken while the calliper rests on the tree, and not after it has been withdrawn.

c. The Tree Compass.

The shape of this instrument will be understood on reference to Fig. 4. The diameter of the tree or log is taken by the two points c and d, while it can be read off at h at the arc $f g$.

In order to produce sufficient stiffness in the arms of the compass, they have to be made of metal which makes the instrument very heavy and unsuited for continued use.

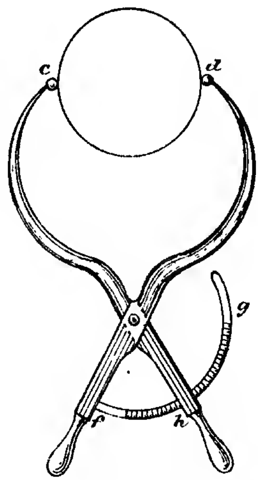


Fig. 4.—The Tree Compass.

d. Dendrometers.

In some cases, certain dendrometers are used to measure the diameter of trees at some height from the ground. The theory is this:—

The angle which is formed by two rays running to the two sides of the tree is measured, as well as the distance of the eye of the observer from the tree. From these data the diameter is calculated. Instead of the angle, the distance $a\ b$ between the two lines of sight can be measured, in which case the diameter is obtained in the following way (Fig. 5 on next page):—

$$C A : C a = A B : a b,$$

and

$$A B = \frac{C A}{C a} \times a b.$$

If, therefore, the instrument gives $a\ b$ and $C\ a$, and the distance $C\ A$ has been measured, the diameter can be calculated.

So far, instruments of this class have not obtained a footing in practice, because those at present available either do not work with sufficient accuracy, or take too much time.

3. Instruments for the Measurements of the Diameter Increment.

The diameter increment of prepared sections is measured with an ordinary rule, or with a pair of compasses and a rule.

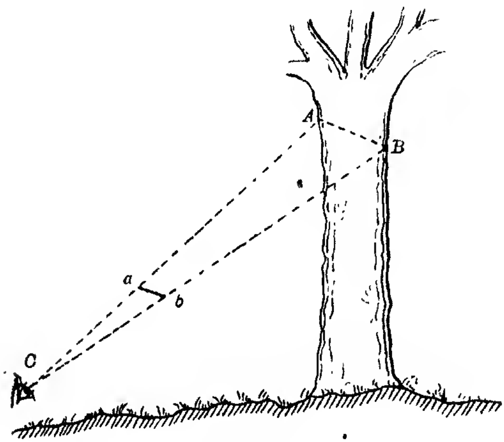


Fig. 5.

Such rules are made of metal or wood, and are sufficiently sub-divided.

If no section is available, as in the case of standing trees, the measurements are made with Pressler's Increment Borer (Fig. 6). This instrument extracts a cylinder of wood from the stem, and it consists of the following parts:—

(*a.*) A hollow borer, *A*, which is slightly conical from the handle towards the point.

(*b.*) A handle, *B*, which serves as a lever when the instrument is in use. It is hollow and receives the borer, wedge, and cradle, when the instrument is not in use (see *E* in figure).

(*c.*) A wedge, *C*, which has a scale marked on one side wherewith to measure the breadth of the concentric rings; it is roughly toothed on the other side to assist in extracting the cylinder of wood.

(*d.*) A cradle *D*, into which the cylinder of wood is placed after extraction, to prevent its breaking.

The borer is used in the following way :—

It is screwed in a radial direction into the tree, at right angles to its axis, to the desired depth, whereby a cylindrical column of wood enters the hollow borer, and is severed from

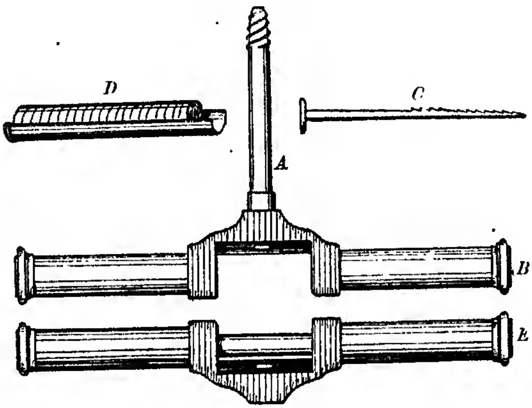


Fig. 6.—Pressler's Increment Borer.

the tree except at its base; then the wedge is inserted between the column of wood and the inner wall of the borer, with its toothed side towards the former, and firmly pressed in. This prevents the cylinder from turning round inside the borer during the following operation. The borer is then screwed backward one or two turns, whereby the base of the cylinder of wood is severed from the tree. The borer is then screwed further in which causes the severed cylinder of wood to be pushed back until it can easily be withdrawn and placed into the cradle. In this way, a column of wood is obtained of about ·2 inch diameter and from 2 to 6 inches long

according to the length of the borer. The breadth of the concentric rings is then measured. If the rings are not distinct, a smooth surface may be prepared with a sharp knife.

4. Instruments for the Measurement of the Length of Felled Trees and Logs.

The length of felled trees and logs is measured with a tape or measuring staff. The former has already been described. The staff varies in length up to about 15 feet; it should be made of hard, straight-grained, well-seasoned wood, and well varnished to protect it against moisture. The ends may usefully be capped with metal plates.

5. Instruments for the Measurement of the Height of Standing Trees.

The instruments which have been designed for measuring the height of standing trees are very numerous, but they are all based upon one of two principles: either they determine the height by means of similar triangles (geometrical height measuring), or they serve to measure the angles of elevation and depression (trigonometrical height measuring).

a. Geometrical Height Measuring.

If a horizontal plane is drawn from the eye of the observer to a tree, it will hit the same, according to the position of the observer, either between the top and the foot of the tree, thus dividing it into two parts, one of which is situated above and the other below the horizontal plane; or above the top; or below the foot. If the observer holds a plumb-line at some distance from his eye, it may be considered parallel to the axis of the tree; hence, by looking at the top and foot of the tree, similar triangles are formed which are used for the determination of the height of the tree.

Let $A\ B$ (Fig. 7) be the height of the tree.

$E\ B$, a ray from the eye of the observer to the top of the tree.

$E\,A$, a ray from the eye of the observer to the foot of the tree.

$E\,C$, a horizontal line.

a, b, and c, the points where the three rays hit the plumb-line. Then the height is determined as follows:—

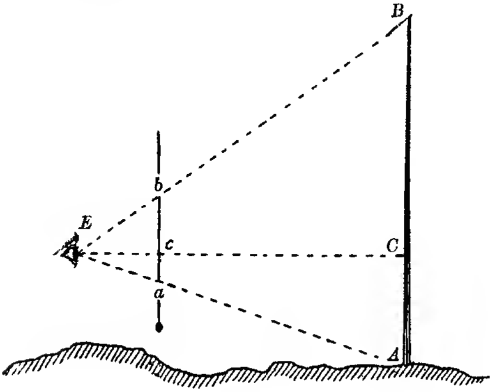


Fig. 7.

(1.) The horizontal line hits the tree between the top and foot. Here the following equation holds good:—

$$B\,C : b\,c = E\,C : E\,c$$

and

$$B\,C = b\,c \times \frac{E\,C}{E\,c}.$$

Again,

$$A\,C : a\,c = E\,C : E\,c$$

and

$$A\,C = a\,c \times \frac{E\,C}{E\,c}.$$

Hence,

$$B\,C + A\,C = A\,B = \text{height of tree} = \left(b\,c + a\,c\right) \times \frac{E\,C}{E\,c}$$

or,

$$H = a\,b \times \frac{E\,C}{E\,c}.$$

(2.) The horizontal plane passes below the foot of the tree. In that case (Fig. 8):—

$$H = B\,C - A\,C = (b\,c - a\,c) \times \frac{E\,C}{E\,c} = a\,b \times \frac{E\,C}{E\,c}.$$

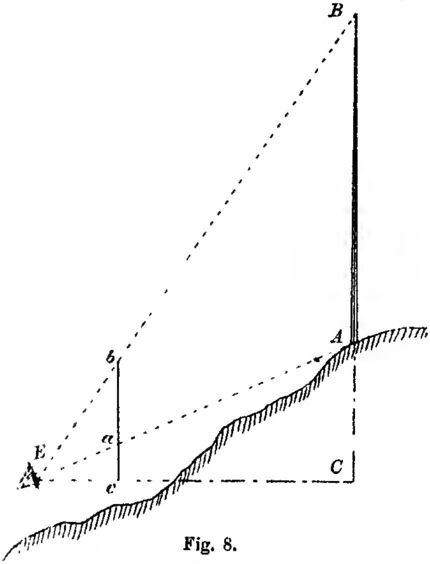


Fig. 8.

(3.) The horizontal plane passes above the top of the tree (Fig. 9). Then—

$$H = A\,C - B\,C = (a\,c - b\,c) \times \frac{E\,C}{E\,c} = a\,b \times \frac{E\,C}{E\,c}.$$

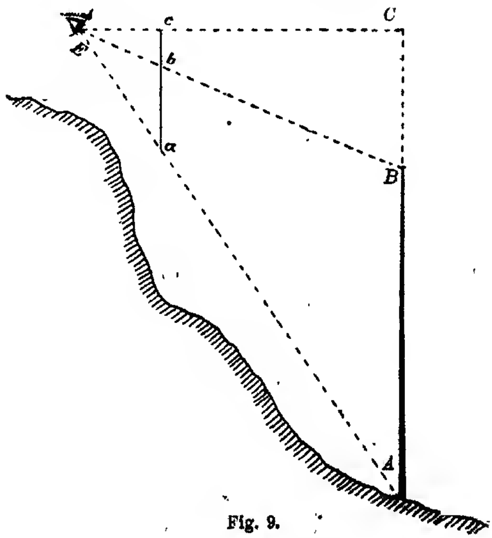


Fig. 9.

In each of the above three cases, two measurements are required, unless the foot of the tree happens to be at the same level as the eye of the observer. The horizontal distance $E\ C$ must be measured, and $a\ c$, $b\ c$ and $E\ c$ are read off upon the instrument.

The measurement of $E\ C$ can be avoided in the following manner (Fig. 10):—

A staff $M\ N$, of a known length $= l$, is placed alongside the tree, so that both its ends can be seen. In this way, the plumb-line gives two further points, m and n, and the similar

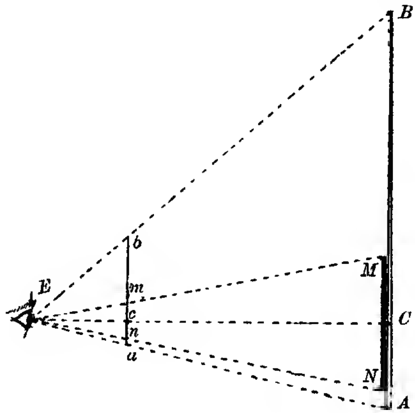


Fig. 10.

triangles, $E\ C\ M$ and $E\ c\ m$, as well as $E\ C\ N$ and $E\ c\ n$, so that the following equations hold good :—

$$E\ C : E\ c = A\ B : a\ b$$

and

$$E\ C : E\ c = M\ N : m\ n;$$

hence,

$$A\ B : a\ b = M\ N : m\ n$$

and

$$A\ B = H = a\ b \times \frac{M\ N}{m\ n} = a\ b \times \frac{l}{m\ n}.$$

The indirect determination of the distance by means of a staff is less accurate than measuring it on the ground, as it is

difficult to read off $m\ n$ with sufficient accuracy, owing to its smallness and the necessarily primitive arrangement of the height-measuring instruments.

If the length of $a\ b$ can be read off at once, the business becomes more simple, and may be expressed as follows:—

If two parallel objects are cut by diverging rays, then the portions of the parallel objects lying between the said rays are proportionate to the lengths of the rays.

Let $E\ A$ (Fig. 11) be the length of ray from the eye of the

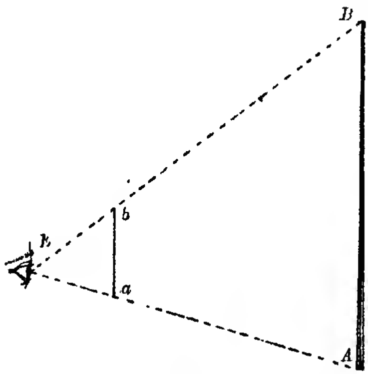


Fig. 11.

observer to the foot of the tree, $E\ a$ that from the eye to the plumb-line, $A\ B = H$ the height of the tree, and $a\ b$ the length of the plumb-line between two rays going from the eye of the observer to the top and foot of the tree, then:—

$$E\ a : E\ A = a\ b : A\ B, \text{ and}$$

$$A\ B = H = \frac{E\ A}{E\ a} \times a\ b.$$

In this case, $E\ A$ can easily be measured following the surface of the ground, whether it be level or slanting, while $E\ a$ and $a\ b$ are read off on the instrument.

Measurements made with the above-mentioned hypsometers

are liable to yield inaccurate results, owing to the following causes :—

(1.) Inaccurate reading owing to the unsteadiness of the plumb-line in windy weather, or in consequence of a shaky hand.

(2.) Inaccurate measurement of the base line.

(3.) Slanting position of the tree.

Other things being equal, the most accurate results are obtained if the distance of the observer from the tree equals the height of the tree.

The inaccuracy of the better hypsometers does not exceed 2 per cent. of the height of the tree.

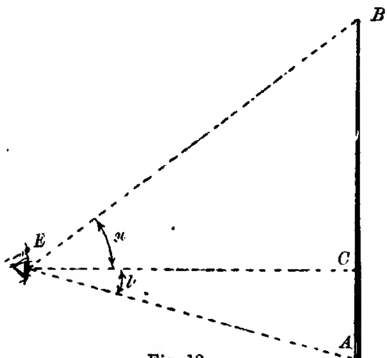


Fig. 12.

b. Trigonometrical Height Measuring.

This is based upon the measurement of the angles of elevation and depression indicated by rays running from the eye of the observer to the top and foot of the tree. In $\triangle$ $E\ C\ B$ (Fig. 12)—

$$B\ C = E\ C \times tan.\ u.$$

and in $\triangle$ $E\ C\ A$

$$A\ C = E\ C \times tan.\ l;$$

hence, $A\ C + B\ C$ = height $= H = E\ C\ (tan.\ l + tan.\ u).$

If the horizontal line of vision passes below the foot of the tree, the above formula becomes—

$$H = E\,C\,(tan.\ u - tan.\ l).$$

If it passes above the tree—

$$H = E\,C\,(tan.\ l - tan.\ u).$$

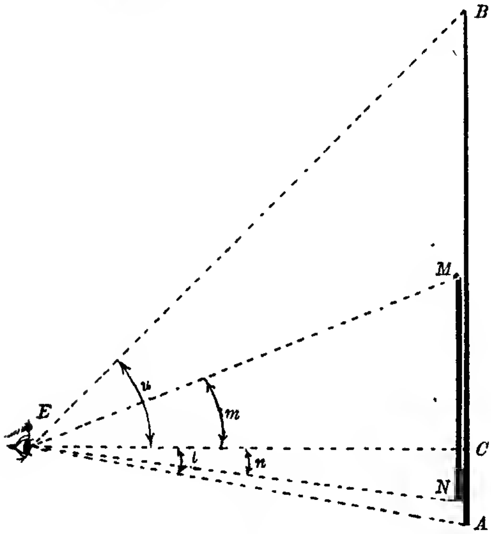


Fig. 13.

In each of these cases the measuring of the horizontal line $E\,C$ can be avoided by placing a staff of known length alongside the tree. In that case (Fig. 13)—

$$M\,C = E\,C \times tan.\ m;\ N\,C = E\,C \times tan.\ n$$

and

$$M\,N = E\,C\,(tan.\ m + tan.\ n);$$

hence,

$$E\,C = \frac{M\,N}{tan.\ m + tan.\ n}.$$

By introducing this value into the former equation, the height is obtained as—

$$H = M\,N \times \frac{(tan.\ l + tan.\ u)}{tan.\ m + tan.\ n}.$$

All instruments which measure vertical angles are suited for trigonometrical height measuring. For practical purposes, it is desirable that the instrument should not require a stand; it is a further advantage if, besides the angles, the corresponding tangents are marked on it.

c. Description of some of the more useful Instruments.

The number of hypsometers, based upon the above theories, is very large, some being used with stands and others without. Only the latter are really useful for forest operations. Of these the following three will be described :—

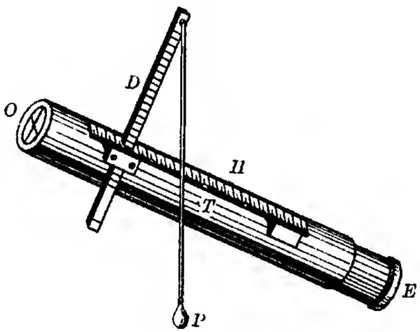


Fig. 14.—Weise's Hypsometer.

Weise's Instrument.—It consists of (1) a tube *T* with an objective in the shape of a cross at one end *O* and an eye-piece *E* at the other. (2) A scale fastened longitudinally to the tube (called the height scale, *H*, Fig. 14) which is toothed on one side, has the zero point some distance from its end, and is graduated from it in both directions. (3) A second scale, *D*, moving at the zero point of the height scale and at right angles to it (called the distance scale). From the upper, or zero, point of this scale depends a plumb-line *P*.

When not used, the distance scale and plumb-line are kept in the tube.

In nsing the instrument, a position is chosen from which both the top and foot of the tree can be seen; then the horizontal distance from the point of observation to the tree is measured, and the distance scale drawn out until it indicates at the zero point of the height scale the number of units in the distance; then the tube is raised and directed towards the top of the tree, taking care that the up and down line of the objective keeps a vertical position. As soon as the horizontal line of the cross covers the top of the tree, the tube is gently turned from left to right, thereby causing the plumb-line, which hitherto swung free, to be caught by the toothed edge

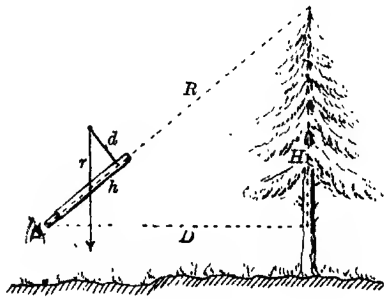


Fig. 15.

of the height scale. The instrument is then taken down and the number of units from the zero point to the point where the plumb-line was arrested read off. This number gives the number of feet (or yards, as the case may be) from the horizontal of the eye of the observer to the top of the tree. To this must be added (or deducted) the difference in height between the eye of the observer and the foot of the tree, which is obtained in the same way, by directing the tube towards the foot of the tree, and reading the number of units on the height scale towards O. An improvement of the instrument would be the addition of a guide for the plumb-line.

The theory of the instrument rests upon the similarity o

the triangles with the sides R H D and r h d; that is to say, the following equation holds good:—

$$d : h = D : H$$

and

$$H = D \times \frac{h}{d}.$$

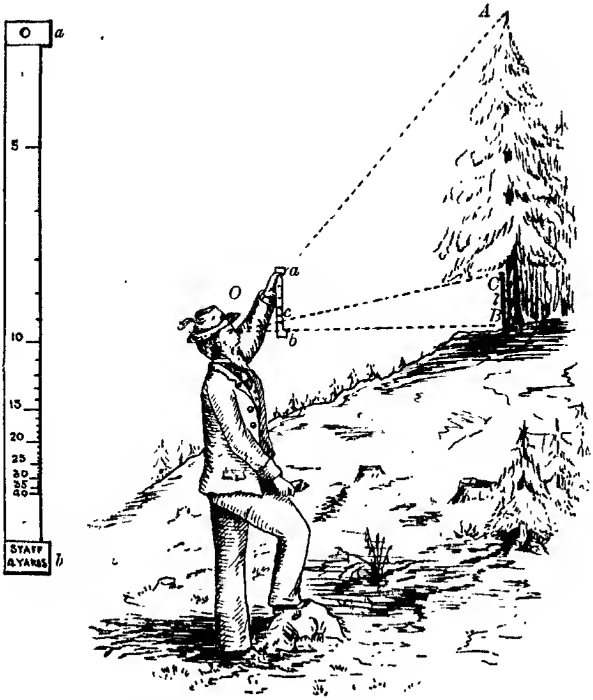


Fig. 16.—Christen's Hypsometer.

If, therefore, the units of the scales, which give h and d, are of the same size, and d is so fixed that its units are the same number as the units of the measured distance D, it follows that the above formula gives the height above or below the horizontal, as the case may be.

Christen's Instrument.—It consists of a piece of metal (see Fig. 16) with protruding upper and lower edges (see a and b).

The instrument is based upon the theory explained on page 17, which avoids the measurement of a base line. A staff of known length $= l$, say 4 yards, is placed alongside the foot of the tree. The instrument is then held in a vertical position at some distance from the observer's eye, and moved backward and forward, until the top of the tree is seen along the upper edge a, and the foot along the lower edge b; then the point is marked where a ray from the eye to the top of the staff hits the instrument at c. In this way similar triangles are formed in which the following equation holds good:—

$$A\,B = \frac{a\,b}{b\,c} \times l, \text{ and } b\,c = \frac{a\,b}{A\,B} \times l.$$

If now $a\,b = 12$ inches, and $l = 4$ yards, and successive values for $A\;B$, the height of tree, are introduced, corresponding values of $b\;c$ are obtained and can be marked on the instrument. In this way, the heights can be read off straight on the instrument. For convenience sake, the marks on the instrument are cuts, so that the place on the scale, where the top of the staff cuts in, may be more easily fixed.

The instrument has the disadvantage that the marks are very close one to another for heights over 30 yards. This has lately been obviated, to some extent, by lengthening the instrument, and making it with a clasp in the middle. When out of use, it can be folded together.

It is evident that, instead of using a staff 4 yards long, one of, say, 2 yards can be used. In that case, the height read off on the instrument must be divided by 2.

The instrument works well for heights up to 30 yards; for higher trees it is not sufficiently accurate.

Brandis' Hypsometer (Fig. 17) is based upon the trigonometrical method of height measuring. It consists of a tube with an objective, O, at one end and an eye-piece, E, in the shape of a horizontal slit, at the other. Attached to this tube is a wheel which is weighted on one side (see at a) and swings between two pivots, so that it always maintains the same position when at rest. Oscillations can be arrested by a

stop (see at S in figure). That point of the wheel which corresponds with the horizontal line of vision is marked as zero, and from this point the wheel is graduated to 60° up and down (see at b). A lens is fastened alongside the eye-piece to facilitate the reading of the angle on the wheel. By directing the tube to any point, the angle can be easily read off on the wheel, which preserves the same position while the

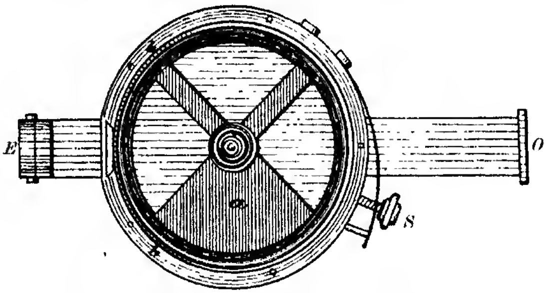


Fig. 17.—Brandis' Hypsometer and Clinometer.
(The front lid removed, so as to show the wheel.)

instrument is being raised or lowered. The wheel is placed in a firm metal case.

In using the instrument, any convenient position, where the top and foot of the tree can be seen, is chosen, the angles to the top and foot of the tree read off, and the distance from the eye of the observer to the tree measured. The height is then found by the formula (see Fig. 12)—

$$\text{Height} = H = E\,C\,(tan.\ u + tan.\ l).$$

Brandis, for convenience of calculation, changed this formula in the following manner :—

$$E\,C = E\,A \times cos.\ l :$$

$$H = E\,A \times cos.\ l\,(tan.\ u + tan.\ l)$$

$$H = E\,A \times cos.\ l \times \frac{sin.\ u \times cos.\ l + sin.\ l \times cos.\ u}{cos.\ u \times cos.\ l}$$

$$H = E\,A\,\frac{sin.\ (u + l)}{cos.\ u}.$$

A table accompanies the instrument, in which the heights, corresponding to various distances and upper plus lower angles, are given. In order to reduce this table as much as possible, it gives only upper angles from 40° to 50° with intervals of 2°, and lower angles from 0° to 25° with intervals of 5°. This necessitates placing a staff, on which feet are marked by alternate colours, alongside the tree, so as to read off the distance between the lower ray of the lower angle and the foot of the tree, a distance which has to be added to the height taken from the table.

The instrument is at the same time an admirable clinometer with which the angles of slopes can be measured and roads laid out. The author has used the instrument extensively, both for the measurement of the height of trees, and for the laying out of forest roads; he has arrived at the conclusion that it is decidedly the most useful of similar instruments which are known to him. The instrument works accurately, and much quicker than the reader would imagine; besides, its strong construction renders it admirably adapted for forest work.

6. Instruments for the Direct Measurement of the Volume.

For this purpose, the xylometer, either alone or in combination with a scale, is used. The method is based upon the fact that a submerged body displaces a volume of water equal to the volume of the body, and the instrument used is called a xylometer. It consists of a graduated vessel, Fig. 19 in which the wood is submerged. Before and after submersion the position of the water is noted, and the difference gives directly the volume. The method is employed for the measurement of irregular pieces, such as root wood and fagots. To obviate the necessity of submerging large quantities of wood, the whole is first weighed, and only a portion submerged. Let the weight of the whole be $= W$, that of the submerged

portion $= w$, the volume of the former $= V$, of the latter v, then :—

$$W : w = V : v$$

and

$$V = \frac{v}{w} \times W.$$

Instead of having a graduated vessel, the latter may be filled up to an opening, then the wood is submerged, the out-

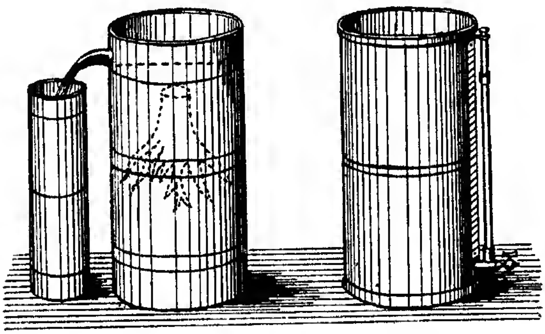


Fig. 18. Fig. 19.

flowing water caught in a separate vessel and measured (Fig. 18).

The xylometer is used in scientific investigations, and when the object is to obtain coefficients to be used in calculating the solid contents of stacked wood.

NOTE.—Pressler's Increment Borer can be obtained from Herr Moritz Perles, Verlagshandlung, Wien, Austria; Brandis' Hypsometer from Herr Max Wolz, Bonn, Germany. All the other instruments can be procured through Herr Wilhelm Spoerhase, 37, Stein Strasse, Giessen Germany.

CHAPTER II.

MEASUREMENT OF FELLED TREES.

THE methods of measuring the various dimensions of felled trees have been explained in Chapter I. In this place, the measurement of the volume will be dealt with.

Each tree consists of a stem or trunk, branches, and roots. These have peculiar shapes of their own which differ considerably; hence, they must be considered separately.

1. VOLUME OF THE STEM.

If the stem, or trunk, of a tree had a regular or distinct shape, its volume could be calculated direct by means of a formula corresponding to that particular shape. As a matter of fact, the stem shows different shapes in different parts of the tree.

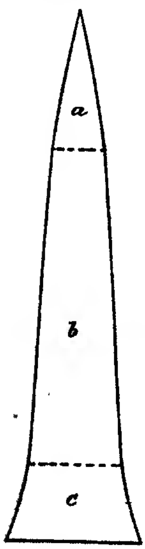


Fig. 20.

Again, the shape of trees differs widely according to species, the ages of the trees, and the conditions under which they have grown up, whether in the open or in a crowded wood. At the same time, trees of the same species and age, grown under the same conditions, generally show shapes which are nearly identical. Moreover, experience has shown that each part of the stem shows approximately a constant form. Thus, the uppermost part, *a* (Fig. 20), of an undivided stem has generally the shape of a cone, the lowest part, *c*, that of a neiloid (truncated semi-cubical paraboloid), while the bulk between these extremes approaches in shape a truncated Appollonian paraboloid, or a cylinder.

If h = the height, or length,
S = the lower cross section,
s = the upper cross section, and
s_m = the middle cross section (Fig. 21),

the volume of each of the above-mentioned truncated solids is, according to Simpson's rule—

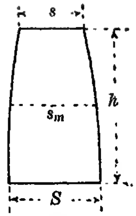


Fig. 21.

$$V = \frac{S + 4 \times s_m + s}{6} \times h.$$

This formula reduces—

For the cylinder to $V = S \times h$

For the cone to $V = \frac{S \times h}{3}$

And for the truncated Ap. paraboloid to $V = \frac{S + s}{2} \times h$, or,
$= s_m \times h$.

By means of these formulæ it would be possible to calculate the volume of each part of the stem, provided its particular shape had first been ascertained. This, however, would be a tedious business, and it is necessary to search for a more simple procedure.

It has been found that by far the greater portion of the stem approaches in shape that of a truncated paraboloid, and that, if the stem is divided into a number of pieces of moderate length, each can, without committing any appreciable error, be considered as a truncated paraboloid. Of the two formulæ,

$$V = \frac{S + s}{2} \times h, \text{ or } V = s_m \times h,$$

the latter is the more convenient, and experience has shown that it is even more accurate than the former.

According to this method, the volume of the whole stem is obtained by means of the following formula (see Fig. 22):—

$$\text{Volume of stem} = s_1 h_1 + s_2 h_2 + s_3 h_3 + \ldots,$$

where $s_1, s_2, s_3 \ldots$ are the sectional areas taken in the middle of successive paraboloids, and $h_1, h_2, h_3 \ldots$ the corresponding

heights or lengths. If the pieces are made of equal length, the above formula changes into the following:—

$$\text{Volume of stem} = (s_1 + s_2 + s_3 + \ldots)\, h.$$

This formula is used in all scientific investigations, and the degree of accuracy with which it works depends on the length of the pieces.

For the purposes of determining the yield of woods and for the sale of logs, the formula is further simplified by considering each log as one paraboloid; in other words, the

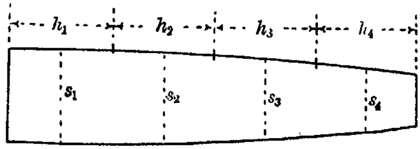


Fig. 22.

volume is calculated from the middle section of the log multiplied by its length, according to the formula—

$$V = S_m \times H,$$

where S_m represents the area of the circle, or sectional area, in the middle, and H the total length of the log. Experience has shown this formula to give sufficiently accurate results for all practical purposes.

The sectional area is obtained, either by measuring the girth, or the diameter. If g = girth, d = diameter, and r the radius, the section is—

$$S = \frac{g^2}{4\pi} = {\cdot}0796 \times g^2,$$

or

$$S = \frac{\pi\, d^2}{4} = {\cdot}785 \times d^2 = r^2 \times \pi,$$

and

$$V = {\cdot}0796 \times g^2 \times H = {\cdot}785 \times d^2 \times H = r^2 \times \pi \times H.$$

In practical work, the sectional areas are taken from specially prepared tables; there are also tables which give directly the volume of logs according to their mean girth and

length, or their mean diameter and length. (See Appendix I., page 333.)

All these calculations are made on the assumption that the section represents a circle. This is, however, rarely the case. As a rule, the degree of divergence from the circular shape depends on:—

(1.) The part of the stem; the lowest and uppermost parts differ most.

(2.) The age of the tree; young trees are more regularly shaped than old ones.

(3.) The species.

(4.) The conditions under which the tree has grown up; in crowded woods the shape is more regular than in the case of trees grown in the open; exposure to strong winds, slanting position, and the nature of the soil also affect the shape.

Generally, the sections of trees approach the shape of an ellipse, the greater axis of which lies, in the same locality, as a rule in a constant direction. Where trees are much exposed to wind, the greater axis lies generally in the direction of the prevailing wind: in Western Europe, therefore, from west to east or from south-west to north-east.

The inaccuracy caused by measuring the girth, and calculating therefrom the sectional area, has been found to amount, on an average, to about 7 per cent.; where only one diameter is measured, the error may be the same or even more; where two diameters at right angles are measured and the mean taken, the error generally does not exceed 2 per cent. of the true amount.

In Britain and in India, the sectional area in the middle is calculated by the method of the quarter-girth, that is to say by the formula—

$$S = \left(\frac{g}{4}\right)^2 = \cdot 0625 \times g^2.$$

In comparing this with the real sectional area $= \cdot 0796 \times g^2$, it is found that the quarter-girth method gives only 78½ per

cent. of the true basal area and volume, omitting 21½ per cent. The method is based upon the assumption that this amount represents the waste incurred in squaring the timber. Quantities calculated by the exact method can be converted into the quantities corresponding to the other by deducting 21½ per cent. of the volume. If the bark has been omitted in the measurements in both cases, no further correction is required. In measurements according to the quarter-girth, it is usual to deduct for the bark one inch out of every twelve of the quarter-girth, before the latter is squared. Hence, the deduction on account of bark comes to $12 \times 12 - 11 \times 11 = 144 - 121 = 23$ square inches out of every 144. In other words, a reduction of 16 per cent. is made on account of bark. This may be too much in some cases and too little in others. It is a much better plan, in the case of felled logs, to remove a ring of bark, and to measure the girth over tho wood only.

Generally speaking, it may be said that the volume calculated according to the quarter-girth method is equal to three-fourths of the volume in the round.

2. Volume of Branch and Root Wood.

In some cases, pieces of branch and root wood are of a sufficiently regular shape to measure and calculate their volume in the manner given above. As a general rule, however, such wood requires a different treatment. Its volume is ascertained by shaping it according to custom and stacking it in a space of regular geometrical form, say, 100 cubic feet. Such a space would contain a certain amount of wood and of air. In order to obtain the proportion of each, the volume of some stacks of the different classes of wood, such as split wood, round billets, branch wood, fagots, or root wood, is carefully ascertained. This can be done by measuring each piece separately, an operation of considerable difficulty, and one which takes much time. A more expeditious way is to submerge the material in a xylometer and ascertain the volume by

measuring the quantity of the displaced water. From the data thus obtained, average coefficients are calculated and used on all subsequent occasions.

It is evident that different descriptions of wood give different coefficients. The solid contents of stacked wood depend on many things, amongst which may be mentioned :—

(1.) Shape and nature of the pieces: thick, smooth and straight pieces give more solid contents than thin, bent, uneven pieces.

(2.) Length of pieces: short pieces pack better than long ones; hence, they give a higher percentage of solid contents.

(3.) Method of stacking: careful stacking causes the percentage of solid wood to be considerably increased.

Under these circumstances, the coefficients differ in accordance with local conditions. By way of illustration, the following may be given :—

Split firewood	= ·7
Round firewood billets under 5″ diameter	= ·6
Root and stump wood	= ·5
Fagot wood stacked (not bound) . .	= ·2

That is to say, 100 cubic feet of stacked split firewood contain 70 cubic feet of solid wood and 30 cubic feet of air, etc.

3. Volume of the Bark.

In many cases it is desirable to ascertain the volume of the bark, especially when it is sold separately, as in the case of tanning bark. This can be done stereometrically or xylometrically. In the former case, the pieces of wood are measured before and after barking, the difference giving the volume of the bark. If a xylometer is used the bark can be measured separately, or the pieces of wood are immersed before and after barking.

According to species, age, and locality, the bark comprises

from 4 to 20 per cent. of the total volume. Schwappach found on a number of trees the following results:—

Oak	= 15—20%	Alder	= 16—19%
Ash	= 12—14	Lime	= 16—19
Elm	= 9—11	Aspen	= 9—13
Birch	= 13—17	Scotch Pine	= 10—16

Tanning bark is usually sold by weight; other bark is sold according to measurement, like firewood.

In Britain, figures are sometimes locally obtained which indicate the proportion between the quantity of timber and that of tanning bark. Such figures vary according to local conditions.

CHAPTER III.

MEASUREMENT OF STANDING TREES.

1. Ocular Estimate.

Originally, the volume of standing trees was estimated. Such an estimate takes into consideration the special shape or form of each tree, and fixes the volume accordingly. The accuracy of purely ocular estimates depends entirely on the person who makes them. To be only approximately correct, the estimator requires great practice and occasional opportunities to compare his estimates with actual measurements after the trees have been felled. Even then, the results are subject to considerable errors, unless the estimator practises his art constantly. Mistakes of 25 per cent. are of common occurrence, and they may reach up to 100 per cent. in the case of an inexperienced estimator.

The uncertainty of purely ocular estimates led to the measurement of diameter (or girth) and height; this done, the basal area near the ground can be calculated, multiplied by the height, and an estimate made of the actual volume of the tree. It stands to reason that such an estimate is less dependent on the individuality of the estimator than that mentioned above, since he has only to estimate the proportion which exists between the actual volume and that of an imaginary body constructed out of the height and the sectional area near the base, a matter which he must decide according to the peculiar shape of the tree. By degrees, it was considered desirable to collect data regarding the form of various trees which might be utilised in subsequent estimates, and thus foresters arrived at the method next to be described.

2. Estimate of Volume by means of Form Factors.

a. Definition and Classification of Form Factors.

By "form factor" is understood the proportion which exists between the volume of a tree and that of a regularly-shaped body of the same base and height as the tree. The form factor means, therefore, a coefficient, with which the volume of the regularly-shaped (geometrical) body must be multiplied in order to obtain the volume of the tree.

Any regularly-shaped body, the volume of which can easily be calculated by means of a mathematical formula, is suited for the above purpose. In practice, only the cone and cylinder have been employed, and at the present time only the latter is used. Let s be the area of the basal section of the tree, h its height, f the form factor, and v the volume, then—

$$\text{Volume of cylinder} = s \times h,$$
$$\text{Volume of tree} = v = s \times h \times f,$$

and

$$\text{Form factor} = f = \frac{v}{s \times h}.$$

The volume of the stem of a tree by itself is always smaller than that of the corresponding cylinder; hence, the form factor for the stem only is always smaller than 1. If the volume of the branches is added, the form factor is sometimes greater than 1, especially during the early youth of the tree.

Various kinds of form factors are used in forestry, of which the following may be mentioned:—

(1.) Stem form factors, which refer only to the volume of the stem above ground.

(2.) Tree form factors, which refer to stem and branches, omitting root wood.

(3.) Timber form factors, which refer only to the parts of the tree classed as timber, whether they are taken from the stem or branches, omitting all other material.

Form factors for branch wood, fagots, or root wood only are,

as a rule, not used; their volume is ascertained by utilising the results of actual fellings and determining their proportion to the volume of timber.

As it would be highly inconvenient to measure the diameter, or girth, of the tree close to the surface of the ground where it is usually cut, it has been agreed to take the measurement at a convenient height. According to whether that point is fixed or variable, the following kinds of form factors may be distinguished:—

(1.) *Absolute Form Factors.* — The diameter (or girth) is measured at any convenient height above the ground, and the form factor refers only to the part *a* of the tree above that point (Fig. 23), while the volume of the piece *b* below it is ascertained by separate measurement and added to the rest. This is evidently troublesome and takes extra time.

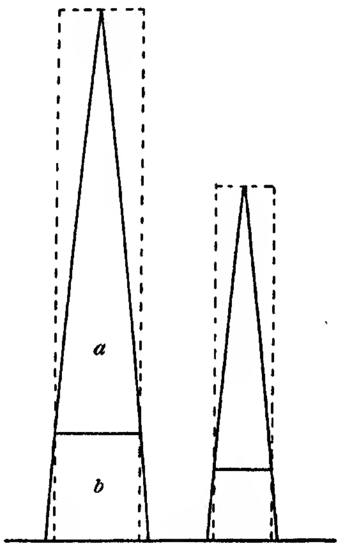


Fig. 23. Fig. 24.

(2.) *True or Normal Form Factors.*—The diameter (or girth) is measured at a constant proportion of the height of the tree, say, $\frac{1}{10}$th, $\frac{1}{20}$th, etc. (Figs. 23 and 24). In this case, the height of the ideal cylinder is equal to the height of the tree. Such form factors, it was believed, would have the advantage that all trees of the same shape would have the same form factor, since they have been measured at a height which bears in all cases the same proportion to the total height. There are, however, various drawbacks to the employment of these form factors. In the first place, the height of the tree must be

determined before the point of measurement can be fixed; secondly, the latter may be very inconvenient in the case of very tall, as well as very short trees; thirdly, it has been found from actual measurements that the factors thus obtained are by no means so regular as had been supposed, that is to say, trees of different heights show by no means the same form factor if measured at a constant proportion of the height.

(3.) *Form Factors based on Measurements made at Height of Chest, called Artificial Form Factors.*—The diameter (or girth) is measured at the most convenient height from the ground—namely, at chest height of an ordinary man. (In Germany and France now generally fixed at 1·3 meters = about 4 feet 3 inches.) The height of the ideal cylinder is equal to the height of the tree. Owing to the measurements being taken at an absolutely constant height, the form factors of two trees, which show the same shape but differ in height, cannot be the same. It follows that, in using such form factors for calculating the volume of trees, the height of the latter must be taken into consideration. Nevertheless, in practice, these are the only form factors now used.

b. Determination of Form Factors.

At first, form factors were estimated, taking into consideration all points which affect them, such as species of tree, height, age, free or crowded position, etc. Such an operation requires much skill and practice, and in fact it comes practically to the same thing as estimating the volume direct. To eliminate such uncertainty, tables have been prepared which give the form factors for different species, heights, and ages, such tables being based upon the results obtained by the measurement of numerous felled trees. Of late years, it has been recognised that the variations due to age can be omitted, except where great accuracy is required, as in scientific investigations.

The following tables show the form factors, taken from the Yield Tables in Appendix III., on pages 344 to 377, and obtained from other sources.

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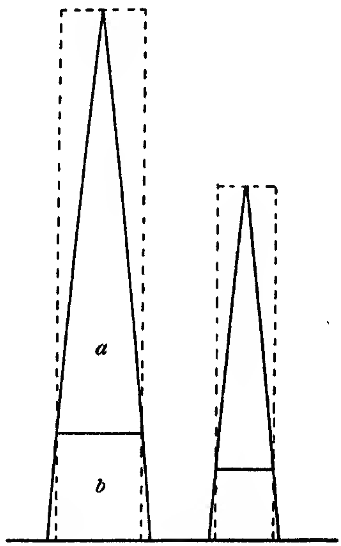

Fig. 23. Fig. 24.

(2.) *True or Normal Form Factors.*—The diameter (or girth) is measured at a constant proportion of the height of the tree, say, $\frac{1}{10}$th, $\frac{1}{20}$th, etc. (Figs. 23 and 24). In this case, the height of the ideal cylinder is equal to the height of the tree. Such form factors, it was believed, would have the advantage that all trees of the same shape would have the same form factor, since they have been measured at a height which bears in all cases the same proportion to the total height. There are, however, various drawbacks to the employment of these form factors. In the first place, the height of the tree must be

In using these tables, it must not be forgotten that they give the averages of numerous measurements; hence, they do not give reliable results in calculating the volume of a single tree. Their application should be restricted to the calculation of the volume of a number of trees; in other words, of whole woods, where the differences between the several trees are likely to compensate each other. (See example on pages 60 and 61.)

3. Estimate of Volume by means of Volume Tables.

If, instead of giving the form factors only, they are multiplied by the corresponding heights and basal areas, the volumes of the trees are obtained which can be arranged into so-called "volume tables." The latter may be defined as tables which give the volume of single trees arranged according to species, age, diameter, and height of tree.

These tables rest upon the assumption that trees of the same species, which have reached in the same time an equal height and diameter, show also an equal volume, and that trees of the same species, diameter, and height show volumes which differ with the age of the trees; in other words, the volume becomes greater with advancing age, although the height and the diameter at chest-height may be the same. Foresters say the trees become less tapering, or more full bodied, or the point where the real tapering commences moves higher up the tree with advancing age.

In order to use such tables, it is necessary to ascertain the diameter at chest-height, the total height, and the approximate age of the tree, when the volume corresponding to these data can be obtained from the tables. It must, however, not be forgotten that the tables give only averages, and consequently only true results, if used for determining the volume of a number of trees, or of whole woods.

Tables of this kind have been prepared in Germany and France for many years past. The numerous measurements made by the German Forest Statistical Association, extending

over about 35 years, have yielded a rich crop of data which have been arranged in tables according to species and dimensions. They are, no doubt, of great use, but they cover many pages, and it takes time to look up the particular dimensions. In the author's opinion, the method of taking the volume of the cylinder (equal to the height multiplied by the basal area) from the table on pages 336—337, and multiplying it by the form factor, works quicker and suffices for all practical purposes.

Example.—An oak has a height of 80 feet, and a diameter of 24 inches at 4 feet 3 inches from the ground. The table on page 336 gives—

Volume of cylinder . . . = 251·3
The form factor for timber (page 39) = ·52
Volume of tree in the round . . = 251·3 × ·52 = 130·7

or, according to quarter-girth measurement,

Volume = 251·3 × ·38 = 98 cubic feet.

4. Measurement of Standing Trees by Sections.

Analogous to the measurement of felled trees by sections, the volume of standing trees can be ascertained by determining the diameter (or girth) at various heights from the ground. For this purpose, a man must be sent up the tree which is a cumbrous procedure, or the several diameters must be determined indirectly. The latter, as has been explained in Chapter I. (page 12), is subject to great inaccuracies.

Where the diameter at half height is wanted, it is often estimated from the diameter at height of chest. The method is a rough one, but much used in Britain, France, and Belgium. As far as the author is aware, no uniform method of estimating the girth or diameter at half height, from the girth or diameter at, say, height of chest, has been recognised in Britain. Consequently, the actual estimate depends on the individuality of the estimator. No wonder, then, that different estimators obtain different results. The calculation of the volume by means of form factors, which represent the average of numerous measurements of trees lying on the ground, leaves no latitude to the estimator. He measures the diameter

of the tree at height of chest, and the height; he takes the basal area and form factor out of a little table which he carries in his pocket, and obtains the volume by a simple multiplication.

Example.—An oak tree, grown in a fairly stocked wood, has a diameter of 12 inches at height of chest and a total height of 70 feet. On reference to the table on page 334, it is found that the basal area, corresponding to a diameter of 12 inches, is $s = 0{\cdot}7854$, and on reference to page 39 the form factor for a height of 70 feet will be found to amount to ·52 for timber in the round, or ·39 ($= \frac{3}{4}$ of ·52) for quarter-girth measurement. Hence, the volume $V = {\cdot}7854 \times 70 \times {\cdot}52 = 28{\cdot}5$ cubic feet in the round, or $V = {\cdot}7854 \times 70 \times {\cdot}39 = 21{\cdot}4$ cubic feet quarter-girth measurement. The product of $\cdot7854 \times 70$ can be obtained from the table on page 337 without multiplication.

On the other hand, according to British custom, the estimator has to do three things: (1.) to measure the girth at height of chest, say, = 38 inches; (2.) the length of serviceable timber, say 50 feet; and (3.) to estimate the girth at 25 feet from the ground. Supposing he estimates a decrease of 20 per cent., then the girth at 25 feet comes to about 30 inches, and the volume, according to the quarter-girth measurement, amounts to—

$$V = \frac{(7{\cdot}5)^2 \times 50}{144} = 19{\cdot}5 \text{ cubic feet.}$$

If he estimates the decrease of the girth at 15 per cent., the girth at 25 feet from the ground would be 32 inches, and

$$V = \frac{(8)^2 \times 50}{144} = 22{\cdot}2 \text{ cubic feet.}$$

If he estimates the decrease at 25 per cent., the girth at 25 feet comes to about 28 inches, and

$$V = \frac{(7)^2 \times 50}{144} = 17{\cdot}0.$$

In fact, the volume thus estimated would range from 17 to 22·2 cubic feet, and the only chance of obtaining the exact amount depends on the skill of the estimator, thus introducing a factor of considerable uncertainty.

The most urgent need for calculating the volume of trees grown in Britain in fairly well-stocked woods is the collection of data from which form factors can be calculated, whether for determining the volume in the round, or according to quarter-girth measurement. The method is, however, not applicable in the case of hedgerow trees, or others grown under similar conditions. In the latter case, each tree must be measured, or estimated, separately.

CHAPTER IV.

DETERMINATION OF THE VOLUME OF WHOLE WOODS.

This chapter may be divided into three sections, according to whether the measurements extend over the whole wood, or over only a selected portion of it, or whether the volume is estimated.

SECTION I.—MEASUREMENTS EXTENDING OVER THE WHOLE WOOD.

The method demands a uniform treatment of the whole wood, but a distinction may be drawn between the measurement of all trees and that of selected trees, called sample, or type, trees.

A. Measurement of all Trees.

Each tree is measured separately and its volume ascertained in one of the ways described in Chapter III. By adding up the volumes of the several trees, that of the whole wood is obtained.

As the method takes much time, it is, in practice, only employed when the total number of trees is small, or when the wood is of an irregular description. In all other cases, the following system is chosen, as it works more rapidly.

B. Determination of Volume by means of Sample, or Type, Trees.

The volume of a wood consists of the sum of the volumes of the individual trees. The volume of each tree is calculated according to the formula—

$$v = s \times h \times f,$$

where s represents the basal area at a certain height, h the total height, and f the form factor of the tree. In all cases where s, h, and f differ from tree to tree, nothing remains but to ascertain them separately for each tree as indicated under A. In the case of regularly grown woods, however, there are always a number of trees which show, at any rate approximately, the same basal area, height, and form factor, so that they can be thrown together and dealt with in a uniform manner; in other words, all trees of a wood which show the same base, height, and form factor are joined into one class; the volume of one tree (or of a few trees) is ascertained and the volume of the whole class obtained by multiplying the former by the number of trees in the class. If every class is dealt with in the same way, the volume of the whole wood is obtained by adding together the volumes of the several classes.

So far, however, little or no advantage is gained, because it would be necessary to ascertain the base, height, and form factor of each tree, in order to put it into its proper class, and when this has been done, the volume of each tree may just as well be calculated separately. Moreover, in crowded woods the height is not always easy to measure, and the form factor could only be estimated, unless it is taken from a table. Only the basal area is easily ascertainable by measuring either the diameter or the girth.

Here, experience had to be called in, which fortunately showed that, in regularly-grown crowded woods, the height and form factor are approximately functions of the diameter of the tree; in other words, trees of the same diameter have approximately the same height and form factor. At any rate, this is found to hold good to a sufficient extent, so as to justify a classification according to diameter classes only.

In open woods, however, the height and form factor vary within much wider limits, so that, besides diameter classes, at any rate height classes also must be formed. Hence, the two cases must be dealt with separately.

I.—The Height is a Function of the Diameter.

1. Description of the General Method.

a. Formation of Diameter Classes.

The number of classes depends on the difference between the largest and smallest trees of a wood, and the desired degree of accuracy. As a rule, all classes are given the same extent, that is to say, either 1 inch, 2 inches, 3 inches, etc., or part of an inch. For the purpose of forest working plans in Europe, each class comprises 1 or 2 inches: in India frequently, as yet, 6 inches.

The calliper, used in measuring the diameters, should have a rounded-off scale, as described in Chapter I., that is to say, in the case of inch classes, the first should comprise the space from $\frac{1}{2}$ to $1\frac{1}{2}$ inch; the second that from $1\frac{1}{2}$ to $2\frac{1}{2}$ inches, etc. For scientific investigations the classes may be further reduced to a part of an inch.

b. Height and Manner of Measurement.

All trees must be measured at the same height, the latter being so chosen that the place of measurement falls above the irregular swelling frequently observed near the foot of the tree; at the same time, the height should not be so great that it becomes difficult for an ordinary-sized man to measure accurately. Whenever practicable, the height should be the height of chest of an average man.

In executing the measurement, all the precautions indicated in Chapter I. must be duly taken, so as to obtain as accurate results as possible. More especially, any irregularity in the shape of the sections must be duly considered. Where the section differs systematically from that of a circle, either two diameters at right angles must be measured, or the direction of measurement changed from time to time. For instance, after a certain number of stems have been measured with the face of the measurer to the east, an equal number must then be measured with the face of the measurer towards the north

or south. Or the change can be made at alternate trees. In this manner, average diameters are obtained.

c. The Booking of the Measurements.

In measuring the diameter, the gaugers call out each measurement, and in mixed woods also the species; the book-keeper enters each announcement, repeating it at the time, so as to prevent mistakes.

A book-keeper may work with one or two gaugers, the party taking a narrow strip of wood at the time; each tree is marked as soon as measured, preferably with chalk.

The booking can be done in a variety of ways, as the following samples will show:—

Diameter in Inches measured at 4' 3" from the Ground.	Species.			Number of Trees.			
	Beech.	Oak.	Ash.	Beech.	Oak.	Ash.	Total.
8				18	9	3	30
9				23	13	8	44
10				37	13	9	59
11	••••• ••••• ••••• ••••• ••••• ••	•••••••••• ••••	••••• ••	27	14	7	48
			Grand Total	105	49	27	181

The first two methods of booking are least liable to errors.

d. Selection and Number of Sample Trees.

As the volume of the whole class is to be calculated from that of the sample tree, it is necessary to select for the latter

a tree which represents the average of the class: in other words, the samplo tree should have the mean height, as near as possible a circular section, a fairly straight and not a forked stem, and an average extent of crown. Even with the greatest care, it is not always possible to avoid errors in the selection; hence, it is generally advisable to take several sample trees for each class, and to ascertain the average of their volume for the calculation of the volume of the class. The actual number of sample trees depends on the desired degree of accuracy, and the total number of trees in the class. At the same time, the felling of many sample trees is undesirable; hence, their number should be kept within reasonable limits.

A further requirement is that the sample tree should show a basal area which corresponds exactly to the mean section of the class. Such a tree is found only in exceptional cases; hence, it is necessary to take a tree as near as possible to the true section, and to modify the volume in proportion of the basal areas of the true and approximate, or false, sample trees. Let v = volume of true sample troe, v' that of the approximate sample tree, s and s' the corresponding basal areas, then v is found by the formula—

$$v : v' = s : s'$$

and

$$v = v' \times \frac{s}{s'}.$$

c. Determination of the Volume of Sample Trees.

The volume of the sample trees is determined, either by felling and measuring them on the ground, or by means of form factors or volume tables.

If the trees are felled, the stem and all straight pieces of branches, in fact all regularly shaped parts, are divided into pieces of moderate length, from 3 to 10 feet, according to the desired degree of accuracy, and the volume of each section is ascertained separately by the formula:—

Volume = area of circle in the middle × by the length of the section:

$$v = s_m \times h.$$

The volume of all irregular pieces, including root and branch wood, is ascertained, either by the xylometric method, or by proportionate figures, or by measuring their volume stacked and multiplying it by known reducing factors, if such are available.

The xylometric method has been explained in Chapter I.

Proportionate figures are obtained from actual fellings. If it has been found that in the felling of a wood every 100 cubic feet of timber are accompanied by, say, 20 cubic feet of firewood, that proportion can be applied to other woods of a similar description.

The determination of the volume of sample trees by means of form factors or volume tables can be highly recommended whenever suitable data are available, because they give averages, and that is just what is wanted in this case. Experience has shown that form factors and volume tables are applicable for a considerable distance outside the locality for which they have been prepared.

f. Calculation of the Volumes of the Classes and of the Whole Wood.

Here several cases may occur:

(1.) One sample tree has been measured in each class, the dimensions of which are exactly the average of the class. In that case, the volume of the class is obtained by multiplying the volume of the sample tree by the number of trees in the class. If—

V = volume of whole wood,

$V_1, V_2, V_3 \ldots$ = volumes of classes 1, 2, 3 . . .

$v_1, v_2, v_3 \ldots$ = volumes of mean sample trees of successive classes,

$n_1, n_2, n_3 \ldots$ = numbers of trees in successive classes,

then—

$$V = V_1 + V_2 + V_3 + \ldots = v_1 \times n_1 + v_2 \times n_2 + v_3 \times n_3 + \ldots$$

(2.) The sample trees in the several classes differ in basal area somewhat from the mean basal areas.

If the volumes of the approximate sample trees are v_1', v_2', v_3' . . . and the corresponding basal areas $= s_1'$, s_2', s_3', ... then

$$V_1 = \frac{v_1' \times s_1}{s_1'} \times n_1; \; V_2 = \frac{v_2' \times s_2}{s_2'} \times n_2; \; V_3 = \frac{v_3' \times s_3}{s_3'} \times n_3 \ldots$$

As

$s_1 \times n_1 = S_1 =$ total basal area of the first class,
$s_2 \times n_2 = S_2 =$ total basal area of the second class, etc.,

the volume of the wood is:

$$V = \frac{v_1' \times S_1}{s_1'} + \frac{v_2' \times S_2}{s_2'} + \frac{v_3' \times S_3}{s_3'} + \ldots$$

(3.) Several sample trees are measured in each class. In that case—

$$V = \frac{(v_1' + v_1'' + v_1''' + \ldots) \times S_1}{s_1' + s_1'' + s_1''' + \ldots} + \frac{(v_2' + v_2'' + v_2''' + \ldots) \times S_2}{s_2' + s_2'' + s_2''' + \ldots} + \ldots$$

g. Clubbing together several Classes, leading to the Method of the Arithmetical Mean Sample Tree.

In order to shorten the method described above and to reduce the number of sample trees to be felled, several, or all classes may be clubbed together into a group.

Let

n_1, n_2, n_3 . . . be the numbers of trees in the several classes
s_1, s_2, s_3 . . . „ basal areas „ „ „ „
h_1, h_2, h_3 . . . „ heights „ „ „ „
f_1, f_2, f_3 . . . „ form factors „ „ „ „

and

s, h, f the basal area, height, and form factor of the mean tree of the classes thrown together,

then the following equation holds good:—

$$V = n_1 \times s_1 \times h_1 \times f_1 + n_2 \times s_2 \times h_2 \times f_2 + \ldots$$
$$= (n_1 + n_2 + \ldots) \times s \times h \times f.$$

If it is now assumed, that $h_1 \times f_1 = h_2 \times f_2 = h_3 \times f_3 = \ldots = h \times f$, then the above equation becomes:

$$n_1 \times s_1 + n_2 \times s_2 + \ldots = (n_1 + n_2 + \ldots)\, s,$$

and

$$s = \frac{n_1 \times s_1 + n_2 \times s_2 + \ldots}{n_1 + n_2 + \ldots} = \frac{S}{N},$$

where S = basal area of all trees of the group, and
N = total number of trees ,, ,,

In words, the basal area of the average tree is equal to the arithmetical mean of the basal area of all trees contained in the group.

The volume of the group is then:

$$V = v \times N,$$

where v represents the volume of the arithmetical mean sample tree with a basal area $= s$.

If no tree can be found with the basal area s, another as near as possible to it is chosen of a section s', and the volume of the group is obtained by the formula:

$$\mathbf{V} = \frac{\mathbf{v'} \times \mathbf{s}}{\mathbf{s'}} \times \mathbf{N} = \frac{\mathbf{v'} \times \mathbf{S}}{\mathbf{s'}},$$

since $s \times N = S$ = the basal area of all trees in the group.

If several approximately mean sample trees are taken, the formula changes into the following:—

$$\mathbf{V} = \frac{(\mathbf{v'} + \mathbf{v''} + \mathbf{v'''} + \ldots) \times \mathbf{S}}{\mathbf{s'} + \mathbf{s''} + \mathbf{s'''} \ldots}.$$

The above method rests on the assumption that $h_1 f_1 = h_2 f_2 = h_3 f_3 = \ldots = h \cdot f$. This, however, is not absolutely correct, though it holds good approximately in regularly grown woods. It follows that the degree of accuracy decreases with the increase in the number of classes which are clubbed together into a group, the least accuracy being obtained by joining all classes into one group. In this latter case, the method is known as "the method of the arithmetical mean sample tree."

Example.—In order to illustrate this and the methods to be described hereafter, one acre of Scotch pine wood, 70 years old, was measured, and

twenty-four sample trees of various diameters felled and measured. Only timber down to 3 inches diameter at the small end has been included in the account. The wood is situated at Cæsar's Camp, on gravelly sand with a fair layer of humus, showing a yield capacity of about middling quality. The following list shows the dimensions and the volume of the sample trees :—

LIST OF TWENTY-FOUR SAMPLE TREES.

Number.	Diameter. Inches.	Height. Feet.	Basal Area. Square Feet.	Volume. Solid Cubic Feet.
1	4·00	38	·087	1·80
2	5·25	40	·150	3·27
3	6·40	45	·223	4·41
4	6·50	42	·230	4·52
5	7·50	49	·307	6·92
6	7·50	37	·307	6·62
7	8·50	44	·394	7·26
8	9·25	56	·467	12·13
9	9·40	46	·482	10·81
10	9·60	50	·503	11·63
11	10·70	40	·624	12·37
12	10·70	64	·624	17·14
13	11·00	57	·660	16·19
14	11·50	56	·721	17·43
15	11·60	58	·734	19·17
16	12·10	48	·799	20·41
17	12·10	62	·799	22·40
18	13·10	63	·936	25·93
19	13·60	56	1·009	21·87
20	15·00	48	1·227	27·79
21	15·00	59	1·227	28·40
22	16·40	63	1·467	38·53
23	17·00	64	1·576	39·50
24	17·50	74	1·670	49·43

The example on pages 52 and 53 illustrates the procedure which has just been described.

2. MODIFICATIONS OF THE GENERAL METHOD.

It has been shown above that the volume of a wood is represented by the formula :—

$$V = V_1 + V_2 + V_3 + \ldots = v_1 \times \frac{S_1}{s_1} + v_2 \times \frac{S_2}{s_2} + v_3 \times \frac{S_3}{s_3} + \ldots$$

It is obvious that, as long as the fractions $\frac{S_1}{s_1}, \frac{S_2}{s_2}, \frac{S_3}{s_3} \ldots$ differ, the volumes of the sample trees in the several classes must be measured separately. In order to avoid this

CALCULATION OF VOLUME BY INCH CLASSES, FOUR GROUPS, AND FOR ONE ACRE OF SCOTCH

Diameter in Inches.	Number of Trees.	Basal Area in Square Feet.	Method of Inch Classes.				Method of				
			Diameter of Sample Tree.	Basal Area of Sample Tree.	Volume of Sample Tree in Cubic Feet.	Volume of Class in Cubic Feet.	Number of Group.	Number of Trees in Group.	Basal Area of Group.	Mean Sample Tree. Basal Area.	Mean Sample Tree. Diameter
4	4	·349	4·0	·087	1·80	7					
5	12	1·637	5·25	·150	3·27	36					
6	26	5·104	6·4	·223	4·41	101	I.	83	18·047	·218	6·3
7	41	10·957	7·5	·307	6·92	247					
8	42	14·661	8·5	·394	7·26	270					
9	51	22·532	9·25	·467	12·13	585	II.	144	65·008	·451	9·1
10	51	27·815	9·6	·503	11·63	643					
11	34	22·439	11·0	·660	16·19	550					
12	16	12·566	12·1	·799	20·41	321	III.	59	43·301	·734	11·6
13	9	8·296	13·1	·936	25·93	230					
14	6	6·414	13·6	1·009	21·87	139					
15	5	6·136	15·0	1·227	28·40	142	IV.	16	19·892	1·243	15·1
16	3	4·189	16·4	1·467	38·53	110					
17	2	3·153	17·0	1·576	39·50	79					
Total...	302	146·248				3,460					

inconvenience, it has been proposed to fix the number of sample trees in each class so that

$$\frac{S_1}{s_1} = \frac{S_2}{s_2} = \frac{S_3}{s_3} = \ldots = \text{a constant} = c,$$

when the above formula reduces to :—

$$V = (v_1 + v_2 + v_3 + \ldots) \times c.$$

By following this method, it is not necessary to keep the sample trees separate; they can be thrown together and measured in one lot. This is a great convenience and saves

THE ARITHMETICAL MEAN SAMPLE TREE OF THE WHOLE WOOD, PINE WOOD, 70 YEARS OLD.

Four Groups.				Method of Arithmetical Mean Sample Tree.							
Real Sample Trees.			Volume of Group.	Number of Trees.	Basal Area of all Trees.	Mean Sample Tree of Wood.		Real Sample Trees.			Volume of Wood.
Diameter	Basal Area.	Volume.				Basal Area.	Diameter	Diameter	Basal Area.	Volume.	
6·4	·223	4·41									
6·5	·230	4·52									
Total	·453	8·93	356								
9·25	·467	12·13									
9·40	·482	10·81									
Total	·949	22·94	1,571					9·25	·467	12·13	
								9·40	·482	10·81	
								9·60	·503	11·63	
11·5	·721	17·43		302	146·248	·484	9·4	Total	1·452	34·57	3,482
11·6	·734	19·17									
Total	1·455	36·60	1,089								
15·0	1·227	27·79									
15·0	1·227	28·40									
Total	2·454	56·19	455								
		Total...	3,471								

$$V = \frac{34 \cdot 57 \times 146 \cdot 248}{1 \cdot 452} = 3{,}482 \text{ cubic feet.}$$

much time. The volume of all sample trees multiplied by the constant c gives the volume of the wood. In this way, a number of modifications of the general method have been elaborated, of which the following may be mentioned :—

a. Draudt's Method.

Draudt selects in each class the same percentage of sample trees, thus ensuring, that—

$$\frac{S_1}{s_1} = \frac{S_2}{s_2} = \frac{S_3}{s_3} = \ldots$$

Let

$$V = v_1 \times n_1 + v_2 \times n_2 + v_3 \times n_3 + \ldots$$

If p per cent. of the trees in each class are taken as sample trees, the proportion of these to the total number of trees is $= \frac{p}{100} = \cdot 0p$. By multiplying each term in the above equation by this coefficiont, the following equation is obtained:—

$$V \times \cdot 0p = v_1 \times n_1 \times \cdot 0p + v_2 \times n_2 \times \cdot 0p + v_3 \times n_3 \times \cdot 0p + \ldots$$

Here, $v_1 \times n_1 \times \cdot 0p$ represents the volume of the sample trees in the first class, $v_2 \times n_2 \times \cdot 0p$ that of second class, etc. If the volume of all the sample trees, $v_1 \times n_1 \times \cdot 0p + v_2 \times n_2 \times \cdot 0p + \ldots = v$, then

$$V \times \cdot 0p = v$$

and

$$V = \frac{v}{\cdot 0p} = \frac{v \times 100}{p}.$$

It generally happens that the number of sample trees in each class contains a fraction of one. Such fractions are eliminated by considering ·51 as a full sample tree and by neglecting ·50 and under, taking care that the total number of sample trees is as nearly as possible correct. The result of this operation is that the original proposition is no longer absolutely maintained; in other words, the volume of the actual sample trees does no longer represent the true value $\frac{V \times p}{100}$. To avoid this inaccuracy, Draudt introduces the basal area of the whole wood $= S$ and of the sample trees $= s$, by saying:—

$$s : S = v : V,$$

and

$$V = v \times \frac{S}{s}.$$

This formula is used to calculate, not only the whole volume, but also that of timber and firewood separately.

The advantages of Draudt's method are that the sample trees can all be worked up together.

Its drawbacks are:—

(1.) That in rounding off the number of sample trees in each class, inaccuracies are likely to be introduced; and

(2.) that frequently no sample trees at all are taken from classes which contain only a small number of trees.

The larger the wood, that is to say, the greater the number of trees in the several classes, the more accurately the method works.

Example.—The following example will further explain the method. The data are the same as those used in the previous example, but they have been multiplied by ten all round, in order to obtain a larger number of trees.

CALCULATION OF VOLUME ACCORDING TO DRAUDT'S METHOD.

Area = 10 Acres.

Diameter	No. of Trees.	Basal Area.	Percentage = 1 % multiplied by Number of Trees.	Sample Trees.				Volume of Wood.
				Number rounded off.	Basal Area should be.	Basal Area really is.	Volume.	
Inches.		Square Feet.			Square feet.	Square Feet.	Cubic Feet.	Cubic Feet.
4	40	3·49	·40					
5	120	16·37	1·20	1	·136	·150		
6	260	51·04	2·60	3	·588	·669		
7	410	109·57	4·10	4	1·068	1·228		
8	420	146·61	4·20	4	1·396	1·576		
9	510	225·32	5·10	5	2·210	2·335		
10	510	278·15	5·10	5	2·727	2·515		
11	340	224·39	3·40	3	1·980	1·980		
12	160	125·66	1·60	2	1·570	1·598		
13	90	82·96	·90	1	·922	·936		
14	60	64·14	·60	1	1·069	1·009		
15	50	61·36	·50	1	1·227	1·227		
16	30	41·89	·30					
17	20	31·53	·20					
	3,020	1462·48	30·20	30	14·893	15·223	357·60	34,355

$$V = \frac{357{\cdot}60 \times 1462{\cdot}48}{15{\cdot}223} = 34{,}355. \qquad \text{Per Acre} = 3{,}436$$

b. Urich's Method.

In order to avoid the inaccuracy caused by rounding off fractions of sample trees, Urich proposes to arrange the trees in such groups that each sample tree represents the same number of trees. For this purpose, he places the same number of trees in each group and then calculates the arithmetical mean sample tree for each. For the rest, he proceeds on the same lines as Draudt, so that he obtains the volume of the whole wood by the same formula, namely:—

$$V = v \times \frac{S}{s}.$$

Urich's method is thus a combination of Draudt's method

Example:—

CALCALATION OF VOLUME,

Diameter.	Number of Trees.	Groups. No.	Groups. Diameter. Inches.	Groups. Number of Trees. Detailed.	Groups. Number of Trees. Total.	Groups. Basal Area. Square Feet. Detailed.	Groups. Basal Area. Square Feet. Total.
4	40	I.	4	40		3·49	
5	120		5	120		16·37	
6	260		6	260		51·04	
7	410		7	335	755	89·53	160·43
8	420						
9	510	II.	7	75		20·04	
10	510		8	420		146·61	
11	340		9	260	755	114·87	281·52
12	160						
13	90	III.	9	250		110·45	
14	60		10	505	755	275·43	385·88
15	50						
16	30	IV.	10	5		2·72	
17	20		11	340		224·39	
			12	160		125·66	
			13	90		82·96	
			14	60		64·14	
			15	50		61·36	
			16	30		41·89	
			17	20	755	31·53	634·65
	3,020				3,020		1462·48

The volume is obtained by the formula:—

with that of the arithmetical mean sample tree for each group.

The degree of accuracy depends on the number of groups which are formed, and on the number of sample trees measured in each. Too large a number of groups is, however, inconvenient, as it involves repeated separation of the original diameter classes.

The method has the disadvantage that the basal areas must be calculated before the sample trees can be selected. Urich proposes to avoid this by estimating the sizes of the sample trees in the several groups, a procedure which may lead to inaccuracies.

ACCORDING TO URICH'S METHOD.

Mean Sample Trees.		Real Sample Trees.					Volume of Wood.
Basal Area.	Diameter.	Number.	Diameter.	Basal Area. Detailed.	Basal Area. Total.	Volume. Cubic feet.	Cubic feet.
·212	6·2	1 2	6·4 6·5	·223 ·230	·453		
·373	8·2	3 4	7·5 8·5	·307 ·394	·701		
·511	9·7	5 6	9·4 9·6	·482 ·503	·985		
·841	12·4	7 8	12·1 12·1	·799 ·799	1·598		
				Total...	3·737	88·36 Per Acre	34,313 =3,431

$$V = \frac{88{\cdot}36 \times 1462{\cdot}48}{3{\cdot}737} = 34{,}313.$$

c. Robert Hartig's Method.

In Draudt's and Urich's methods, each sample tree represents the same number of trees. As the volume increases rapidly with the diameter, it follows that a sample tree in a class or group of small diameter represents a much smaller volume than one in a class or group with a large diameter. For this reason, Robert Hartig argues that the number of sample trees in each class or group should be proportionate to the volume which it contains, and not to the number of trees. As the volume is fairly proportionate to the basal area, Robert Hartig forms groups which contain equal basal areas. He divides the total basal area of the wood by the number of groups which he proposes to form; this gives the basal area

The following example will further explain the method :—

MEASUREMENT OF VOLUME ACCORDING

Diameter.	Number of Trees.	Basal Area.	Groups.					
			No.	Diameter.	Number of Trees.		Basal Area.	
Inches.		Square Feet.			Detailed	Total.	Detailed.	Total.
4	40	3·49	I.	4	40		3·49	
5	120	16·37		5	120		16·37	
6	260	51·04		6	260	1337	51·04	365·52
7	410	109·57		7	410		109·57	
8	420	146·61		8	420		146·61	
9	510	225·32		9	87		38·44	
10	510	278·15						
11	340	224·39	II.	9	423	751	186·88	365·77
12	160	125·66		10	328		178·89	
13	90	82·96						
14	60	64·14	III.	10	182		99·26	
15	50	61·36		11	340	575	224·39	365·28
16	30	41·89		12	53		41·63	
17	20	31·50						
			IV.	12	107		84·03	
				13	90		82·96	
				14	60	357	64·14	365·91
				15	50		61·36	
				16	30		41·89	
				17	20		31·53	
Total...	3,020	1462·48				3,020		1462·48

to be allotted to each group. After having placed in each a sufficient number of trees to give that basal area, he calculates the mean sample tree for each group and selects an equal number of these for each.

The formula for R. Hartig's method is as follows:—

$$V = v_1 \times \frac{S_1}{s_1} + v_2 \times \frac{S_2}{s_2} + v_3 \times \frac{S_3}{s_3} + \ldots$$

As $\frac{S_1}{s_1}, \frac{S_2}{s_2}, \frac{S_3}{s_3}, \ldots$ are not equal the one to tho other, it follows that the sample trees must be measured and the volumo of each group calculated separately. By adding together the volumes of the groups, the volume of the whole wood is obtained. This makes the method more laborious than those of Draudt and Urich.

TO ROBERT HARTIG'S METHOD.

Mean Sample Trees.		Real Sample Trees.					Volume of Groups and of whole Wood.	Remarks.
Basal Area.	Diameter	No.	Diameter	Basal Area. Detailed.	Basal Area. Total.	Volume.		
·273	7·1	1 2	7·5 7·5	·307 ·307	·614	13·54	8060	$\frac{13{\cdot}54 \times 365{\cdot}52}{{\cdot}614} =$ 8,060 cubic feet.
·487	9·4	3 4	9·4 9·6	·482 ·503	·985	22 14	8333	
·635	10·8	5 6	10·7 10·7	·624 ·624	1·248	29·51	8637	
1·025	13·7	7 8	13·1 13·6	·936 1·009	1·945	47·80	8992	
						Total...	34022	Per Acre = 3,402 cubic feet.

3. Determination of Volume by means of Form Factors or Volume Tables.

Instead of felling and measuring sample trees, their volume can be ascertained by means of form factors, or taken from volume tables. This applies to all the methods given above. In all these cases the volume is ascertained according to the general formula :—

$$V = S \times H \times F.$$

How the basal areas of the trees of a class or a wood are obtained, has already been explained. The mean height of a number of trees, or of a whole wood, is ascertained in the following way :—

$$V = S \times H \times F = s_1 \times h_1 \times f_1 + s_2 \times h_2 \times f_2 + s_3 \times h_3 \times f_3 + \ldots$$

Example :—

Calculation of Volume by

					According to Form Factors			
					By Inch Classes.		By Four	
Diameter	Number of Trees.	Basal Area. Square feet.	Height ascertained from 24 Sample Trees. Feet.	Basal Area multiplied by Height $c \times d$.	Form Factors.	Volume.	Basal Area.	Basal Area multiplied by Height.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*
4	4	·349	38	13·26	·43	6		
5	12	1·637	40	65·48	·44	29	18·047	775·22
6	26	5·104	42	214·37	·45	96		
7	41	10·957	44	482·11	·45	217		
8	42	14·661	46	674·41	·46	310		
9	51	22·532	48	1081·56	·46	498	65·008	3146·72
10	51	27·815	50	1390·75	·46	640		
11	34	22·439	52	1166·83	·47	548		
12	16	12·566	54	678·56	·47	319	43·301	2309·97
13	9	8·296	56	464·58	·47	218		
14	6	6·414	58	372·01	·47	175		
15	5	6·136	60	368·16	·47	173	19·892	1201·68
16	3	4·189	62	259·72	·47	122		
17	2	3·153	64	201·79	·47	95		
					Total...	3446		

hence

$$H = \frac{s_1 \times h_1 \times f_1 + s_2 \times h_2 \times f_2 + \ldots}{S \times F}.$$

If it is assumed that the form factors are the same throughout, the above formula reduces to the following:—

$$H = \frac{s_1 \times h_1 + s_2 \times h_2 + \ldots}{S};$$

in words: "the mean height is equal to the total volume of cylinders erected over the trees divided by the total basal area." This formula holds good in the case of the several trees of one class, as well as for calculating the mean height of several classes, or of a whole wood.

If a somewhat smaller accuracy suffices, one of the following methods may be adopted:—A number of trees are selected which show about an average diameter and height, their

MEANS OF FORM FACTORS.

FOR II., OR AVERAGE, QUALITY.							
Groups.			By One Group.				
Mean Height $\frac{i}{h}$	Form Factors.	Volume $h \times k \times l$, or $i \times l$.	Basal Area.	Basal Area multiplied by Height.	Mean Height $\frac{o}{n}$	Form Factor.	Volume $n \times p \times q$, or $o \times q$.
k	*l*	*m*	*n*	*o*	*p*	*q*	*r*
42·95	·45 =	349	146·25	7433·39	50·83	·46 =	3419
48·45	·46 =	1447					
53·35	·47 =	1086					
60·42	·47 =	565					
	Total...	3447					

heights are accurately measured, and the mean taken which represents the mean height of the class, or wood. Or, the height of the tree with the average diameter (or basal area) is taken as the mean height of the wood. It remains to be noted that the heights obtained by means of these simplified methods are generally a foot or two smaller than those obtained by the above formula.

How the form factors of single trees are ascertained, has been described above. Similarly, form factors for whole woods can be determined according to the formula:—

$$F = \frac{V}{S \times H}.$$

If volume tables are used, the calculation is made according to the formula—

$$V = n \times s \times h \times f.$$

Here $s \times h \times f$, equal to the volume of the mean tree, is taken direct from the tables.

4. Comparative Accuracy of the several Methods.

If the results obtained in the examples used above are put together, the following data are obtained:—

Method.		Volume. Cubic feet.	Difference in %
The general method—			
Each inch class calculated separately	=	3460	
Inch classes arranged in four groups	=	3471	+ ·3
All inch classes thrown together into one group	=	3482	+ ·7
Draudt's method	=	3436	− ·7
Urich's method	=	3431	− ·8
Hartig's method	=	3402	− 1·7
Calculation with form factors—			
According to inch classes	=	3446	− ·4
According to four groups	=	3447	− ·4
All inch classes thrown together	=	3419	− 1·2

These data show that form factors for Scotch pine, obtained from measurements in North Germany, are quite applicable to woods in the South of England. Experience has also proved that they are applicable to the Midland Counties, and probably also to districts further north.

It will be seen that, for the methods described above, the greatest difference amounts to 1·7 per cent. Under these circumstances, it appears that any of the methods meets the requirements of measurements for the preparation of working plans, and that much more depends on the care bestowed upon the operation than on the particular method followed. If the actual fellings show greater differences than the calculations justify, they are frequently due to extraneous causes, such as the felling of the trees at some distance above the ground, careless working up of the material, inaccurate measurement of the fall, theft of material, etc.

Guided by investigations made by various authors, it may be said that, in the majority of cases, the difference between the calculation made according to any one of the above-mentioned methods and the results of actual fellings keeps within 2 per cent., that the maximum error in the case of the method of the arithmetical mean sample tree may be placed at 10 per cent. and in the case of all the other methods at 5 per cent.

In all cases where special accuracy is required, as for instance for scientific investigation, or for the determination of the sale value of woods, the classes, or groups, should be small and the number of sample trees large. In this way greater accuracy is likely to be obtained, than by making a distinction between the different methods which have been described. Above all, the utilisation of form factors is strongly recommended, especially as there are now callipers to be had which, when measuring the diameter, give the basal area of all the trees, which have been measured. Of these, Wimmenauer's calliper can be specially recommended. Where form factors are not yet available, efforts should be made to

collect the necessary data for their preparation at the earliest possible date. Their application obviates the cutting down of sample trees, and the determination of the volume takes only a fraction of the time required in the case of any other method. In the absence of form factors, or volume tables, Urich's method is the most useful.

II. The Height is not a Function of the Diameter.

If it is found that, in the case of equal diameters, the heights differ considerably, then height classes must be formed in addition to diameter classes.

In some cases, it happens that the different height classes are separated according to area, for instance where a marked change in the quality of the locality occurs, due to change in the soil or subsoil, aspect, etc. In such cases, the wood is

BEECH.

Diameter in Inches.	I. Height Class.	II. Height Class.	Totals.		
			I. Height Class.	II. Height Class.	Total.
8	𝍸 𝍸 𝍸 𝍷𝍷𝍷	𝍸 𝍸 𝍷𝍷	18	12	30
9	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	𝍸 𝍸 𝍸 𝍷𝍷𝍷	34	18	52
10	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷	47	33	80
11	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍷𝍷𝍷𝍷	𝍸 𝍸 𝍸 𝍸	39	20	59
12	𝍸 𝍸 𝍸 𝍷𝍷	𝍸 𝍸 𝍷	17	11	28
		Grand Total	155	94	249

divided into as many parts as different height classes appear, and each is treated as a separate wood.

If the several height classes are mixed over the whole area, as in irregular selection forests, then the diameter and height must be measured in each case. Where only two height classes are adopted, the height may be estimated, while the diameter is measured, and the tree placed in the one or the other height class. Each of the latter is then considered as a separate wood and its volume ascertained according to one of the methods already described.

Cases where more than two height classes, in addition to diameter classes, are called for, are very rare. Generally, the distinction of height classes is a matter of considerable difficulty, unless the heights are measured. It is necessary only where a very high degree of accuracy is aimed at.

The example on page 64 shows the manner of booking in the case of two height classes.

SECTION II.—DETERMINATION OF VOLUME BY MEANS OF SAMPLE PLOTS.

1. General.

Instead of measuring all trees in a wood, a certain part of the area may be selected, the volume on it ascertained, and from it the volume of the whole wood calculated. Such a part is called a sample, or type, plot. It may be defined as a portion of a wood which contains an average volume of material per unit of area.

Having ascertained the volume of the sample plot, that of the whole wood can be calculated in two ways: either according to area, or according to the number of trees on the sample plot and in the wood.

Let A = area of wood,

a = area of sample plot,

V = volume of wood,

v = volume of sample plot,

then the following proportion is assumed to exist:—

$$v : V = a : A$$

and

$$V = \frac{v \times A}{a}.$$

Again, if

N = number of trees in the wood,
n = number of trees on the sample plot,

then

$$v : V = n : N,$$

and

$$V = \frac{v \times N}{n}.$$

In the former case, it is necessary to ascertain the areas, and in the latter the number of trees, in both sample plot and wood. As, however, the counting of all trees gives hardly less trouble than measuring them, the second method yields only a small saving of labour; it would be adopted only when the area of the wood is not known, or cannot readily be ascertained.

2. Selection of Sample Plots.

The proportion given above will hold good only if the sample plot represents a fair average of the whole wood, so that it can be considered as a model of it; in other words, if a measurement of the trees on it yields an average basal area of stems per unit of area, an average height, and the same form factors. Hence, the sample plots must be selected accordingly.

Here several cases must be distinguished:—

(*a*.) The quality of the wood is the same throughout the area. In this case, the sample plot may be selected anywhere, as long as the density of stocking represents an average. In very large woods, it may become desirable to take several sample plots and calculate the mean.

(*b*.) Several qualities occur which are clearly separated according to area. Here, each quality is treated

separately and one or more sample plot taken in each part (Fig. 25).

(*c*.) Several qualities exist which change gradually from one to the other. In this case, the sample plot may take the shape of a strip which runs through the whole wood, so as to include a due proportion of each quality (Fig. 26). As this is difficult to accomplish,

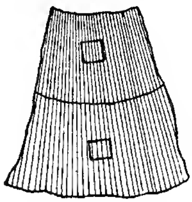

Fig. 25.

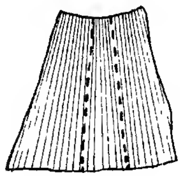

Fig. 26.

it is generally better to follow the method given under (*b*), to divide the wood into several parts and to take a sample plot in each.

3. Extent and Shape of Sample Plots.

The sample plot must be of sufficient extent to contain the different classes of trees in the same proportion as the wood. Hence, its size depends on the degree of regularity of the stocking; the more uniform this is, the smaller may be the sample plot. It follows that they may be made smaller in young, fully stocked, than in old, irregularly stocked woods.

Very small sample plots have the disadvantage that proportionately too many trees fall into the boundary lines.

The absolute extent of the sample plot depends on the desired degree of accuracy. In mature woods, it should not be less than 5 per cent. of the whole area, but in young woods it may be much less.

The best shape would be that which includes the greatest area as compared with the boundary, in other words, a circle.

As this is impracticable, it is usual to give to the sample plot the shape of a square, or of a rectangle approaching a square.

4. Measurement of Volume on Sample Plots.

This can be done according to any one of the methods described above. As here a conclusion is drawn from the volume of a small area to that of the whole wood, it is desirable to measure the volume on the sample plot as accurately as possible.

5. Merits of the Method of Sample Plots.

The method of sample plots works quickly, and it affords a great saving of time and expense as compared with the measurement of whole woods. On the other hand, its accuracy depends on the degree to which the sample plot represents an average of the whole wood. Hence, it only yields accurate results in regular grown, young and middle-aged woods, less so in old, irregularly stocked areas, or where the quality changes frequently. The method is chiefly useful where extensive areas have to be assessed, or where the value of the produce is small; in fact, where a high degree of accuracy is either impossible to attain, or not required. Where only small areas have to be measured, or where the value of a forest has to be ascertained for the purpose of sale, when a high degree of accuracy is wanted, the whole wood should be measured.

SECTION III.—DETERMINATION OF THE VOLUME BY ESTIMATE.

Instead of measuring the trees on the whole or a part of the area, the volume can be estimated in various ways, of which the following deserve to be mentioned:—

1. Estimating the Volume of the Wood as a Whole.

This method, being the oldest and roughest of all, consists of going through the wood and estimating the volume, either of the whole wood, or per unit of area if the total area is

known. The estimator must consider differences in the density of stocking, the average volume per tree, the differences in the quality of the locality, and, if for the whole wood at once, its area or number of trees. It stands to reason that the method requires great experience and practice on the part of the estimator, and even then considerable mistakes may be made.

2. Estimating by Trees.

Under this method, each tree is estimated separately, the volume of the wood being obtained by adding together the volumes of the several trees. With great care, an experienced estimator can obtain fairly accurate results, but, if done carefully, the operation takes almost as much time as if the diameters of all trees and the height of some of them are measured; in the latter case, the volume can be calculated by means of form factors or volume tables, a procedure which yields far more reliable results.

The method is only justified in open woods consisting chiefly of old trees, such as standards in high forest or in coppice with standards, or where a low degree of accuracy meets the requirements of the case. In such cases, the estimate may extend over the whole area, or over a sample plot only.

3. Estimating according to the Results of Past Fellings.

Where fellings have been made and the fall accurately measured, the results can be used to estimate the standing crop in similar woods. In such cases, it is necessary to take into consideration any differences in the age, density of stocking, height, etc.

Frequently, fellings made in strips cleared for roads or rides give useful data for estimating the crop of the adjoining woods. In all such cases, the estimate is based on the volume per unit of area.

4. Estimating the Volume by means of Yield Tables.

In the same way as volume tables of single trees are constructed which give the average volume of trees arranged

according to diameter, height, form factor, and age, so tables can be compiled, on the basis of extensive measurements, which show the volume of whole woods according to species, age, quality of locality, etc. (see Appendix III.).

If tables are available which are suited to a particular part of a country, it is necessary to ascertain in the wood to be estimated—

(1.) The quality class of the locality.

(2.) The density of the crop.

(3.) The age of the crop.

The first is judged by the height of the trees; the second by ascertaining the basal area of the trees on a sample plot, or it may be estimated; the third by counting the concentric rings on stumps, or on a few trees cut close to the ground, unless the age is known from records. Based upon these data, the volume can be taken from the yield table. If, for a certain age, the basal area given in the table differs from that of the wood, the volume of the table must be modified accordingly; a second correction may be necessary owing to the difference in the height. If, however, basal area and mean height of the wood have been ascertained, it is much shorter to calculate the volume by means of form factors or volume tables.

Where a sufficient number of suitable yield tables are available, where the staff is experienced in estimating the density of stocking, and where the age is known, the determination of the volume becomes a very simple matter. In some continental countries, no measurements are required in the case of all woods of fairly regular growth. This shows the great importance of the systematic collection of statistics bearing on the development of woods from their formation up to maturity.

CHAPTER V.

THE AGE OF TREES AND WOODS.

It is of importance to know, not only the actual dimensions of the trees and their volume, but also the time which has been necessary to produce them. To solve this question, the age of single trees, as well as that of whole woods, must be ascertained.

1. Determination of the Age of Single Trees.

a. Standing Trees.

All trees increase annually in diameter and also by the elongation of the leading shoots and branches, at any rate up to a certain age. The diameter increment produces every year an additional concentric ring, and the new leading shoot leaves marks which are more or less distinguishable according to species and age. These facts yield data by which the age can be determined in the majority of cases, but not in all, when no records are available which give the age. Accordingly, the following methods of determining the age may be distinguished:—

i. Determination from Existing Records.

Reliable records yield the best results, if they refer to individual trees. In the case of trees which form part of a wood, they are not always accurate, as many woods are not even-aged.

ii. Determination by Estimate.

As a general rule, it may be assumed that the larger the tree the older it is. Taking, therefore, into consideration the conditions under which a tree has grown up, its age can be estimated within 10 or 20 years as long as height-growth

continues. In the case of very old trees, the limit of accuracy is much wider. At all times, this method requires much practice and experience, and even then it yields only approximately correct results.

iii. Determination by the Number of Annual Shoots.

In the case of species which loavo cloar marks of the successive annual shoots, the age can be ascertained by counting these shoots from the top downwards and adding a proportionate number of years for the lowest part of the stem, where the marks are no longer distinguishable. This method is, in Europe, only applicable to the various species of pine up to a certain age, less so in the case of firs and not at all in that of larch or of tbe ordinary broad-leaved species.

iv. Determination by means of Pressler's Increment Borer.

As explained in Chapter I., with this instrument a narrow cylinder of wood can be extracted from tho stem, on which the concentric rings may be counted. The instrument does not, however, work satisfactorily beyond a depth of 6 inches, so that the centre can only be reached if the diameter of the tree does not exceed 12 inches. Even then, it is frequently difficult to hit off the centre, as tbe trees generally grow more or less eccentric.

b. Felled Trees.

By far tbe best metbod is to count the concentric rings on a stump, and, if necessary, to fell a tree for the purpose. At the same time, this is not always an easy operation, and in some cases it is altogether impracticable. It is easiest in the case of the so-called ring-porey broad-leaved species, and in conifers which produce a darker coloured summer, or autumn, wood than tbat formed in spring. Frequently, false rings appear. These may be distinguished from true rings by finding that they do not run right round the tree (Hornbeam, Alder). In the case of suppressed trees, the true rings are

frequently so narrow, either all round, or in parts, that they are difficult to distinguish.

The business may be facilitated by smoothing the surface, making a slanting cut, or applying colouring matters (as indigo, alizarine ink, Prussian blue, alcohol coloured with aniline, sulphuric acid, etc.). Such colouring, however, does not always produce the desired effect.

The number of rings thus counted represents only the age of the tree above the place where it has been cut. To the number so obtained, the number of years which the tree took to reach that height must be added. If absolute accuracy is required, the stool must be split open along the centre and the rings counted to the starting-point.

In this way, the *physical* age of the tree can be ascertained, provided that each concentric ring represents a year's growth. It is, however, by no means certain, whether this is always the case, as temporary interruptions of growth may cause two rings to be formed in one year, as, for instance, the destruction of the leaves by insects and the subsequent sending forth of a second crop of leaves, fire running through a wood, or even late frost. Moreover, there are trees in the tropics on which the concentric rings do not exist, or cannot be distinguished.

Another point is that a distinction must be made between the *physical* and *economic* age of a tree. By the latter is understood the actual growing age, leaving out of consideration any years during which the tree may have been at a standstill, owing, for instance, to heavy shade from above.

2. Determination of the Age of Whole Woods.

a. Even-aged Woods.

If the age of such woods is not known from authentic records, it can be ascertained by determining the age of a tree by one of the methods indicated above. If a tree is felled for the purpose of counting the concentric rings, it is desirable to avoid exceptionally thick trees, as such trees may represent former advance growth.

As whole woods are rarely established in one year, owing to failures and subsequent repairing, or, in the case of natural regenerations, owing to two or more seed years being necessary for the complete stocking of the area, it is generally desirable to examine several trees and take the mean.

b. Uneven-aged Woods.

In many cases, woods are less even-agod than has been indicated above. The differonces in the age of the several component parts of the wood may bo very considerable, as regeneration may have extended over a long period. In such cases, the mean age must be ascertained.

By the "mean age" of an unoven-aged wood is understood that period which an evon-aged wood requires to produce the same volumo as the unoven-aged wood.

Let V be the volume of the wood;
$a_1, a_2, a_3, \ldots$ the ages of the several age classes;
$v_1, v_2, v_3, \ldots$ the volumos of the several age classos;
I, the mean annual increment of an even-aged wood of the same volume as the uneven-aged one;
A, the mean age, or the age of an even-aged wood of the same volume as the uneven-aged one;

Then, according to the above definition, the following equation holds good:—

$$v_1 + v_2 + v_3 + \ldots = I \times A,$$

and

$$A = \frac{v_1 + v_2 + v_3 + \ldots}{I} = \frac{V}{I}.$$

As the even-aged and uneven-aged woods are assumed to have the same volume, it follows that I must be equal to the sum of the mean increments of the several age classes of the uneven-aged wood, that is to say:—

$$I = \frac{v_1}{a_1} + \frac{v_2}{a_2} + \frac{v_3}{a_3} + \ldots$$

By substituting this expression for I in the above equation, the latter becomes—

$$A = \frac{v_1 + v_2 + v_3 + \dots}{\frac{v_1}{a_1} + \frac{v_2}{a_2} + \frac{v_3}{a_3} + \dots} \quad . \quad . \quad . \quad (1.)$$

This formula is known as that of Smalian and C. Heyer. It says in words: The mean age of a wood is obtained by dividing the volume of the whole wood by the sum of the mean annual increments of the several age classes. The method may be simplified by assuming that the age is approximately proportionate to the diameter; hence, the diameter classes may be taken as the age classes. The above formula is chiefly used when the age classes are irregularly mixed over the area.

If the age classes are found on different parts of the area, the following formula may be used, where m_1, m_2, m_3, . . . represent the areas of the several age classes:—

$$A = \frac{m_1 \times a_1 + m_2 \times a_2 + m_3 \times a_3 + \dots}{m_1 + m_2 + m_3 + \dots} \quad . \quad (2.)$$

This formula was first given by Gümpel. It gives good results if the differences in age are small and the age itself is close to that at which the increment culminates, as it then changes but slowly.

André bases the calculation upon the number of trees in the several age classes. If they are n_1; n_2; n_3; . . . . , his formula runs thus:—

$$A = \frac{n_1 \times a_1 + n_2 \times a_2 + n_3 \times a_3 + \dots}{n_1 + n_2 + n_3 + \dots} \quad . \quad (3.)$$

All these formulæ are somewhat troublesome. Formula (1) demands a knowledge of the volume and increment; (2) of the areas occupied by each age class; formula (3) requires the number of trees in each age class. In practice, the mean age is frequently taken as equal to the average age of the sample trees, or of the age classes, according to the formula:—

$$A = \frac{a_1 + a_2 + a_3 + \dots}{n} \quad . \quad . \quad . \quad (4.)$$

where n represents the number of sample trees, or age classes, as the case may be.

Finally, the age of the arithmetical mean sample tree can be taken as the mean age of the wood.

Example:—

Let

$$v_1 = 4{,}000 \quad a_1 = 50 \quad m_1 = 2 \text{ acres} \quad n_1 = 1{,}500$$
$$v_2 = 9{,}000 \quad a_2 = 60 \quad m_2 = 3 \text{ ,,} \quad n_2 = 1{,}600$$
$$v_3 = 7{,}000 \quad a_3 = 70 \quad m_3 = 2 \text{ ,,} \quad n_3 = 800$$
$$v_4 = 4{,}000 \quad a_4 = 80 \quad m_4 = 1 \text{ ,,} \quad n_4 = 300$$

Mean age according to formula :—

Years.

$$(1)\ A = \frac{4{,}000 + 9{,}000 + 7{,}000 + 4{,}000}{\frac{4{,}000}{50} + \frac{9{,}000}{60} + \frac{7{,}000}{70} + \frac{4{,}000}{80}} = \frac{24{,}000}{380} = 63{\cdot}2$$

$$(2)\ A = \frac{2 \times 50 + 3 \times 60 + 2 \times 70 + 1 \times 80}{2 + 3 + 2 + 1} = 62{\cdot}5$$

$$(3)\ A = \frac{1{,}500 \times 50 + 1{,}600 \times 60 + 800 \times 70 + 300 \times 80}{1{,}500 + 1{,}600 + 800 + 300} = 59{\cdot}8$$

$$(4)\ A = \frac{50 + 60 + 70 + 80}{4} = 65$$

CHAPTER VI.

DETERMINATION OF THE INCREMENT.

During every growing season, a tree increases by the elongation of the top shoot, side branches, and roots, and by the laying on of a new layer of wood and bark throughout its extent. Thus, the height and diameter (or basal area), as well as the spread of the crown, increase constantly up to a certain age, producing an increase of volume called the *increment*. By adding up the increment of the several trees in a wood, that of the whole is obtained.

The increment may refer to one or more growing seasons, and accordingly a distinction must be made between —

(1.) The current annual increment, or that laid on in the course of one year.

(2.) The periodic increment, or that laid on during a number of years.

(3.) The total increment, or that laid on from the origin of a tree or wood up to a certain age, frequently that when the tree, or wood, is cut over.

(4.) The mean annual increment, or that which is obtained by dividing the increment laid on during a given period by the number of years in the period. If the mean annual increment is calculated for a portion of the total age, it is called the *periodic* mean annual increment, if for the total, or final age of the tree or wood, it is called the *final* mean annual increment.

In determining the increment of whole woods, it must be remembered that a certain number of trees disappear from

time to time owing to thinnings and natural causes. All such removals must be taken into account in determining the total increment laid on.

The determination of the increment may refer to tho past (backward) or to tho future (forward). As the former deals with actually existing quantities, the determination can be made with a comparatively high degree of accuracy; the latter, on the other hand, is to a considerable extent based on speculation; hence, less reliable.

SECTION I.—DETERMINATION OF THE INCREMENT OF SINGLE TREES.

1. Height Increment.

a. Of the Past.

The height increment of standing trees can, in some cases, be ascertained by the whorls formed in successivo years. This, however, refers to a limited number of species. In the majority of cases, it is necessary to cut down a tree for the purpose of investigating the height increment.

The height increment of the last fow, say n, years can be ascertained, in the case of some conifers, by measuring the length of the last n shoots. In the case of all other specios, the height increment of the tree during the last n years is ascertained by cutting off a certain length and counting the rings; if they are less than n in number, an additional piece must be cut off, and so on until that spot has been found where the section shows n rings. If the section of the first cut shows more than n rings, then another cut higher up is made, until again the section shows n rings. The length above that point gives the height growth of the last n years.

In all cases, where a complete knowledge of the height increment during the several periods of life is required, the tree should be divided into a number of sections, the length

of which depends on the desired degree of accuracy. The concentric rings are then counted at the end of each section, and, from the data thus obtained, the height of the tree at successive periods of life can be ascertained, either by calculation, or interpolation.

Generally, graphic interpolation gives the better results, as it equalises accidental irregularities. In this case, the abscissæ represent the ages, and the ordinates the corresponding heights. By connecting the points thus indicated by a steady curve, the height at successive ages can easily be read off.

Example.—See analysis of a Scotch pine tree, at p. 82.

b. *Height Increment of the Future.*

The expected height increment for a number of years to come can be estimated from the increment of the immediate past. In doing this, the rate of increment during the past must be studied, and especially the time ascertained when the current annual increment of the species usually culminates. If the increment immediately before the time of inquiry was still rising, it may continue to do so or not, according to whether the maximum has been reached or not. If it is already falling, it will continue to do so, and in that case the rate at which it is likely to fall must be estimated. In this way, the probable increment for a limited number of years (say 10) can be estimated with satisfactory accuracy. This is best done by constructing a height curve of the past, and elongating it for the required period, so as to form a continuous curve.

2. Diameter Increment.

a. *Of the Past.*

This can refer to wood and bark, or to wood only.

The increment of wood and bark laid on by standing trees an be ascertained by repeated measurements of the same tree,

a certain number of years being allowed to pass between every two moasuremonts. The latter are made with the calliper, care boing taken to mark the place of measurement without causing an unusual swelling at that part of the tree. Where immediate results are required, the increment can be ascertained with Pressler's increment borer. The number of years, for which it can be ascertained, depends on the length of the cylinder which can be extracted, and on the rate of growth. As most troes grow irregularly, it is necessary to ascertain the increment at opposite sides, or at four sides, and to take the mean. These investigations rest on the assumption that the concentric rings are distinguishable, and that each ring represents one year's growth.

The increment can be ascertained with much greater accuracy by felling a tree and measuring the breadth of the desired number of rings on the section, the latter being laid at right angles to the axis of the stem. The measurements are made with a scale subdivided to a sufficient degree. This is either laid on tho section and the breadths read off, or the latter are taken off with a pair of compasses, and the dimensions then taken from the scale. In either case, care must be taken to obtain averages by measuring along two, four, or more radii, arranged at equal distances over the section, and then taking the mean of the several readings.

In the case of standing trees, the increment can only be ascertained for a limited number of years. If a tree is felled, the increment can be ascertained for the several periods of its life, say, for every five, ten, or more years. The result can be graphically represented and a mean curve of increment constructed, from which the increment for any desired intervals can easily be determined. By repeating the above operation at successive heights from the ground, the increment can be ascertained in the several parts of the stem. (See example below.)

b. Diameter Increment of the Future.

This is estimated from the increment of the immediately preceding period, taking into consideration how far the future diameter increment may be affected by the method of treatment, more especially the proposed degree of thinning; the stronger the latter, the greater is the increment likely to be.

3. Area Increment.

The increment in basal area is calculated from that of the diameter. Let D be the mean diameter of the whole section, d the diameter of the same section n years ago, then

Basal increment during n years =

$$\frac{D^2 \times \pi}{4} - \frac{d^2 \times \pi}{4} = \frac{(D^2 - d^2) \times \pi}{4} = (D^2 - d^2) \times {\cdot}785.$$

The basal increment can be ascertained for a limited number of years only, or for the several periods of the life of a tree. An estimate of the future increment is based upon that of the immediate past, taking into consideration the proposed treatment as in the case of the diameter increment.

4. Volume Increment.

a. Of the Past.

The past volume increment of a tree during a certain period of years, n, is equal to the difference of volumes at the commencement and end of the period. These volumes can be ascertained by examining a series of sections at various heights of the tree, or by basing the calculation upon measurements made at the middle section, or by using form factors.

i. Determination of the Increment by Sections.

If the increment of only a limited number of years, n, is desired, it can be ascertained by means of the increment borer. The breadth of n rings is ascertained at regular intervals along

the stem, and the difference between the present volume and that *n* years ago calculated.

The investigation of the progress of increment throughout the life of a tree is called a *stem analysis.* It consists of a combination of a height and a diameter analysis.

Tho tree having been divided into a suitable number of sections, each is cut through in the middle, tho number of concentric rings counted, and the diameter at the several ages measured. The measurements are best plotted, so that a representation of a longitudinal section through the tree is obtained. For this purpose, the heights of the several cross sections from the ground are marked on a vertical lino, which represents the axis of the stem; also the heights which the tree had obtained at successive periods of its life. Next, the radii, or diameters, of the cross sections are marked on horizontal lines, and the points thus obtained connected by a series of lines which represent the stem curves at the several stages during the life of the tree. From the data thus obtained, the increment throughout the several periods of the life of the tree can be calculated. As the thickness of the bark at former periods cannot be ascertained, these investigations can refer only to the increment in wood, exclusive of bark.

Example:—

Analysis of a Scotch Pine Tree.

The tree was cut up into nine pieces, which gave the following cross sections:—

Section I.	taken at	foot of tree,	showing	97 concentric rings.
II.	„	5 feet above ground,	„	95 „ „
III.	„	15 „ „ „	„	89 „ „
IV.	„	25 „ „ „	„	85 „ „
V.	„	35 „ „ „	„	80 „ „
VI.	„	45 „ „ „	„	72 „ „
VII.	„	55 „ „ „	„	64 „ „
VIII.	„	64 „ „ „	„	34 „ „
IX.	„	68 „ „ „	„	26 „ „

Top = 9 feet long. Total hoight = 77 feet.

Height of Section. Feet.	Number of Rings.	Number of Years which the Tree took to reach that Height.
0	97	0
5	95	2
15	89	8
25	85	12
35	80	17
45	72	25
55	64	33
64	34	63
68	26	71
77	0	97

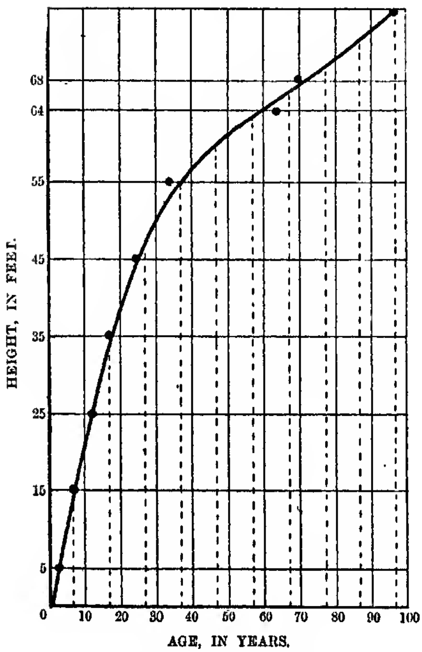

Fig. 27.—Graphic representation of the *Height* Increment.

Note.—The tree grew in a very favourable locality, but it was overtopped by other trees at the age of about 40 years; hence the reduced height growth after that period.

Radius of Section I. at foot of tree, In Inches.	Radius of Section II. at 5′ from the ground, in Inches.	Radius of Section III. at 15′ from the ground, In Inches.	Radius of Section IV. at 25′ from the ground, in Inches.
Total = 11·50	Total = 8·82	Total = 6·92	Total = 6·38
97 = 10·56	95 = 8·32	89 = 6·78	85 = 6·21
87 = 9·88	85 = 7·86	79 = 6·45	75 = 5·94
77 = 9·22	75 = 7·34	69 = 6·16	65 = 5·62
67 = 8·50	65 = 6·77	59 = 6·04	55 = 5·30
57 = 7·65	55 = 6·25	49 = 5·46	45 = 4·99
47 = 6·71	45 = 5·74	39 = 4·95	35 = 4·41
37 = 5·74	35 = 5·06	29 = 4·24	25 = 3·80
27 = 4·94	25 = 4·34	19 = 3·50	15 = 2·81
17 = 3·83	15 = 3·38	9 = 2·25	5 = 1·30
7 = 1·85	5 = 1·30		
Radius of Section V. at 35′ from the ground, in Inches.	**Radius of Section VI. at 45′ from the ground, in Inches.**	**Radius of Section VII. at 55′ from the ground, in Inches.**	**Radius of Section VIII. at 64′ from the ground, in Inches.**
Total = 6·03	Total = 5·81	Total = 3·54	Total = 2·12
80 = 5·96	72 = 5·75	64 = 3·16	34 = 2·07
70 = 5·71	62 = 5·40	54 = 2·98	24 = 1·66
60 = 5·28	52 = 4·87	44 = 2·40	14 = ·88
50 = 4·80	42 = 4·31	34 = 1·85	4 = ·24
40 = 4·37	32 = 3·74	24 = 1·40	
30 = 3·85	22 = 3·05	14 = 1·03	
20 = 2·95	12 = 1·90	4 = ·41	**Radius of Section IX. at 68′ from the ground, in Inches.**
10 = 1·35	2 = ·35		Total = 1·43
			26 = 1·39
			16 = 1·02
			6 = ·50

CALCULATION OF THE VOLUME OF THE TREE AT DIFFERENT AGES.

Number of Section.	Diameter, in Inches.	Basal Area, in Square Feet.	Length, in Feet.	Volume, in Cubic Feet.	Number of Section.	Diameter, in Inches.	Basal Area, in Square Feet.	Length, in Feet.	Volume, in Cubic Feet.
Whole Tree, including Bark; age = 97 Years.					*Whole Tree, without Bark; age = 97 Years.*				
1	17·6	1·69	10	16·9	1	16·6	1·50	10	15·0
2	13·8	1·04	10	10·4	2	13·6	1·01	10	10·1
3	12·8	·89	10	8·9	3	12·4	·84	10	8·4
4	12·1	·80	10	8·0	4	11·9	·77	10	7·7
5	11·6	·73	10	7·3	5	11·5	·72	10	7·2
6	7·1	·27	10	2·7	6	6·9	·26	10	2·6
7	4·2	·10	8	·8	7	4·1	·09	8	·7
		Total Timber =		55·0			Total Timber =		51·7
8	2·9	·05	$\frac{8}{3}$*	·15	8	2·8	·04	$\frac{8}{3}$	·12
		Total Timber and Fuel =		55·15			Total Timber and Fuel =		51·82

* The top is considered as representing a cone, the volume of which = basal area × one-third of the height. Minimum size of timber = 3 inches.

CALCULATION OF THE VOLUME OF THE TREE AT DIFFERENT AGES.—*cont*

Number of Section.	Diameter, in Inches.	Basal Area, in Square Feet.	Length, in Feet.	Volume, in Cubic Feet.
		Tree 87 Years Old.		
1	15·7	1·34	10	13·4
2	12·9	·91	10	9·1
3	11·9	·77	10	7·7
4	11·4	·71	10	7·1
5	10·8	·64	10	6·4
6	6·0	·20	10	2·0
7	3·3	·06	8	·5
			Total Timber =	46·2
8	2·0	·02	$\frac{5}{3}$	·03
			Total Timber and Fuel =	46·23
		Tree 77 Years Old.		
1	14·7	1·18	10	11·8
2	12·3	·83	10	8·3
3	11·2	·68	10	6·8
4	10·6	·61	10	6·1
5	9·7	·51	10	5·1
6	4·8	·13	10	1·3
			Total Timber =	39·4
7	1·8	·02	8	·16
8	1·0	·01	$\frac{1}{3}$	—
			Total Timber and Fuel =	39·56
		Tree 67 Years Old.		
1	13·5	·99	10	9·9
2	12·1	·80	10	8·0
3	10·6	·61	10	6·1
4	9·6	·50	10	5·0
5	8·6	·40	10	4·0
6	3·7	·07	10	·7
			Total Timber =	33·7
7	2·0	·02	$\frac{6}{3}$	·04
			Total Timber and Fuel =	33·74
		Tree 57 Years Old.		
1	12·5	·85	10	8·5
2	10·9	·65	10	6·5
3	10·0	·55	10	5·5
4	8·7	·41	10	4·1
5	7·5	·31	10	3·1
			Total Timber =	27·7
6	2·8	·04	10	·4
7	1·0	·01	$\frac{4}{3}$	·01
			Total Timber and Fuel =	28·11
		Tree 47 Years Old.		
1	11·5	·72	10	7·2
2	9·9	·53	10	5·3
3	8·8	·42	10	4·2
4	7·7	·32	10	3·2
5	6·1	·20	10	2·0
			Total Timber =	21·9
6	2·1	·02	10	·20
7	·6	·002	$\frac{1}{3}$	—
			Total Timber and Fuel =	22·10
		Tree 37 Years Old.		
1	10·1	·56	10	5·6
2	8·5	·39	10	3·9
3	7·6	·32	10	3·2
4	5·9	·19	10	1·9
5	3·8	·08	10	·8
			Total Timber =	15·4
6	2·4	·03	$\frac{7}{3}$	·07
			Total Timber and Fuel =	15·47
		Tree 27 Years Old.		
1	8·7	·41	10	4·1
2	7·0	·27	10	2·7
3	5·6	·17	10	1·7
			Total Timber =	8·5
4	2·7	·04	10	·40
5	1·6	·01	$\frac{7}{3}$	·02
			Total Timber and Fuel =	8·92
		Tree 17 Years Old.		
1	6·8	·25	10	2·5
2	4·5	·11	10	1·1
			Total Timber =	3·6
3	2·6	·04	10	·40
4	1·2	·01	$\frac{5}{3}$	·02
			Total Timber and Fuel =	4·02
		Tree 7 Years Old.		
			Total Timber =	0·0
1	2·6	·04	10	·4
2	1·2	·01	$\frac{4}{3}$	·01
			Total Timber and Fuel =	0·41

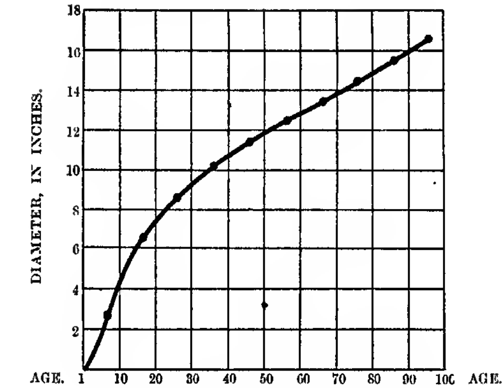

Fig. 28.—Graphic representation of the *Diameter* Increment at 5 feet from the ground.

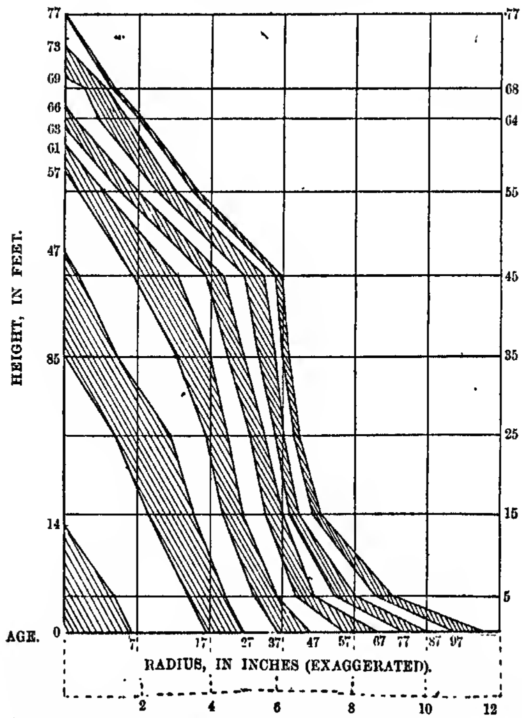

Fig. 29.—Graphic representation of a *Tree Analysis* (Vertical section of one-half of the Tree).

Recapitulation.

The stem of the tree had, at the age of 97 years:

A total volume of = 55·15 cubic feet.

Of this was

$$\text{Bark} \begin{cases} = 55{\cdot}15 - 51{\cdot}82 = 3{\cdot}33 \text{ cubic feet.} \\ = 6 \text{ per cent. of total volume.} \end{cases}$$

Leaving

Timber (over 3″ diameter)	=	51·7 cubic feet.
Firewood	=	·12 „ „

Total Timber and Firewood = 51·82 cubic feet.

By graphically representing the volume of wood at the several ages, Fig. 30 is obtained, which, with the previous diagrams, gives the following data:—

Age of Tree.	Height, in Feet.	Diameter without Bark, at 5′ above ground, Inches.	Volume without Bark, in Cubic Feet.	Periodic Increment, for every Ten Years. Cubic Feet.
0	—	—	—	
				1·8
10	20	4·0	1·8	
				4·2
20	38	7·4	6·0	
				6·0
30	51	9·1	12·0	
				6·0
40	58	10·6	18·0	
				6·0
50	62	11·9	24·0	
				5·9
60	64	12·8	29·9	
				5·4
70	67	13·9	35·3	
				6·7
80	70	15·0	42·0	
				6·0
90	74	16·0	48·0	
				3·8
97	77	16·6	51·8	
			Total	51·8

ii. Determination of the Increment by the Middle Section.

If somewhat less accurate results suffice, the volumes can be ascertained by multiplying the basal area in the middle by the height. Let V be the volume of the tree at the present time,

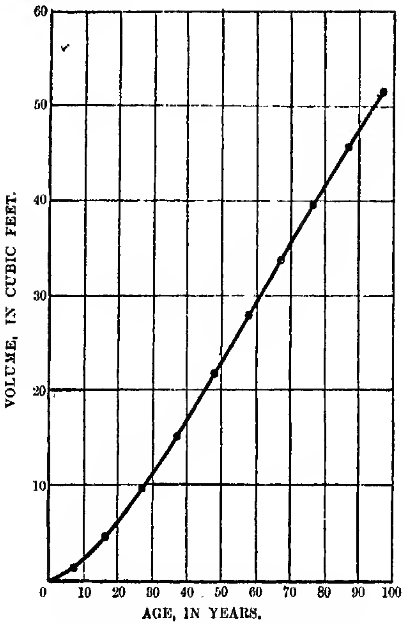

Fig. 30.—Graphic representation of the *Volume*.

v that n years ago, H and h the corresponding heights, Fig. 31 and S and s the corresponding basal areas at $\frac{H}{2}$ and $\frac{h}{2}$, then

$$I = V - v = S \times H - s \times h.$$

The height and the basal area at $\frac{H}{2}$ can easily be measured

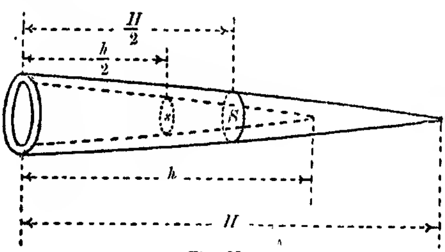

Fig. 31.

The height h, n years ago, if it cannot be ascertained by external marks, is ascertained by cutting off a piece from the top downwards, and repeating the operation, until the point has been ascertained, where the basal area contains n concentric rings; then the basal area s, at $\frac{h}{2}$, is ascertained. The breadth of the last n rings can be obtained with the increment borer. In either case, the diameter increment must be measured in several places, so as to obtain the mean.

In order to simplify the operation, Pressler proposed to cut off a length corresponding to n years' height growth in the

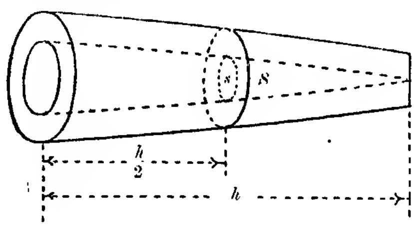

Fig. 32.

first place, and then to measure the basal area in both cases at $\frac{h}{2}$. (See Fig. 32.) He obtains the increment according to the formula:

$$I = S \times h - s \times h = (S - s)\, h.$$

The error due to omitting the top is said to be compensated for by S being taken somewhat below $\frac{H}{2}$.

iii. Determination of the Increment by Form Factors.

Let S be the basal area of a tree taken at chest height; s the basal area of the tree n years ago taken at the same height; H and h the corresponding heights, and F and f the corresponding form factors, then the increment—

$$I = S \times H \times F - s \times h \times f.$$

In tho case of a standing tree, H is measured with a height measurer, h is estimated or taken from tables of height growth, tho proportion between the outer and inner diameters being utilised to determine h. S is obtained by measuring the diameter with a calliper, and s with the assistance of an increment borer. F and f must be obtained from form factor tables, or estimated. If F is taken as $= f$, the formula becomes:

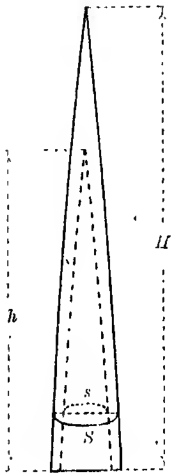

Fig. 33.

$$I = (S \times H - s \times h) \times F.$$

The method can give only approximately correct results, because h has to be estimated. It must also not be overlooked that form factor tables give only averages; hence, the method is not adapted to the measurement of a single tree, but only to that of a large number of trees. Even in the latter case, the results can be only approximately correct.

b. Volume Increment of the Future.

The increment which a tree may be expected to lay on in the future can be estimated from its own past increment, especially that of the immediate past.

The increment is represented by the formula:

$$I = S \times H \times F - s \times h \times f,$$

where $s \times h \times f$ represents the present volume, and $S \times H \times F$ that to be expected after n years. The formula shows that, in order to obtain fairly accurate results, it is necessary to estimate S, H and F from s, h and f. How this should be done as regards basal area and height has been explained above. The form factor F may be obtained from tables if

such are available; otherwise it must be estimated, or it may be taken as equal to f.

Instead of estimating the separate factors, the volume increment of the next n years may be estimated direct from that laid on during the last n years, taking into consideration how far the latter should be modified with regard to the age of the tree, locality, future treatment of the wood, especially the proposed degree of thinning, etc.

According to Pressler's method, the probable increment can be ascertained by estimating the probable diameter increment and then proceeding by the formula:

$$I_n = (S - s)\, h,$$

where s represents the present section in the middle, S the expected section in the same spot after n years, and h the present height. The method applies only to felled trees; hence, it necessitates the felling of one or more sample trees.

SECTION II.—DETERMINATION OF THE INCREMENT OF WHOLE WOODS.

It has been shown that, in the case of single trees, the accumulation of the volume, as well as the increase of the factors which lead up to it, height, diameter, or basal area increment, can be followed backwards with a considerable degree of accuracy. This is not the case as regards whole woods, because trees die or are taken away in thinnings. Investigations made on sample trees selected in a wood show only the gradual development of the individuals existing at the time of examination, but they throw no light on that of those trees which have disappeared in course of time since the wood was created. Height growth alone makes an exception. An analysis of a number of sample trees will indicate the mean height of these trees during previous periods, which may be taken as the upper height of the wood at those times. These would, of course, not represent the mean heights at the several ages, because it may safely be assumed

that the now existing trees were, as a rule, always the leading trees. Investigations have proved that the mean height of woods can be deduced from the upper height. For instance, in the case of the Scotch pine the difference ranges from about 3 to 5 per cent. according to the age of the wood. But no such relation has as yet been found as regards the basal area or the volume, and to evolve the former amounts of these out of the present quantities is more or less speculative. Under these circumstances, one of the two methods about to be described may be followed.

A. Determination of the Future Increment according to the Mean Annual Increment of the Past.

The present volume of the wood is ascertained and divided by its age, the quotient giving the mean annual increment calculated on the growing stock present at the time of measurement. According to the age of the wood, it may be assumed that the mean annual increment will be laid on for a number of years to come, or a somewhat diminished or increased increment.

The method gives fair results, if the calculation is made for the time when the mean annual increment culminates and even for older woods; it is less accurate in the case of younger woods. Moreover, it is only applicable for a limited number of years.

B. Determination of the Increment by means of Yield Tables.

I.—Yield Tables Generally.

1. Definition of Yield Tables.

It has already been explained that the progress of height, diameter, basal area, and volume increment can be represented by curves constructed on the principle that the successive ages are marked as abscissæ, and that the corresponding ordinates represent the height, diameter, basal area, or volume. Such curves indicate the appropriate quantities for

any age up to a certain limit, generally the highest rotation likely to be adopted. Instead of employing curves, the data which they represent are read off and arranged in tables, and these are called *Yield Tables.*

By a yield table is understood a tabular statement which gives the course of the development of a wood from early youth up to a certain age, either from year to year, or for intervals of a certain number of years.

2. Object and Contents of Yield Tables.

Yield tables are used for a great variety of purposes, as:

(*a.*) Determination of the volume of woods.
(*b.*) „ „ „ increment of woods.
(*c.*) „ „ „ quality of localities or growing stock.
(*d.*) „ „ „ most profitable species, method of treatment, and rotation.
(*e.*) „ „ „ value of the soil, growing stock, or both.
(*f.*) „ „ „ yield of forests.

In order to meet all these requirements, yield tables should show, per unit of area (acre):

(1.) The number of trees.
(2.) The mean diameter of trees.
(3.) The basal area of trees.
(4.) Tho height of the wood.
(5.) The volume, which may bo found in a fully-stocked wood at successive ages; also the yield of thinnings.
(6.) The current annual and mean annual increment.
(7.) The form factors.

Separate yield tables must be prepared for

(*a.*) Each species.
(*b.*) Each method of treatment, as high forest, coppice woods, and combination forest.
(*c.*) Each quality of locality.

The volume is given divided into the different classes of wood, as timber, firewood, etc. The volume of thinnings, or intermediate yields generally, is entered separately from that of final yields.

Yield tables are prepared only for "normal" woods, that is to say, for woods which are fully stocked, taking into consideration the species, quality of locality, and the adopted method of treatment. Such woods are produced if no extraordinary influences have interfered with their progress, such as natural phenomena, faulty treatment, etc.

3. Local and General Yield Tables.

If a yield table has been prepared for a particular district of limited extent, it is called a *local* yield table; if for a whole province or county, a *general* yield table.

The question, what territorial limits should be assigned to the applicability of a yield table is still under discussion, but so much is certain that, in the preparation of such tables, a considerable extent of country can be thrown together without incurring any appreciable inaccuracy. It has been proved, for instance, that yield tables prepared for North Germany are quite applicable to the south and centre of England.

4. Quality Classes.

Localities of different quality, or yield capacity, produce woods which follow in their development different laws. The law of increment of the one cannot be evolved out of that of the other. The preparation of each yield table should, therefore, be based on data obtained from localities of precisely the same quality. As a rule, however, a large number of different qualities exist; hence, in practical forestry, some concession must be made by being satisfied with a limited number. These rarely exceed five, and frequently three are quite sufficient. The best quality is generally designated as I. quality, though the reverse would be better.

In proceeding to construct yield tables, it is obviously of the first importance to have a ready method by which the quality of a locality may be determined. It has been explained in Silviculture that the several factors of the locality, such as the chemical and physical conditions of the soil and subsoil, the climate, etc., do not enable the forester to determine the quality of tho locality for forest purposes with any degree of accuracy, and that the only satisfactory indication is given by the wood which has been produced on it. In other words, a locality which produces, in a given time, a large volume, is of good quality; one which produces a small volume, of inferior quality. The volume, then, is in the first place the surest indication of the quality of the locality.

As it is, however, a somewhat cumbrous process to ascertain the volume when searching for a certain quality, the question arises, whether one or more of the elements, from which the volume is calculated, would not do equally good service. It has been shown above that the volume $V = s \times h \times f$.

Of these three elements, the form factor moves between comparatively narrow limits, and it is not suited for the prosent purpose, apart from the fact that the volume would first have to be ascertained in order to determine the form factor. Basal area and height together give a sure indication of the quality, that is to say, two woods which show the same basal area and height may safely be assumed to have the same volume; hence, the localities which have produced them would be of the same quality.

If only one indicating element is used, the height is far preferable to the basal area. While two woods of the same species and equal basal areas may have very different heights, experience has shown that two normal woods of the same height have approximately equal basal areas. It follows that the height is, next to the volume itself, the best indicator of the quality of the locality, and it is easily ascertained. Great height growth means good quality, small height growth inferior quality of locality.

Neither the mean diameter nor the number of treos can be used for the above purpose, as they are not in due proportion to the volume. Nor can the product of the number of trees multiplied by the mean diameter be used.

5. Methods of Constructing Yield Tables.

The following methods have been proposed :—

a. Annual or Periodical Measurement of the Growing Stock of one and the same Wood; in the second case, the Intermediate Values are found by Interpolation.

The method givos absolute certainty that all figures of the yield table are derived from the same quality class, but as tho preparation of the table would take a century and moro, tho method has only theoretical value. Moroover, accidents may happen which would render the wood unfit for further observation.

b. Annual or Periodical Measurement of the Growing Stock of a limited number of Woods of different Ages.

In order to save time, it has been proposed to select several woods differing in age by a certain number of years, say 20, and to obtain from the measurements of each, extending over 20 years, part of the yield table. To mako sure that the quality of the several woods is the same, it is nocessary that they should have the same volume at the samo age; in other words, the wood now 40 years old should have had, when 20 years old, the same volume as the present 20 years old wood has; again, the 60 years old wood the same volume, when it was 40 years old, as the present 40 years old wood, etc. Or, to put the matter differently, the 20 years old wood should have, when it becomes 40 years old, the same volume as the 40 years old wood has now, etc. In addition, tho progress of the increment should be steady throughout.

Although it is difficult to select localities on these lines which are of exactly the same quality, or woods which will

develop in the same manner, there can be no doubt that ultimately satisfactory yield tables can be obtained only by observing and periodically measuring suitable woods for a series of years. Hence, the method is actually followed. For each quality class, and age gradation, several sample plots are selected, and these are periodically measured and the mean taken. In this way, yield tables will ultimately be obtained. It is necessary to take several plots for each quality and age gradation, so as to obtain average results, and because one or other may become unfit for the purpose in consequence of unforeseen events.

c. Measurement of a large number of Woods of different Ages once so that Yield Tables are obtained immediately.

Until yield tables, prepared as indicated under *b*, become available, others for immediate use are required. These are obtained by measuring fully-stocked sample plots in a sufficient number of woods, representing all ages with moderate intervals. Out of the data thus obtained, steady curves and tables are prepared. A separate set of woods is required for each quality class, and the great difficulty consists in selecting for each set localities of the same quality. For this purpose various methods have been suggested. Most of these start from an indicating wood, while one, specially elaborated by Baur,* starts from a different principle; the latter will be dealt with in detail further on, as it is the most practical method.

i. Selection of Woods for each Quality Class by Means of an Indicating Wood.

The method is based upon the fact that the older wood has been evolved out of the younger, in other words that the older wood had at one time the same volume as the younger. Hence, it should be possible, by analysing a number of sample trees, to ascertain the volume, or basal area, height, and form factor, which the trees of a mature wood had at the several periods of

* Late Professor of Forestry at the University of Munich.

their life. Guided by the data thus obtained, woods are selected, the dominant trees of which show the same dimensions as those which the mature trees had at the same age. Such woods are assumed to give true representations of what the now mature wood was at the same ages. When a sufficient number of woods of various ages has been selected, sample plots with normal stocking are measured in them, and the data worked up into a yield table for the corresponding quality class. The same procedure is followed for all other quality classes.

Various authors have gradually elaborated this system, first Seutter as early as 1799, then Hossfeld in 1823. Huber, in 1847, was the first to give a regular method of working with an indicating wood. He calculated the mean tree of a normal, mature wood, analysed it and searched for younger normal woods, the mean trees of which possess the same dimensions as the mean tree of the mature wood had at the same ages. His method is, however, wrong, because the mean tree of the mature wood was not the mean tree at all former stages of life.

Theodor Hartig, and afterwards Robert Hartig, analysed the largest trees of the mature wood, and then searched for younger woods, an equal number of the largest trees of which show the same dimensions as the largest trees of the mature wood had at the same ages. Such woods are considered as having been produced on localities of the same quality, so that they can be used for the preparation of one yield table.

The system pre-supposes that the largest trees of the mature wood were amongst the largest trees at all previous periods of the wood's life. Although this holds good generally, exceptions occur. Besides, the method is very troublesome in execution.

ii. Baur's Method of Preparing Yield Tables.

After a sufficient number of normal sample plots on all sorts of qualities and ages have been carefully measured (at least 80

for each quality class), the volumes are marked as ordinates over the corresponding ages as abscissæ (Fig. 34).

Next, two curves are drawn, so that the lower touches the lowest points and the upper the highest points indicating these volumes. Then, the area thus confined is divided into as many equal strips as there are quality classes to be distinguished. The woods falling into each strip are considered as belonging to the same quality class. By drawing a mean curve through each strip, the mean volume curve for the quality is obtained from which the volume table is prepared for successive years. In a similar way, mean curves for the height, basal area, and number of trees are constructed for each quality class. The method is of easy application, and it yields good results.

Baur's method, combined with that described under (*b*), has been utilised in the preparation of yield tables for Germany. The work was commenced some 35 years ago on a prearranged systematic plan, and has led to the compilation of a number of tables. Those which appeared best adapted for the conditions in Southern and Central England will be found in Appendix III. There is no pretence on the part of the author to maintain that the tables absolutely apply to the conditions found in the United Kingdom, but there is sufficient proof to show that the data given in them hold good approximately for woods in the south and centre of England, and that they may safely be used until tables have been prepared based on data collected in this country. In may be said that the most urgent need of British forestry is the collection of statistics, by means of which the financial results of the industry can be estimated. Until such statistics become available, no substantial progress is likely to be made, unless the results obtained elsewhere are temporarily admitted.

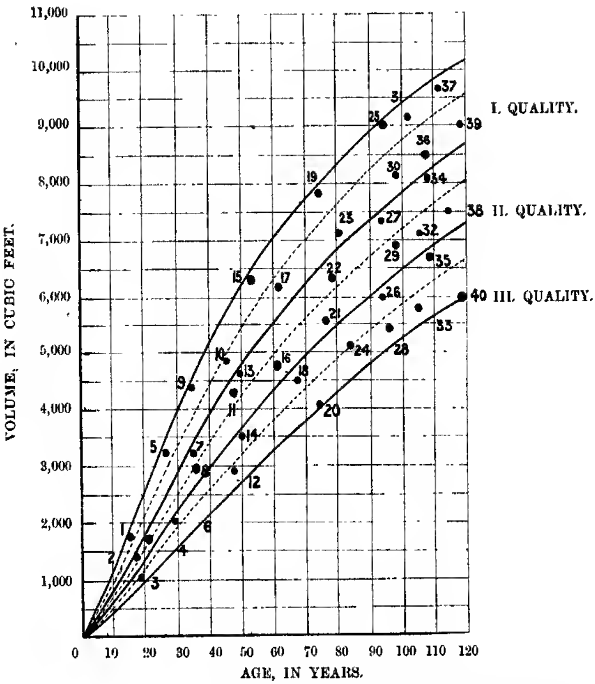

Fig. 34.—Graphic Representation of the *Volume per Acre* of 40 different Woods and their allotment to Three Quality Classes, according to Baur's method.

EXAMPLE OF PREPARING A YIELD TABLE ACCORDING TO BAUR'S METHOD.

Scotch Pine : 3 Quality Classes to be distinguished.

Woods Measured as follows :—

No.	Age. Years.	No. of Trees.	Basal Area, sq. ft.	Mean Height, feet.	Volume in solid cub. ft.	No.	Age. Years.	No. of Trees.	Basal Area, sq. ft.	Mean Height, feet.	Volume in solid cub. ft.
1	15	...	62	16	1800	21	76	295	173	70	5500
2	17	...	60	14	1100	22	79	265	177	72	6300
3	18	...	61	13	1100	23	81	245	192	86	7200
4	21	...	81	20	1700	24	85	290	156	62	5030
5	27	1100	130	33	3300	25	94	190	196	93	9000
6	29	2100	99	25	2050	26	94	240	150	67	6000
7	34	1480	133	35	3250	27	94	218	177	80	7300
8	35	1670	113	32	2800	28	96	248	150	69	5300
9	35	910	156	46	4150	29	97	200	176	82	6950
10	46	620	165	55	4800	30	99	170	194	93	8200
11	47	740	150	47	4230	31	104	160	192	94	9200
12	48	860	132	40	2900	32	106	169	177	86	7150
13	49	680	154	52	4700	33	106	220	160	69	5700
14	50	750	132	44	3500	34	108	173	179	86	8100
15	54	450	182	69	6400	35	109	210	152	72	6700
16	62	450	169	65	4700	36	109	151	196	96	8500
17	62	369	184	73	6200	37	112	148	194	98	9700
18	68	420	148	56	4450	38	115	150	176	88	7500
19	74	270	192	83	7800	39	118	145	194	98	9000
20	74	350	146	61	4000	40	120	186	157	75	6000

WOODS SEPARATED INTO QUALITY CLASSES. (FIG. 34.)

No.	Age.	No. of Trees.	Basal Area.	Mean Height.	Volume.	No.	Age.	No. of Trees.	Basal Area.	Mean Height.	Volume.
		I. Quality.				13	49	680	154	52	4700
1	15	...	62	16	1800	16	62	450	169	65	4700
5	27	1400	130	33	3300	21	76	295	173	70	5500
9	35	910	156	46	4450	22	79	265	177	72	6300
10	46	620	165	55	4800	27	94	218	177	80	7300
15	54	450	182	69	6400	29	97	200	176	82	6950
17	62	369	184	73	6200	32	106	169	177	86	7150
19	74	270	192	83	7800	34	108	173	179	86	8100
23	81	245	192	86	7200	38	115	150	176	88	7500
25	94	190	196	93	9000				*III. Quality.*		
30	99	170	194	93	8200	3	18	...	61	13	1100
31	104	160	192	94	9200	6	29	2400	99	25	2050
36	109	151	196	96	8500	12	48	860	132	40	2900
37	112	148	194	98	9700	14	50	750	132	44	3500
39	118	145	194	98	9000	18	68	420	148	56	4450
						20	74	350	146	61	4000
		II. Quality.				24	85	290	156	62	5030
2	17	...	60	14	1400	26	94	240	150	67	6000
4	21	...	84	20	1700	28	96	248	150	69	5300
7	34	1480	133	35	3250	33	106	220	160	69	5700
8	35	1670	113	32	2800	35	109	210	152	72	6700
11	47	740	150	47	4230	40	120	186	157	75	6000

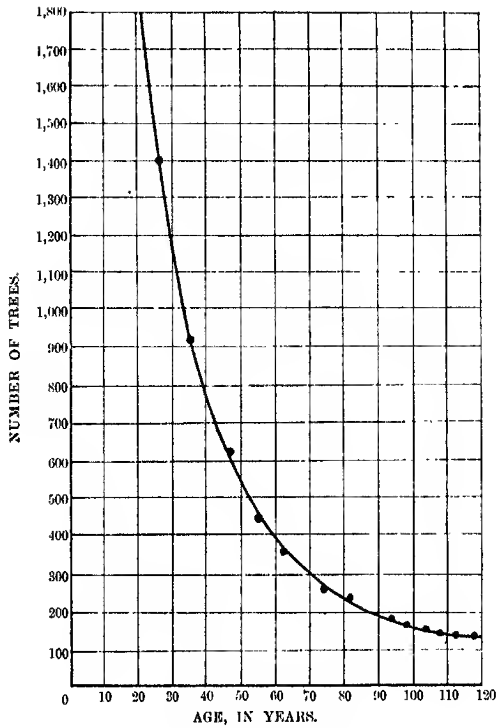

Fig. 35.—Graphic Representation of the *Number of Trees* per Acre.

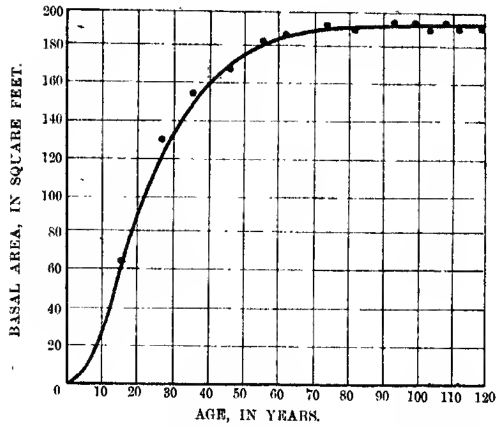

Fig. 36.—Graphic Representation of the *Basal Areas* per Acre.

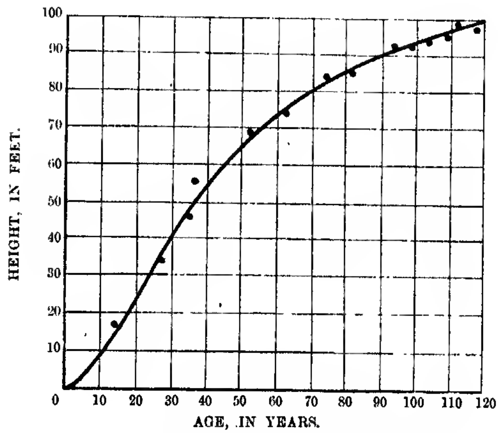

Fig. 37.—Graphic Representation of the *Height Growth*.

YIELD TABLE FOR THE SCOTCH PINE, I. QUALITY.

Derived from the Curves in Figs. 34, 35, 36 and 37.

Age.	Number of Trees.	Basal Area. Square Feet.	Mean Height. Feet.	Volume. Cubic Feet, Solid.	Increment.	
					Current Annual. Cubic Feet.	Mean Annual. Cubic Feet.
10	...	30	10	900	90	90
20	2000	92	23	2100	120	105
30	1200	133	40	3300	120	110
40	770	160	54	4500	120	112
50	520	175	64	5400	90	108
60	380	186	73	6250	85	104
70	300	190	80	6950	70	99
80	250	192	86	7600	65	95
90	200	193	90	8200	60	91
100	160	194	94	8650	45	86
110	150	194	97	9100	45	83
120	140	194	100	9500	40	79

II.—Determination of the Increment of Woods by Means of Yield Tables.

If yield tables are available, and it is desired to estimate the increment of a wood forward or backward, it is necessary to decide in the first place which of the quality classes of the tables corresponds with that of the given wood; in other words, it must be ascertained to which quality class the wood belongs.

The best way of doing this is, to measure the volume of a normal sample plot in the wood and compare it with the volumes given in the tables for the same age and the different quality classes. If it agrees with one of these volumes, the two are of the same quality class, and the increment shown in the table applies also to the wood in question.

If the volume of the wood does not agree with any of the volumes in the tables, then that quality class is selected which comes nearest to it, and the increment is ascertained in proportion to the two volumes. Let v_a be the present volume of the wood, V_a the nearest volume given in the table; V_{a+n} the volume given in the same table for the year $a+n$; and

v_{a+n} the desired volume of the wood in the year $a + n$, then the following equation may be assumed to hold good (see Fig. 38):—

$$V_a : v_a = V_{a+n} : v_{a+n} \text{ and } v_{a+n} = V_{a+n} \times \frac{v_a}{V_a}$$

and

$$I = \text{Increment in } n \text{ years} = v_{a+n} - v_a = \frac{V_{a+n} \times v_a}{V_a} - v_a$$

$$\mathbf{I} = \mathbf{v_a} \left(\frac{\mathbf{V_{a+n}}}{\mathbf{V_a}} - 1 \right).$$

This method rests upon the assumption that the selected yield table correctly represents the progressive increment of

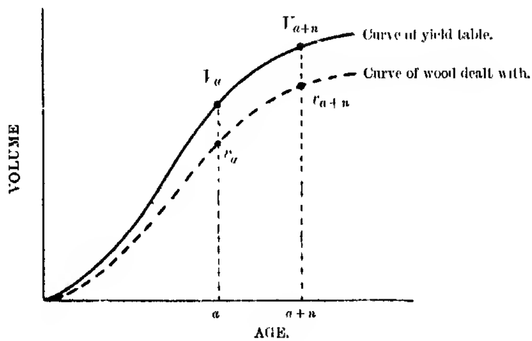

Fig. 38.

the wood of which the increment is to be ascertained. As this is only approximately the case, the degree of accuracy of the method depends—

(*a*.) On the degree to which v_a approaches V_a,

(*b*.) On the difference of ages, that is to say, the difference between a and $a + n$; the smaller this is, the more accurate will be the result.

In following the above method, it is essential to measure the volume on a "normal" sample plot, because only then can the true quality class be ascertained by means of the volume. If no fully stocked sample plot is available, that nearest to it should be selected, and the proportion between the actual and

normal stocking ascertained. The actual volume must then be augmented in the same proportion, before it is used for the determination of the quality class and the selection of the yield table. At the same time, this procedure is subject to errors, as it is not always easy to determine correctly the proportion between the actual and normal stocking.

Generally, the method is bettor adapted to woods which have passed middle age than to younger woods, as in the latter the factors of the locality have not in all cases found full expression. In tho case of very young woods, it is altogether useless to measure the volume for the purpose of selecting the proper yield table. For such woods, the quality class must be determined by means of an older wood growing in the vicinity on a locality of similar quality; the same procedure is followed in the case of blanks. If no such older wood is available, the soil and climate must be examined and the best possible estimate of the quality made accordingly.

As the measurement of the volume takes much time, and as it is difficult to estimate the exact proportion between the actual and normal stocking, it has been proposed to select the proper yield table for a wood by means of one factor of the volume. It has already been explained that of all such, as number of trees, diameter, form factors, basal area and height, the last is the most suitable. Indeed, actual investigation has proved that, in the case of all woods of middle age and upwards, the volume of two woods, other conditions being the same, is fairly proportionate to their mean heights. The mean height is, therefore, an excellent indication of the quality class; it, as well as the age, is comparatively easy to ascertain. In selecting the appropriate yield table, the mean height is used in the same way as has been described for the volume. If the height agrees with one of the heights given in the yield table for the same age, the increment can be read off directly. If it differs, the nearest is selected and the increment of the table modified in proportion to the difference between the actual height and that given in the table. Should,

moreover, the wood not be fully stocked, then the increment given in the table must be further modified in the manner indicated above.

The height by itself is no true indicator of the quality for very young woods; for such, as well as for blanks, other woods growing in the vicinity must be utilised, or the soil and climate examined.

Example.—A Scotch pine wood has a height of 53 feet when 60 years old, and a volume of 3,800 cubic feet; find the probable increment for the next ten years.

YIELD TABLES.

(See Appendix III.)

Quality.	Height at 60 Years.	Volume in the Year 60.	Volume in the Year 70.
I.	73	6,960	—
II.	52	4,660	5,070
III.	30	2,170	—

The wood belongs to the II. Quality.

According to the formula—

$$I_n = (v_{a+n} - v_a) \times \frac{h_a'}{h_a} \times \frac{v_a'}{v_a}$$

$$I_{60-70} = (5{,}070 - 4{,}660) \times \frac{53}{52} \times \frac{3{,}800}{4{,}660} = 341 \text{ cubic feet}$$

APPENDICES.

APPENDIX I.

A. Area of circles for diameters ranging from 1 inch to 60 inches.

B. Sum of the areas of circles for diameters ranging from 1 inch to 48 inches,

and,

Volume of cylinders for diameters ranging from 1 inch to 48 inches and any length.

Example:

Find the area of 24 circles of 15 inches diameter.

	Square Feet.
Area of 20 circles = 10 × 2·4544 =	24·544
Area of 4 circles =	4·9088
Total =	29·4528

or

Find the volume of a log 24 feet long with a mean diameter of 15 inches.

	Cubic Feet.
Volume of a log 20 feet long = 10 × 2·4544 =	24·544
Volume of a log 4 feet long =	4·9088
Total =	29·4528

A. Area of Circles for Diameters

Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.
1·0	0·0055	**2·0**	0·0218	**3·0**	0·0491	**4·0**	0·0873	**5·0**	0·1364
1	·0067	1	·0240	1	·0524	1	·0917	1	·1418
2	·0079	2	·0264	2	·0559	2	·0963	2	·1474
3	·0092	3	·0289	3	·0594	3	·1009	3	·1532
4	·0107	4	·0314	4	·0631	4	·1056	4	·1590
5	·0123	5	·0341	5	·0669	5	·1105	5	·1650
6	·0140	6	·0369	6	·0707	6	·1154	6	·1710
7	·0158	7	·0398	7	·0747	7	·1205	7	·1772
8	·0177	8	·0428	8	·0788	8	·1257	8	·1835
9	·0197	9	·0459	9	·0830	9	·1310	9	·1899
11·0	0·6600	**12·0**	0·7854	**13·0**	0·9218	**14·0**	1·0690	**15·0**	1·2272
1	·6721	1	·7986	1	·9360	1	1·0843	1	1·2437
2	·6842	2	·8118	2	·9504	2	1·0997	2	1·2602
3	·6965	3	·8252	3	·9648	3	1·1153	3	1·2768
4	·7089	4	·8387	4	·9794	4	1·1309	4	1·2936
5	·7214	5	·8523	5	·9941	5	1·1467	5	1·3104
6	·7340	6	·8660	6	1·0089	6	1·1626	6	1·3274
7	·7467	7	·8798	7	1·0237	7	1·1785	7	1·3444
8	·7595	8	·8937	8	1·0387	8	1·1946	8	1·3616
9	·7724	9	·9077	9	1·0538	9	1·2108	9	1·3789
21·0	2·4053	**22·0**	2·6398	**23·0**	2·8852	**24·0**	3·1416	**25·0**	3·4088
1	2·4283	1	2·6638	1	2·9103	1	3·1679	1	3·4361
2	2·4514	2	2·6880	2	2·9356	2	3·1942	2	3·4636
3	2·4745	3	2·7122	3	2·9610	3	3·2207	3	3·4911
4	2·4978	4	2·7366	4	2·9864	4	3·2471	4	3·5188
5	2·5212	5	2·7611	5	3·0120	5	3·2748	5	3·5465
6	2·5447	6	2·7857	6	3·0377	6	3·3006	6	3·5744
7	2·5684	7	2·8104	7	3·0635	7	3·3275	7	3·6024
8	2·5921	8	2·8352	8	3·0894	8	3·3545	8	3·6305
9	2·6159	9	2·8602	9	3·1154	9	3·3816	9	3·6587
40	8·7266	41	9·1684	42	9·6211	43	10·0847	44	10·5592
50	13·6354	51	14·1863	52	14·7480	53	15·3207	54	15·9043
60	19·6350								

The circles of full inches were calculated with logarithms

OF 1 INCH TO 60 INCHES.

Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.	Diam. in inches	Area of circle in square ft.
6·0	0·1963	7·0	0·2673	8·0	0·3491	9·0	0·4418	10·0	0·5454
1	·2029	1	·2750	1	·3579	1	·4517	1	·5564
2	·2096	2	·2828	2	·3668	2	·4617	2	·5675
3	·2164	3	·2907	3	·3758	3	·4718	3	·5787
4	·2234	4	·2987	4	·3849	4	·4820	4	·5900
5	·2304	5	·3068	5	·3941	5	·4923	5	·6014
6	·2376	6	·3151	6	·4034	6	·5027	6	·6129
7	·2448	7	·3234	7	·4129	7	·5132	7	·6245
8	·2522	8	·3319	8	·4224	8	·5238	8	·6362
9	·2597	9	·3404	9	·4321	9	·5345	9	·6481
16·0	1·3963	17·0	1·5763	18·0	1·7671	19·0	1·9689	20·0	2·1817
1	1·4138	1	1·5949	1	1·7868	1	1·9897	1	2·2036
2	1·4314	2	1·6136	2	1·8066	2	2·0106	2	2·2256
3	1·4492	3	1·6324	3	1·8265	3	2·0316	3	2·2477
4	1·4670	4	1·6513	4	1·8465	4	2·0527	4	2·2699
5	1·4849	5	1·6703	5	1·8666	5	2·0739	5	2·2922
6	1·5030	6	1·6894	6	1·8869	6	2·0952	6	2·3146
7	1·5212	7	1·7087	7	1·9072	7	2·1167	7	2·3371
8	1·5394	8	1·7280	8	1·9277	8	2·1382	8	2·3597
9	1·5578	9	1·7475	9	1·9482	9	2·1599	9	2·3825
26·0	3·6870	27·0	3·9761	28·0	4·2761	29·0	4·5869	30	4·9087
1	3·7154	1	4·0056	1	4·3067	1	4·6186	31	5·2414
2	3·7439	2	4·0353	2	4·3374	2	4·6504	32	5·5851
3	3·7725	3	4·0650	3	4·3682	3	4·6823	33	5·9396
4	3·8013	4	4·0948	4	4·3991	4	4·7143	34	6·3050
5	3·8301	5	4·1248	5	4·4301	5	4·7464	35	6·6813
6	3·8591	6	4·1548	6	4·4612	6	4·7787	36	7·0686
7	3·8882	7	4·1850	7	4·4925	7	4·8110	37	7·4667
8	3·9174	8	4·2152	8	4·5238	8	4·8435	38	7·8758
9	3·9467	9	4·2456	9	4·5553	9	4·8760	39	8·2958
45	11·0447	46	11·5410	47	12·0482	48	12·5664	49	13·0954
55	16·4988	56	17·1042	57	17·7206	58	18·3478	59	18·9859

of 7 places; the intermediate values were found by interpolation.

B. Table of the Sum of Circles for Diameters of

Number of Circles, or Length of Cylinder.	Diameter in Inches.							
	1	2	3	4	5	6	7	8
1	0·0055	0·0218	0·0491	0·0873	0·1364	0·1963	0·2673	0·3491
2	0·0110	0·0436	0·0982	0·1746	0·2728	0·3926	0·5346	0·6982
3	0·0165	0·0654	0·1473	0·2619	0·4092	0·5889	0·8019	1·0473
4	0·0220	0·0872	0·1964	0·3492	0·5456	0·7852	1·0692	1·3964
5	0·0275	0·1090	0·2455	0·4365	0·6820	0·9815	1·3365	1·7455
6	0·0330	0·1308	0·2946	0·5238	0·8184	1·1778	1·6038	2·0946
7	0·0385	0·1526	0·3437	0·6111	0·9548	1·3741	1·8711	2·4437
8	0·0440	0·1744	0·3928	0·6984	1·0912	1·5704	2·1384	2·7928
9	0·0495	0·1962	0·4419	0·7857	1·2276	1·7667	2·4057	3·1419
	17	18	19	20	21	22	23	24
1	1·5763	1·7671	1·9689	2·1817	2·4053	2·6398	2·8852	3·1416
2	3·1526	3·5342	3·9378	4·3634	4·8106	5·2796	5·7704	6·2832
3	4·7289	5·3013	5·9067	6·5451	7·2159	7·9194	8·6556	9·4248
4	6·3052	7·0684	7·8756	8·7268	9·6212	10·5592	11·5408	12·5664
5	7·8815	8·8355	9·8445	10·9085	12·0265	13·1990	14·4260	15·7080
6	9·4578	10·6026	11·8134	13·0902	14·4318	15·8388	17·3112	18·8496
7	11·0341	12·3697	13 7823	15·2719	16·8371	18·4786	20·1964	21·9912
8	12·6104	14·1368	15·7512	17·4536	19·2424	21·1181	23·0816	25·1328
9	14·1867	15·9039	17·7201	19·6353	21·6477	23·7582	25·9668	28·2744
	33	34	35	36	37	38	39	40
1	5·9396	6·3050	6·6813	7·0686	7·4667	7·8758	8·2958	8·7266
2	11·8792	12·6100	13·3626	14·1372	14·9334	15·7516	16·5916	17·4532
3	17·8188	18·9150	20·0439	21·2058	22·4001	23·6274	24·8874	26·1798
4	23·7584	25·2200	26·7252	28·2744	29·8668	31·5032	33·1832	34·9064
5	29·6980	31·5250	33·4065	35·3430	37·3335	39·3790	41·4790	43·6330
6	35·6376	37·8300	40·0878	42·4116	44·8002	47·2548	49·7748	52·3596
7	41·5772	44·1350	46·7691	49·4802	52·2669	55·1306	58·0706	61·0862
8	47·5168	50·4400	53·4504	56·5488	59·7336	63·0064	66·3664	69·8128
9	53·4564	56·7450	60·1317	63·6174	67·2003	70·8822	74·6622	78·5394

1 INCH TO 48 INCHES, AND OF THE VOLUMES OF CYLINDERS.

Number of Circles, or Length of Cylinder.	Diameter in Inches.							
	9	10	11	12	13	14	15	16
1	0·4418	0·5454	0·6600	0·7854	0·9218	1·0690	1·2272	1·3963
2	0·8836	1·0908	1·3200	1·5708	1·8436	2·1380	2·4544	2·7926
3	1·3254	1·6362	1·9800	2·3562	2·7654	3·2070	3·6816	4·1889
4	1·7672	2·1816	2·6400	3·1416	3·6872	4·2760	4·9088	5·5852
5	2·2090	2·7270	3·3000	3·9270	4·6090	5·3450	6·1360	6·9815
6	2·6508	3·2724	3·9600	4·7124	5·5308	6·4140	7·3632	8·3778
7	3·0926	3·8178	4·6200	5·4978	6·4526	7·4830	8·5904	9·7741
8	3·5344	4·3632	5·2800	6·2832	7·3744	8·5520	9·8176	11·1704
9	3·9762	4·9086	5·9400	7·0686	8·2962	9·6210	11·0448	12·5667
	25	26	27	28	29	30	31	32
1	3·4088	3·6870	3·9761	4·2761	4·5869	4·9087	5·2414	5·5851
2	6·8176	7·3740	7·9522	8·5522	9·1738	9·8174	10·4828	11·1702
3	10·2264	11·0610	11·9283	12·8283	13·7607	14·7261	15·7242	16·7553
4	13·6352	14·7480	15·9044	17·1044	18·3476	19·6348	20·9656	22·3404
5	17·0440	18·4350	19·8805	21·3805	22·9345	24·5435	26·2070	27·9255
6	20·4528	22·1220	23·8566	25·6566	27·5214	29·4522	31·4484	33·5106
7	23·8616	25·8090	27·8327	29·9327	32·1083	34·3609	36·6898	39·0957
8	27·2704	29·4960	31·8088	34·2088	36·6952	39·2696	41·9312	44·6808
9	30·6792	33·1830	35·7849	38·4849	41·2821	44·1783	47·1726	50·2659
	41	42	43	44	45	46	47	48
1	9·1684	9·6211	10·0847	10·5592	11·0447	11·5410	12·0482	12·5664
2	18·3368	19·2422	20·1694	21·1184	22·0894	23·0820	24·0964	25·1328
3	27·5052	28·8633	30·2541	31·6776	33·1341	34·6230	36·1446	37·6992
4	36·6736	38·4844	40·3388	42·2368	44·1788	46·1640	48·1928	50·2656
5	45·8420	48·1055	50·4235	52·7960	55·2235	57·7050	60·2410	62·8320
6	55·0104	57·7266	60·5082	63·3552	66·2682	69·2460	72·2892	75·3984
7	64·1788	67·3477	70·5929	73·9144	77·3129	80·7870	84·3374	87·9648
8	73·3472	76·9688	80·6776	84·4736	88·3576	92·3280	96·3856	100·5312
9	82·5156	86·5899	90·7623	95·0328	99·4023	103·8690	108·4338	113·0976

APPENDIX II.

TABLES OF COMPOUND INTEREST.

A. Amount to which a capital accumulates with compound interest in n years:—

$$C_n = C_o \times 1{\cdot}0p^n.$$

Let $C_o = £50$; $n = 30$ years; $p = 2\frac{1}{2}$ per cent.; then $C_{30} = 50 \times 2{\cdot}0976 = £104{\cdot}88$.

If $n = 32$ years, then $C_{32} = C_o \times 1{\cdot}025^{30} \times 1{\cdot}025^2 = 104{\cdot}88 \times 1{\cdot}0506 = £110{\cdot}19$.

B. Present value of a capital to be realised after n years:—

$$C_o = \frac{C_n}{1{\cdot}0p^n}.$$

Let $C_n = £80$; $n = 40$ years; $p = 2\frac{1}{2}$ per cent.; then $C_o = 80 \times {\cdot}3724 = £29{\cdot}79$.

C. Present value of a perpetual rental, R, due every n years:—

$$C_o = \frac{R}{1{\cdot}0p^n - 1}.$$

Let $R = £100$; $n = 50$ years; $p = 2\frac{1}{2}$ per cent.; then $C_o = 100 \times {\cdot}4103 = £41{\cdot}03$.

D. Present value of a rental, r, due at the end of every year, altogether n times:—

$$C_o = \frac{r\,(1{\cdot}0p^n - 1)}{1{\cdot}0p^n \times {\cdot}0p}.$$

Let $r = £10$; $n = 30$ years; $p = 2\frac{1}{2}$ per cent.; then $C_o = 10 \times 20{\cdot}9303 = £209{\cdot}303$.

If the rentals refer to the past n years, the amounts in this Table must be multiplied by the corresponding values of $1{\cdot}0p^n$, to be taken from Table A, so as to comply with the formula—

$$C_o = \frac{r\,(1{\cdot}0p^n - 1)}{{\cdot}0p}.$$

In the above example—

$$C_o = 209{\cdot}303 \times 1{\cdot}025^{30} = £439;$$

or—

$$C = \frac{10\,(1{\cdot}025^{30} - 1)}{{\cdot}025} = \frac{10 \times 1{\cdot}0976}{{\cdot}025} = £439.$$

A. Amount to which a Capital of 1 accumulates with Compound Interest in n Years: $C_n = C_o \times 1{\cdot}0p^n$.

No. of Years = n.	Per Cent.						
	2.	2·5.	3.	3·5.	4.	4·5.	5.
1	1·0200	1·0250	1·0300	1·0350	1·0400	1·0450	1·0500
2	1·0404	1·0506	1·0609	1·0712	1·0816	1·0920	1·1025
3	1·0612	1·0769	1·0927	1·1087	1·1249	1·1412	1·1576
4	1·0824	1·1038	1·1255	1·1475	1·1699	1·1925	1·2155
5	1·1041	1·1314	1·1593	1·1877	1·2167	1·2462	1·2763
6	1·1262	1·1597	1·1941	1·2293	1·2653	1·3023	1·3401
7	1·1487	1·1887	1·2299	1·2723	1·3159	1·3609	1·4071
8	1·1717	1·2184	1·2668	1·3168	1·3686	1·4221	1·4775
9	1·1951	1·2489	1·3048	1·3629	1·4233	1·4861	1·5513
10	1·2190	1·2801	1·3439	1·4106	1·4802	1·5530	1·6289
15	1·3459	1·4483	1·5580	1·6753	1·8009	1·9353	2·0789
20	1·4859	1·6386	1·8061	1·9898	2·1911	2·4117	2·6533
25	1·6406	1·8539	2·0938	2·3632	2·6658	3·0054	3·3864
30	1·8114	2·0976	2·4273	2·8068	3·2434	3·7453	4·3219
35	1·9999	2·3732	2·8139	3·3336	3·9461	4·6673	5·5160
40	2·2080	2·6851	3·2620	3·9593	4·8010	5·8164	7·0109
45	2·4379	3·0379	3·7816	4·7024	5·8412	7·2482	8·9850
50	2·6916	3·4371	4·3839	5·5849	7·1067	9·0326	11·4674
55	2·9717	3·8888	5·0821	6·6331	8·6464	11·2563	14·6356
60	3·2810	4·3998	5·8916	7·8781	10·5196	14·0274	18·6792
65	3·6225	4·9780	6·8300	9·3567	12·7987	17·4807	23·8399
70	3·9996	5·6321	7·9178	11·1128	15·5716	21·7841	30·4264
75	4·4158	6·3722	9·1789	13·1985	18·9452	27·1470	38·8327
80	4·8754	7·2096	10·6409	15·6757	23·0498	33·8301	49·5614
85	5·3829	8·1570	12·3357	18·6179	28·0436	42·1585	63·2544
90	5·9431	9·2289	14·3005	22·1122	34·1193	52·5371	80·7304
95	6·5617	10·4416	16·5782	26·2623	41·5114	65·4708	103·0347
100	7·2446	11·8137	19·2186	31·1914	50·5049	81·5885	131·5013
110	8·8312	15·1226	25·8282	43·9986	74·7597	126·7045	214·2017
120	10·7652	19·3581	34·7110	62·0643	110·6626	196·7682	348·9120
130	13·1227	24·7801	46·6486	87·5478	163·8076	305·5750	568·3409
140	15·9965	31·7206	62·6919	123·4949	242·4753	474·5486	925·7674
150	19·4996	40·6050	84·2527	174·2017	358·9227	736·9594	1507·9775
200	52·4849	139·5639	369·3558	972·9039	2550·7498	6656·6863	17292·5808

B. Present Value of a Capital of 1, to be Realised after n Years: $C_0 = \frac{C_n}{1{\cdot}0p^n}$.

No. of Years = n.	Per Cent.						
	2.	2·5.	3.	3·5.	4.	4·5.	5.
1	0·9804	0·9756	0·9707	0·9662	0·9615	0·9569	0·9524
2	·9612	·9518	·9426	·9335	·9246	·9157	·9070
3	·9423	·9286	·9151	·9019	·8890	·8763	·8638
4	·9238	·9060	·8885	·8714	·8548	·8386	·8227
5	·9057	·8839	·8626	·8420	·8219	·8025	·7835
6	·8880	·8623	·8375	·8135	·7903	·7679	·7462
7	·8706	·8413	·8131	·7860	·7599	·7348	·7107
8	·8535	·8207	·7894	·7594	·7307	·7032	·6768
9	·8368	·8007	·7664	·7337	·7026	·6729	·6446
10	·8203	·7812	·7441	·7089	·6756	·6439	·6139
15	·7430	·6905	·6419	·5969	·5553	·5167	·4810
20	·6730	·6103	·5537	·5026	·4564	·4146	·3769
25	·6095	·5394	·4776	·4231	·3751	·3327	·2953
30	·5521	·4767	·4120	·3563	·3083	·2670	·2314
35	·5000	·4214	·3554	·3000	·2534	·2143	·1813
40	·4529	·3724	·3066	·2526	·2083	·1719	·1420
45	·4102	·3292	·2644	·2127	·1712	·1380	·1113
50	·3715	·2909	·2281	·1791	·1407	·1107	·0872
55	·3365	·2572	·1968	·1508	·1157	·0888	·0683
60	·3048	·2273	·1697	·1269	·0951	·0713	·0535
65	·2760	·2009	·1464	·1069	·0781	·0572	·0419
70	·2500	·1776	·1263	·0900	·0642	·0459	·0329
75	·2265	·1569	·1089	·0758	·0528	·0368	·0257
80	·2051	·1387	·0940	·0638	·0434	·0296	·0202
85	·1858	·1226	·0811	·0537	·0357	·0237	·0158
90	·1683	·1084	·0699	·0452	·0293	·0190	·0124
95	·1524	·0958	·0603	·0381	·0241	·0153	·0097
100	·1380	·08465	·05203	·03206	·01980	·01226	·00761
110	·1132	·06613	·03872	·02273	·01338	·00789	·00467
120	·09289	·05166	·02881	·01611	·00904	·00508	·00287
130	·07620	·04036	·02144	·01142	·00611	·00327	·00176
140	·06251	·03153	·01595	·00810	·00412	·00211	·00101
150	·05128	·02463	·01187	·00574	·00279	·00136	·00066
200	·01905	·007165	·002707	·001028	·000392	·000150	·0000578

C.—PRESENT VALUE OF A PERPETUAL RENTAL OF 1, DUE EVERY n YEARS;

$$C_0 = \frac{R}{1 \cdot 0p^n - 1}.$$

Number of Years $= n$.	Per Cent. 2.	2·5.	3.	3·5.	4.	4·5.	5.
1	50·0000	40·0000	33·3333	28·5714	25·0000	22·2222	20·0000
2	24·7525	19·7531	16·4204	14·0400	12·2549	10·8666	9·7561
3	16·3377	13·0054	10·7843	9·1981	8·0087	7·0839	6·3442
4	12·1312	9·6327	7·9676	6·7786	5·8873	5·1943	4·6402
5	9·6079	7·6099	6·2785	5·3280	4·6157	4·0620	3·6195
6	7·9263	6·2620	5·1532	4·3620	3·7690	3·3084	2·9403
7	6·7256	5·2998	4·3502	3·6727	3·1652	2·7711	2·4564
8	5·8255	4·5787	3·7485	3·1565	2·7132	2·3691	2·0944
9	5·1258	4·0183	3·2811	2·7556	2·3623	2·0572	1·8138
10	4·5663	3·5703	2·9077	2·4355	2·0823	1·8084	1·5901
15	2·8913	2·2307	1·7922	1·4807	1·2485	1·0692	0·9268
20	2·0578	1·5659	1·2405	1·0103	0·8395	0·7084	·6049
25	1·5610	1·1710	0·9143	0·7335	·6003	·4986	·4190
30	1·2325	0·9111	·7006	·5535	·4458	·3643	·3010
35	1·0001	·7282	·5513	·4285	·3394	·2727	·2214
40	0·8278	·5934	·4421	·3379	·2631	·2076	·1656
45	·6955	·4907	·3595	·2701	·2066	·1600	·1252
50	·5912	·4103	·2955	·2181	·1638	·1245	·0955
55	·5072	·3462	·2450	·1775	·1308	·0975	·0733
60	·4384	·2941	·2044	·1454	·1050	·0768	·0566
65	·3813	·2514	·1715	·1197	·0848	·0607	·0438
70	·3334	·2159	·1446	·0989	·0686	·0481	·0340
75	·2928	·1861	·1223	·0820	·0557	·0382	·0264
80	·2580	·1610	·1037	·0681	·0453	·0305	·0206
85	·2282	·1397	·0882	·0568	·0370	·0243	·0161
90	·2023	·1215	·0752	·0474	·0302	·0194	·0125
95	·1798	·1059	·0642	·0396	·0247	·0155	·0098
100	·1601	·0925	·0549	·0331	·0202	·0124	·0077
110	·1277	·07081	·04028	·02326	·01356	·00796	·00469
120	·1024	·05447	·02966	·01638	·00912	·00511	·002874
130	·08249	·04205	·02191	·01155	·006142	·003284	·001763
140	·06668	·03255	·01621	·008164	·004141	·002112	·001081
150	·05406	·02525	·01201	·005774	·002794	·001357	·0006636
200	·01942	·007217	·002707	·001029	·0003926	·0001502	·00005783

D.—PRESENT VALUE OF A RENTAL OF 1, DUE AT THE END OF EVERY YEAR, ALTOGETHER n TIMES: $C_0 = \frac{r\,(1{\cdot}0p^n - 1)}{1{\cdot}0p^n \times {\cdot}0p}$.

Number of Years $= n$.	PER CENT.						
	2.	2·5.	3.	3·5.	4.	4·5.	5.
1	0·9804	0·9756	0·9709	0·9662	0·9615	0·9569	0·9524
2	1·9416	1·9274	1·9135	1·8997	1·8861	1·8727	1·8594
3	2·8839	2·8560	2·8286	2·8016	2·7751	2·7490	2·7232
4	3·8077	3·7620	3·7171	3·6731	3·6299	3·5875	3·5459
5	4·7135	4·6458	4·5797	4·5150	4·4518	4·3900	4·3295
6	5·6014	5·5081	5·4172	5·3285	5·2421	5·1579	5·0757
7	6·4720	6·3494	6·2303	6·1145	6·0020	5·8927	5·7864
8	7·3255	7·1701	7·0197	6·8740	6·7327	6·5959	6·4632
9	8·1622	7·9709	7·7861	7·6077	7·4353	7·2688	7·1078
10	8·9826	8·7521	8·5302	8·3166	8·1109	7·9127	7·7217
15	12·8493	12·3814	11·9379	11·5174	11·1184	10·7395	10·3797
20	16·3514	15·5892	14·8775	14·2124	13·5903	13·0079	12·4622
25	19·5235	18·4244	17·4131	16·4815	15·6221	14·8282	14·0939
30	22·3965	20·9303	19 6004	18·3920	17·2920	16·2889	15·3725
35	24·9986	23·1452	21·4872	20·0007	18·6646	17·4610	16·3742
40	27·3555	25·1028	23·1148	21·3551	19·7928	18·4016	17·1591
45	29·4902	26·8330	24·5187	22·4955	20·7200	19·1563	17·7741
50	31·4236	28·3623	25·7298	23·4556	21·4822	19·7620	18·2559
55	33·1748	29·7140	26·7744	24·2641	22·1086	20·2480	18·6335
60	34·7609	30·9087	27·6756	24·9447	22·6235	20·6380	18·9293
65	36·1975	31·9646	28·4529	25·5178	23·0467	20·9510	19·1611
70	37·4986	32·8979	29·1234	26·0004	23·3945	21·2021	19·3427
75	38·6771	33·7227	29·7018	26·4067	23·6804	21·4036	19·4850
80	39·7445	34·4518	30·2008	26·7488	23·9154	21·5653	19·5965
85	40·7113	35·0962	30·6312	27·0368	24·1085	21·6951	19·6838
90	41·5869	35·6658	31·0024	27·2793	24·2673	21·7992	19·7523
95	42·3800	36·1692	31·3227	27·4835	24·3978	21·8828	19·8059
100	43·0984	36·6141	31·5989	27·6554	24·5050	21·9499	19·8479
110	44·3382	37·3549	32·0428	27·9221	24·6656	22·0468	19·9066
120	45·3554	37·9337	32·3730	28·1111	24·7741	22·1093	19·9427
130	46·1898	38·3858	32·6188	28·2451	24·8474	22·1495	19·9648
140	46·8743	38·7390	32·8016	28·3401	24·8969	22·1754	19·9784
150	47·4358	39·0149	32·9377	28·4074	24·9303	22·1921	19·9867
200	49·0473	39·7134	32·2431	28·5421	24·9902	22·2189	19·9988

APPENDIX III.

YIELD TABLES

FOR

1. High forest of *Oak*, compiled by Wimmenauer, chiefly for low lands, with 6 diagrams.
2. High forest of *Beech*, compiled by Schwappach, for Prussia.
3. High forest of *Scotch Pine*, compiled by Schwappach, for North Germany.
4. High forest of *Spruce*, compiled by Schwappach, for Germany.
5. High forest of *Silver Fir*, compiled by von Lorey, for Württemberg.
6. High forest of *Larch*, preliminary table used in Saxony.
7. High forest of *Alder*, preliminary table used in Saxony.
8. High forest of *Birch*, preliminary table used in Saxony.
9. Coppice forest of Alder, Poplar and Willow, table used in Saxony.
10. Coppice forest of Oak, Beech, Ash and Birch, table used in Saxony.

Yield tables 1, 2, 3, 4 and 5 have been prepared as explained on pages 92 to 104. All the yield tables are for one acre of normal, or fully stocked, wood in solid cubic feet. The data in tables 6, 7, 8, 9, and 10 refer to all wood above ground, that is to say, exclusive of roots and stools.

To convert solid cubic feet into cubic feet according to quarter-girth measurement, take three-fourths of the volume given as "Timber over 3 inches diameter at the small end," or two-thirds of the volume given under "Timber and Fuel." These reductions are sufficiently accurate for all practical purposes.

I. Quality means the best which can reasonably be expected on good fertile land.

II. Quality means an average quality.

III. Quality means the lowest quality of locality on which the species can reasonably be grown. It differs very considerably according to species.

YIELD TABLE FOR HIGH FOREST OF *OAK*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		40	1·	12		400				
20	1,920	70	2·4	30	560	1,430				
30	870	89	4·3	46	1,700	2,430	100	430	100	430
40	510	103	6·1	60	2,930	3,570	290	490	390	920
50	320	115	8·2	71	4,000	4,640	390	510	780	1,430
60	236	125	9·9	79	4,960	5,570	440	540	1,220	1,970
70	187	133	11·4	86	5,790	6,430	470	540	1,690	2,510
80	157	140	12·8	92	6,530	7,220	490	570	2,180	3,080
90	133	145	14·2	97	7,220	7,890	500	570	2,680	3,650
100	114	149	15·5	101	7,800	8,540	500	540	3,180	4,190
110	98	153	16·9	105	8,320	9,150	490	540	3,670	4,730
120	85	157	18·4	108	8,790	9,660	490	530	4,160	5,260
130	76	160	19·6	111	9,220	10,130	460	510	4,620	5,770
140	68	163	21·	113	9,620	10,580	460	500	5,080	6,270
150	61	166	22·3	115	10,010	11,000	440	490	5,520	6,760
160	56	168	23·5	117	10,390	11,420	430	470	5,950	7,230

I., or Best, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	400			·83				400	40	40
560	1,430	·27	·20	·68	560	56	28	1,430	103	71
1,800	2,860	·41	·31	·59	1,800	124	60	2,860	143	95
3,220	4,060	·48	·36	·58	3,320	**152**	83	4,490	**163**	112
4,390	5,150	·49	·37	·57	4,780	146	96	6,070	158	121
5,400	6,110	·50	·37	·56	6,180	140	103	7,540	147	126
6,260	6,970	·51	·38	·56	7,480	130	107	8,940	140	128
7,020	7,790	·51	·38	·56	8,710	123	109	10,300	136	**129**
7,720	8,460	·51	·38	·56	9,900	119	110	11,540	124	128
8,300	9,080	·52	·39	·57	10,980	108	**110**	12,730	119	127
8,810	9,690	·52	·39	·57	11,990	101	109	13,880	115	126
9,280	10,190	·52	·39	·57	12,950	96	108	14,920	104	124
9,680	10,640	·52	·39	·57	13,840	89	106	15,900	98	122
10,080	11,080	·52	·39	·57	14,700	86	105	16,850	95	120
10,450	11,490	·52	·39	·58	15,530	83	104	17,760	91	118
10,820	11,890	·53	·40	·58	16,340	81	102	18,650	89	117

YIELD TABLE FOR HIGH FOREST OF *OAK*,

Age. Years.	Major Part of Wood. Stems. Number.	Basal Area. Sq. Ft.	Mean. Diam. Inches.	Height. Feet.	Volume. Solid Cubic Feet. Timber over 3 in. diam. at small end.	Timber and Fuel.	Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet. Periodic Yields. Timber over 3 in. diam. at small end.	Timber and Fuel.	Summary of Yields. Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		30	·7	10		330				
20	3,590	57	1·7	21	100	960				
30	1,485	75	3·	32	760	1,590		300		300
40	820	91	4·5	43	1,630	2,310	90	330	90	630
50	510	103	6·1	52	2,520	3,160	200	360	290	990
60	370	112	7·4	60	3,260	3,890	270	390	560	1,380
70	280	120	8·9	67	3,940	4,570	310	400	870	1,780
80	230	127	10·1	72	4,600	5,230	320	410	1,190	2,190
90	190	134	11·4	77	5,200	5,840	340	410	1,530	2,600
100	160	139	12·5	81	5,770	6,430	350	410	1,880	3,010
110	140	144	13·7	85	6,290	6,960	360	410	2,240	3,420
120	120	148	15·0	89	6,750	7,450	360	410	2,600	3,830
130	105	151	16·2	92	7,170	7,900	360	400	2,960	4,230
140	93	154	17·4	95	7,570	8,330	350	390	3,310	4,620
150	85	156	18·4	98	7,960	8,750	330	370	3,640	4,990
160	77	158	19·4	100	8,320	9,150	310	360	3,950	5,350

II., or Average, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	330			1·10				330	33	33
100	960	·08	·06	·80	100	10	5	960	63	48
760	1,890	·32	·24	·66	760	66	25	1,890	93	63
1,720	2,670	·42	·31	·60	1,720	96	43	2,970	108	74
2,720	3,520	·47	·35	·59	2,810	109	56	4,150	118	83
3,530	4,280	·49	·37	·58	3,820	101	64	5,270	112	88
4,250	4,970	·49	·37	·57	4,810	99	69	6,350	108	91
4,920	5,640	·50	·38	·57	5,790	99	72	7,420	107	93
5,540	6,250	·50	·38	·57	6,730	94	75	8,440	102	94
6,120	6,840	·51	·38	·57	7,650	92	77	9,440	100	94
6,650	7,370	·51	·38	·57	8,530	88	78	10,380	94	94
7,110	7,860	·51	·38	·57	9,350	82	78	11,280	90	94
7,530	8,300	·52	·39	·57	10,130	78	78	12,130	85	93
7,920	8,720	·52	·39	·57	10,880	75	78	12,950	82	92
8,290	9,120	·52	·39	·57	11,600	72	77	13,740	79	92
8,630	9,510	·53	·40	·58	12,270	67	77	14,500	76	91

YIELD TABLE FOR HIGH FOREST OF *OAK*,

Age. Years.	Major Part of Wood. Stems. Number.	Major Part of Wood. Stems. Basal Area. Sq. Ft.	Major Part of Wood. Mean. Diam. Inches.	Major Part of Wood. Mean. Height. Feet.	Major Part of Wood. Volume. Solid Cubic Feet. Timber over 3 in. diam. at small end.	Major Part of Wood. Volume. Solid Cubic Feet. Timber and Fuel.	Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet. Periodic Yields. Timber over 3 in. diam. at small end.	Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet. Periodic Yields. Timber and Fuel.	Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet. Summary of Yields. Timber over 3 in. diam. at small end. Summary of col. *h*.	Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet. Summary of Yields. Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		20	·4	5		250				
20	6,900	39	1·	12		630				
30	3,300	59	1·8	19		970		170		170
40	1,640	74	2·9	26	470	1,340		200		370
50	1,010	85	3·9	33	990	1,770	30	230	30	600
60	670	96	5·1	39	1,540	2,270	100	230	130	830
70	480	104	6·3	45	2,120	2,790	130	260	260	1,090
80	380	112	7·4	51	2,670	3,300	160	260	420	1,350
90	310	118	8·4	56	3,200	3,800	190	260	610	1,610
100	255	124	9·4	60	3,690	4,290	210	260	820	1,870
110	220	129	10·4	64	4,140	4,740	230	290	1,050	2,160
120	190	134	11·4	68	4,570	5,170	230	270	1,280	2,430
130	165	139	12·4	71	4,990	5,590	230	260	1,510	2,690
140	145	142	13·4	74	5,370	5,990	230	260	1,740	2,950
150	130	145	14·3	77	5,730	6,370	200	260	1,940	3,210
160	115	148	15·4	80	6,070	6,750	200	230	2,140	3,440

III., or Lowest, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	250			2·50				250	25	25
	630			1·36				630	38	31
	1,140			·86				1,140	51	38
470	1,540	·24	·18	·70	470	47	12	1,710	57	43
1,020	2,000	·35	·26	·63	1,020	55	20	2,370	66	47
1,640	2,500	·41	·31	·61	1,670	65	28	3,100	73	52
2,250	3,050	·45	·34	·60	2,380	71	34	3,880	78	55
2,830	3,560	·47	·35	·58	3,090	71	39	4,650	77	58
3,390	4,060	·48	·36	·58	3,810	72	42	5,410	76	60
3,900	4,550	·50	·37	·58	4,510	70	45	6,160	75	62
4,370	5,030	·50	·37	·57	5,190	68	47	6,900	74	63
4,800	5,440	·50	·37	·57	5,850	66	49	7,600	70	63
5,220	5,850	·51	·38	·57	6,500	65	50	8,280	68	64
5,600	6,250	·51	·38	·57	7,110	61	51	8,940	66	64
5,930	6,630	·51	·38	·57	7,670	56	51	9,580	64	64
6,270	6,980	·51	·38	·57	8,210	54	51	10,190	61	64

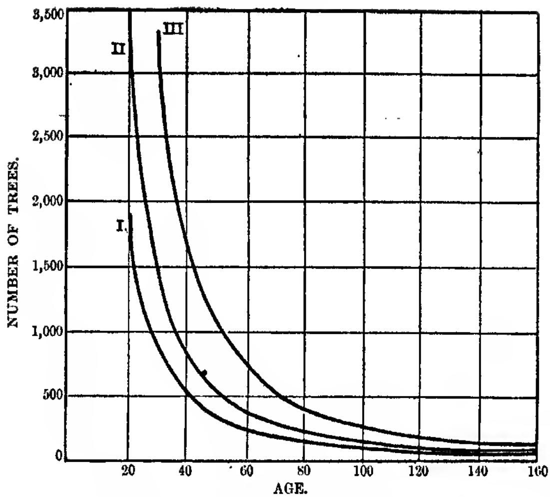

Fig. 54.—Diagram showing the *Number* of Trees per Acre in Oak Woods of I., II., and III. Qualities.

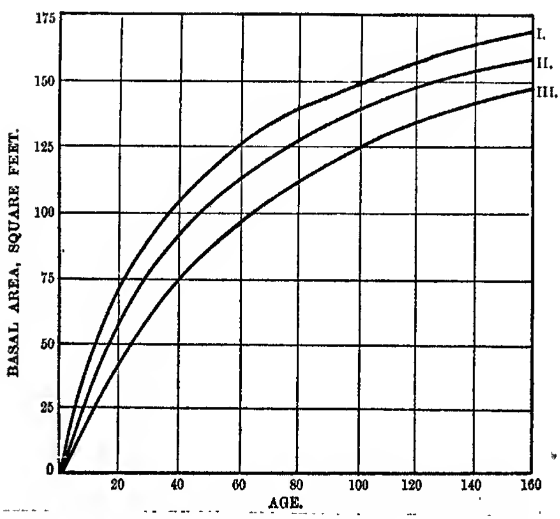

Fig. 55.—Diagram showing the *Basal Area* of Trees per Acre in Oak

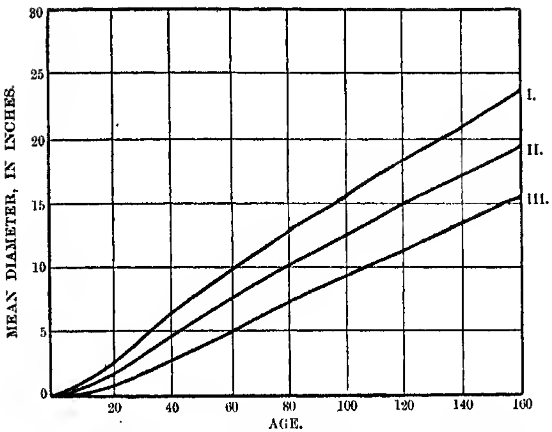

Fig. 56.—Diagram showing the *Mean Diameter* of Trees in Oak Woods of I., II., and III. Qualities.

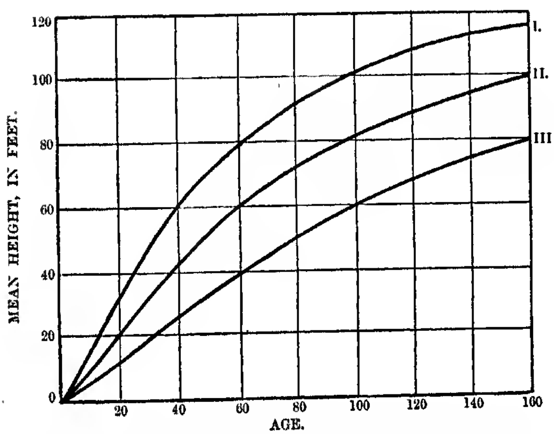

Fig. 57.—Diagram showing the *Mean Height* of Oak Woods of I., II., and III. Qualities.

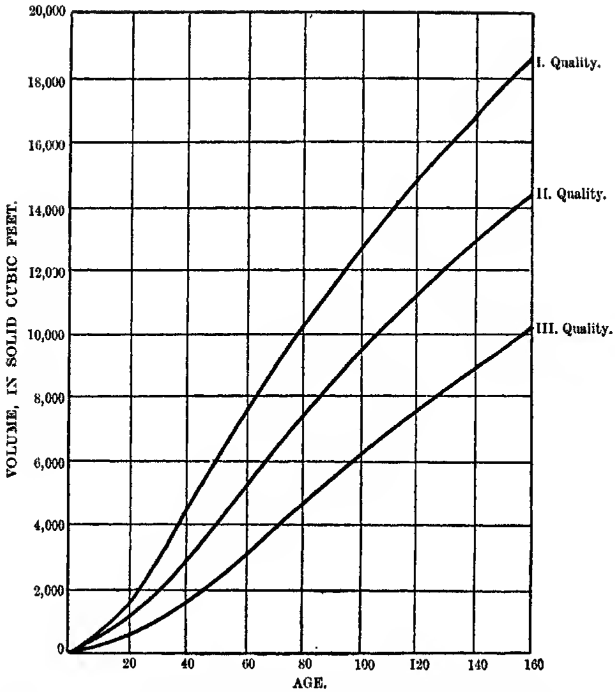

Fig. 58.—Diagram showing the *Total Volume* per Acre of Oak Woods of I., II., and III. Qualities.

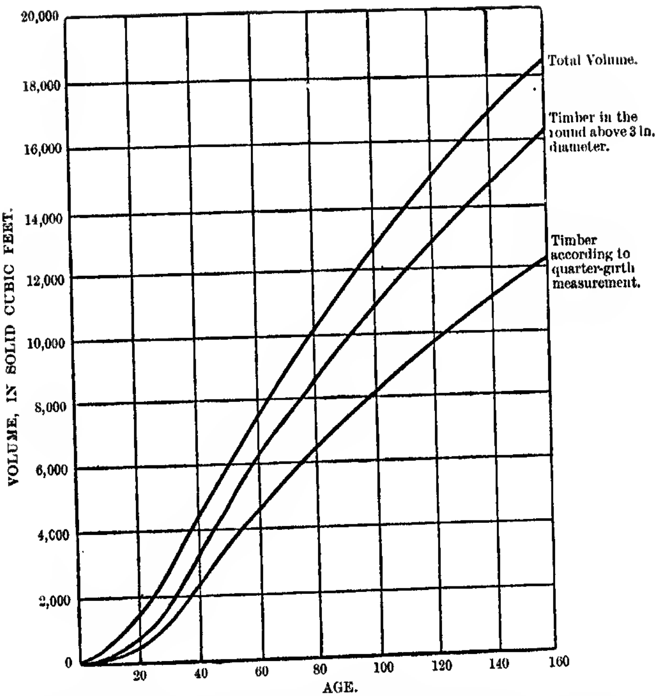

Fig. 59.—Diagram showing the *Total Volume*, the Volume of *Timber over 3 in. diameter* at the small end, and the Volume according to *Quarter-Girth Measurement* per Acre of an Oak Wood of the I. Quality.

YIELD TABLE FOR HIGH FOREST OF *BEECH*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		15	·7	6		150				
20	2,550	40	1·7	18		540				
30	1,550	74	3·	31	690	1,600		200		200
40	940	105	4·5	45	1,940	3,000	130	510	130	710
50	600	131	6·3	56	3,330	4,620	400	660	530	1,370
60	423	148	8·	67	4,570	5,840	590	740	1,120	2,110
70	316	156	9·5	76	5,640	6,970	740	860	1,860	2,970
80	249	161	10·9	85	6,560	7,890	830	960	2,690	3,930
90	201	165	12·2	92	7,360	8,690	830	960	3,520	4,890
100	166	166	13·5	98	8,050	9,400	860	940	4,380	5,830
110	142	167	14·6	104	8,630	10,000	840	940	5,220	6,770
120	126	166	15·6	107	9,120	10,520	830	910	6,050	7,680
130	112	165	16·5	110	9,520	10,950	830	910	6,880	8,590
140	100	165	17·3	113	9,850	11,300	820	900	7,700	9,490

I., or Best, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	150			1·67				150	15	15
	540			·75				540	39	27
690	1,800	·30	·22	·70	690	69	23	1,800	126	60
2,070	3,510	·41	·31	·63	2,070	138	52	3,710	191	93
3,730	5,280	·45	·34	·63	3,860	179	77	5,990	228	120
5,160	6,580	·46	·35	·59	5,690	183	95	7,950	196	133
6,380	7,830	·48	·36	·59	7,500	181	107	9,940	199	142
7,390	8,850	·48	·36	·58	9,250	175	116	11,820	188	148
8,190	9,650	·48	·36	·57	10,880	163	121	13,580	176	151
8,910	10,340	·49	·37	·58	12,430	155	124	15,230	165	152
9,470	10,940	·50	·37	·58	13,850	142	126	16,770	154	152
9,950	11,430	·51	·38	·59	15,170	132	126	18,200	143	152
10,350	11,860	·52	·39	·60	16,400	123	126	19,540	134	150
10,670	12,200	·53	·40	·61	17,550	115	125	20,790	125	148

YIELD TABLE FOR HIGH FOREST OF *BEECH*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		10	·5	5		130				
20		28	1·3	13		400				
30	2,070	53	2·2	23	90	890				
40	1,390	81	3·3	33	940	1,790		230		230
50	970	104	4·4	43	2,000	2,830	90	330	90	560
60	730	121	5·5	52	2,920	3,800	270	370	360	930
70	580	131	6·4	60	3,720	4,660	330	410	690	1,340
80	460	137	7·4	67	4,430	5,400	360	440	1,050	1,780
90	370	138	8·3	73	4,960	5,930	460	540	1,510	2,320
100	300	138	9·2	79	5,340	6,320	560	640	2,070	2,960
110	250	136	10·	84	5,620	6,590	600	690	2,670	3,650
120	210	135	10·9	88	5,860	6,840	570	660	3,240	4,310
130	175	134	11·8	91	6,070	7,090	530	610	3,770	4,920
140	150	132	12·7	93	6,260	7,300	490	570	4,260	5,490

II., or Average, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	130			2·6				130	13	13
	400			1·1				400	27	20
90	890	·07	·05	·73	90	9	3	890	49	30
940	2,020	·35	·26	·67	940	85	24	2,020	113	51
2,090	3,160	·45	·34	·63	2,090	115	42	3,390	137	68
3,190	4,170	·46	·35	·60	3,280	119	55	4,730	134	79
4,050	5,070	·47	·35	·59	4,410	113	63	6,000	127	86
4,790	5,840	·48	·36	·59	5,480	107	68	7,180	118	90
5,420	6,470	·49	·37	·59	6,470	99	72	8,250	107	92
5,900	6,960	·49	·37	·58	7,410	94	74	9,280	103	93
6,220	7,280	·49	·37	·58	8,290	88	75	10,240	96	93
6,430	7,500	·49	·37	·58	9,100	81	76	11,150	91	93
6,600	7,700	·49	·37	·58	9,840	74	76	12,010	86	92
6,750	7,870	·51	·38	·59	10,520	68	75	12,790	78	91

YIELD TABLE FOR HIGH FOREST OF *BEECH*.

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		8	·4	3		90				
20		18	·9	7		210				
30		34	1·6	13		450				
40	2,000	57	2·3	20	400	840		40		40
50	1,510	80	3·1	28	930	1,510		130		170
60	1,200	98	3·7	34	1,530	2,160		170		340
70	1,000	108	4·4	39	2,040	2,690		190		530
80	830	115	5·0	44	2,410	3,090	70	200	70	730
90	700	117	5·5	47	2,670	3,370	130	200	200	930
100	610	118	6·	50	2,860	3,570	150	200	350	1,130
110	530	117	6·4	52	2,990	3,720	160	210	510	1,340
120	470	116	6·7	54	3,030	3,820	170	230	680	1,570
130	430	115	7·	56	3,160	3,900	150	190	830	1,760
140	390	114	7·3	57	3,220	3,970	140	170	970	1,930

III., or Lowest, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	90			4·17				90	9	9
	210			1·59				210	12	11
	450			1·02				450	24	15
400	880	·35	·26	·74	400	40	10	880	43	22
930	1,640	·42	·31	·67	930	53	19	1,680	80	34
1,530	2,330	·46	·34	·65	1,530	60	25	2,500	82	42
2,040	2,880	·48	·36	·64	2,040	51	29	3,220	72	46
2,480	3,290	·48	·36	·61	2,480	44	31	3,820	60	48
2,800	3,570	·49	·37	·61	2,870	39	32	4,300	48	48
3,010	3,770	·49	·37	·61	3,210	34	32	4,700	40	47
3,150	3,930	·49	·37	·61	3,500	29	32	5,060	36	46
3,250	4,050	·49	·37	·61	3,760	26	31	5,390	33	45
3,310	4,090	·49	·37	·61	3,990	23	31	5,660	27	44
3,360	4,140	·50	·37	·61	4,190	20	30	5,900	24	42

YIELD TABLE FOR THE *SCOTCH PINE*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		45	1·5	12		800				
20	1,420	98	3·6	29	1,000	2,200	20	110	20	110
30	1,010	127	4·8	44	2,260	3,440	210	560	230	670
40	720	145	6·1	55	3,520	4,500	510	800	740	1,470
50	510	157	7·5	65	4,560	5,420	660	860	1,400	2,330
60	370	165	9·	73	5,430	6,220	610	740	2,010	3,070
70	290	172	10·4	80	6,170	6,930	510	550	2,520	3,620
80	230	175	11·8	86	6,800	7,550	440	500	2,960	4,120
90	200	178	12·8	91	7,320	8,060	410	460	3,370	4,580
100	170	181	14·	95	7,730	8,460	400	430	3,770	5,010
110	150	182	14·9	99	8,100	8,830	360	390	4,130	5,400
120	140	182	15·4	103	8,430	9,150	310	340	4,440	5,740
130	130	183	16·1	106	8,700	9,420	290	310	4,730	6,050
140	125	183	16·4	108	8,950	9,660	260	290	4,990	6,340

I., or Best, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 8 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	800			1·18				800	80	80
1,020	2,310	·35	·26	·77	1,020	102	51	2,310	151	115
2,470	4,000	·40	·30	·62	2,490	147	83	4,110	180	137
4,030	5,300	·44	·33	·56	4,260	177	106	5,970	186	149
5,220	6,280	·45	·34	·53	5,960	170	119	7,750	178	155
6,040	6,960	·45	·34	·52	7,440	148	124	9,290	154	155
6,680	7,480	·45	·34	·50	8,690	125	124	10,550	126	151
7,240	8,050	·45	·34	·50	9,760	107	122	11,670	112	146
7,730	8,520	·45	·34	·50	10,690	93	118	12,640	97	140
8,130	8,890	·45	·34	·49	11,500	81	115	13,470	83	135
8,460	9,220	·45	·34	·49	12,230	73	111	14,230	76	129
8,740	9,490	·45	·34	·49	12,870	64	107	14,890	66	124
8,990	9,730	·45	·34	·49	13,430	56	103	15,470	58	119
9,210	9,950	·45	·34	·49	13,940	51	100	16,000	53	114

YIELD TABLE FOR THE *SCOTCH PINE*.

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		30	1·	8		500				
20		71	2·	17	400	1,370				
30	1,600	103	3·4	29	1,140	2,220	20	60	20	60
40	1,100	121	4·5	38	2,000	3,030	130	290	150	350
50	760	132	5·6	46	2,820	3,660	330	490	480	840
60	560	140	6·8	52	3,460	4,190	370	470	850	1,310
70	440	145	7·8	58	3,970	4,660	340	410	1,190	1,720
80	360	149	8·7	63	4,390	5,070	330	390	1,520	2,110
90	300	152	9·6	68	4,740	5,430	300	360	1,820	2,470
100	250	152	10·6	72	5,060	5,730	270	300	2,090	2,770
110	220	153	11·3	76	5,340	6,020	240	270	2,330	3,040
120	200	153	11·8	79	5,620	6,290	200	230	2,530	3,270
130	180	154	12·5	82	5,860	6,530	170	200	2,700	3,470

II., OR AVERAGE, QUALITY OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	500			2·08				500	50	50
400	1,370	·33	·25	1·14	400	40	20	1,370	87	68
1,160	2,280	·38	·29	·74	1,160	76	38	2,280	91	76
2,130	3,320	·43	·32	·66	2,150	99	54	3,380	110	84
3,150	4,150	·46	·35	·60	3,300	**115**	66	4,500	**112**	90
3,830	4,660	·48	·36	·58	4,310	101	72	5,500	100	**92**
4,310	5,070	·47	·35	·55	5,160	85	74	6,380	88	91
4,720	5,460	·47	·35	·54	5,910	75	**74**	7,180	80	90
5,040	5,790	·46	·35	·54	6,560	65	73	7,900	72	88
5,330	6,030	·46	·35	·52	7,150	59	72	8,500	60	85
5,580	6,290	·46	·35	·52	7,670	52	70	9,060	56	82
5,820	6,520	·46	·35	·52	8,150	48	68	9,560	50	80
6,080	6,730	·46	·35	·52	8,560	41	66	10,000	44	77

YIELD TABLE FOR THE *SCOTCH PINE*,

Age. Years.	Major Part of Wood.						Minor Part of Wood or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		18	·6	4		200				
20		40	1·4	8		450				
30		76	2·2	14	200	860				
40	2,000	98	3·	21	610	1,330				
50	1,470	105	3·6	26	1,060	1,760	30	110	30	110
60	1,040	107	4·3	30	1,430	2,060	40	110	70	220
70	780	108	5·	34	1,730	2,360	50	100	120	320
80	620	109	5·7	38	1,970	2,570	60	100	180	420
90	510	110	6·3	41	2,170	2,760	70	90	250	510
100	420	110	6·9	44	2,340	2,920	80	90	330	600
110	350	110	7·6	47	2,500	3,060	80	90	410	690

III., OR LOWEST, QUALITY OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 8 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	200			2·78				200	20	20
	450			1·41				450	25	23
200	860	·19	·14	·81	200	20	7	860	41	29
610	1,330	·30	·22	·65	610	41	15	1,330	47	34
1,090	1,870	·39	·29	·64	1,090	48	22	1,870	54	37
1,470	2,170	·45	·34	·64	1,500	41	25	2,280	41	38
1,780	2,460	·47	·35	·64	1,850	35	26	2,680	37	38
2,030	2,670	·48	·36	·62	2,150	30	27	2,990	31	37
2,240	2,850	·48	·36	·61	2,420	27	27	3,270	28	36
2,420	3,010	·48	·36	·60	2,670	25	27	3,520	25	35
2,580	3,150	·48	·36	·59	2,910	24	26	3,750	23	34

YIELD TABLE FOR THE *SPRUCE*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		37	1·	8		500				
20	2,970	97	2·4	20	700	1,800				
30	1,800	170	4·2	35	2,920	4,790	60	410	60	410
40	1,130	207	5·8	51	5,540	7,370	340	690	400	1,100
50	720	229	7·6	65	7,750	9,430	690	910	1,090	2,010
60	510	244	9·4	76	9,550	11,120	890	1,040	1,980	3,050
70	380	254	11·1	85	11,020	12,520	890	1,010	2,870	4,060
80	310	263	12·5	93	12,250	13,710	840	910	3,710	4,970
90	260	271	13·8	99	13,300	14,760	740	800	4,450	5,770
100	220	279	15·2	104	14,250	15,720	640	690	5,090	6,460
110	200	285	16·2	109	15,120	16,590	570	600	5,660	7,060
120	190	291	16·8	112	15,900	17,360	500	530	6,160	7,590

I., OR BEST, QUALITY OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns f and h.	Sum of columns g and i.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns f and j.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns g and k.	Current Annual Increment.	Mean Annual Increment.
l	m	n	o	p	q	r	s	t	u	v
·	500			1·69				500	50	50
700	1,800	·36	·27	·93	700	70	35	1,800	130	90
2,980	5,200	·49	·37	·81	2,980	228	99	5,200	370	173
5,880	8,060	·52	·39	·70	5,940	296	148	8,470	327	212
9,440	10,340	·52	·39	·63	8,840	290	177	11,440	297	229
10,440	12,160	·51	·38	·60	11,530	269	192	14,170	273	236
11,910	13,530	·51	·38	·58	13,890	236	198	16,580	241	237
13,090	14,620	·50	·37	·56	15,960	207	200	18,680	210	234
14,040	15,560	·50	·37	·55	17,750	179	197	20,530	185	228
14,890	16,410	·49	·37	·54	19,340	159	193	22,180	165	222
15,690	17,190	·49	·37	·53	20,780	144	189	23,650	147	215
16,400	17,890	·49	·37	·53	22,060	128	184	24,950	130	208

YIELD TABLE FOR THE *SPRUCE*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		23	·6	4		400				
20		62	1·2	10		1,430				
30	3,310	105	2·4	19	670	2,620				
40	1,950	142	3·7	30	2,110	3,900	30	390	30	390
50	1,230	164	4·9	42	3,670	5,220	210	540	240	930
60	850	180	6·2	52	5,060	6,460	410	640	650	1,570
70	640	191	7·4	61	6,270	7,620	490	640	1,140	2,210
80	510	201	8·5	68	7,320	8,630	470	580	1,610	2,790
90	430	211	9·5	74	8,220	9,520	420	500	2,030	3,290
100	380	219	10·3	79	8,960	10,290	380	440	2,410	3,730
110	350	226	10·9	82	9,630	10,980	340	380	2,750	4,110
120	320	232	11·5	84	10,200	11,590	300	330	3,050	4,440

II., OR AVERAGE, QUALITY OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	400			4·35				400	40	40
	1,430			2·31				1,430	103	71
670	2,620	·34	·25	1·31	670	67	22	2,620	119	87
2,140	4,290	·50	·37	·92	2,140	147	54	4,290	167	107
3,880	5,760	·53	·40	·76	3,910	177	78	6,150	186	123
5,470	7,100	·54	·41	·69	5,710	**180**	95	8,030	**188**	134
6,760	8,260	·54	·41	·65	7.410	170	106	9,830	180	140
7,790	9,210	·54	·41	·63	8,930	152	112	11,420	159	**143**
8,640	10,020	·53	·40	·61	10,250	132	114	12,810	139	142
9,340	10,730	·52	·39	·59	11,370	112	**114**	14,020	121	140
9,970	11,360	·52	·39	·59	12,380	101	113	15,090	107	137
10,500	11,920	·52	·39	·59	13,250	87	110	16,030	94	134

YIELD TABLE FOR THE *SPRUCE*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		15	·3	2		200				
20		38	·7	5		550				
30		61	1·2	9		1,100				
40	3,970	87	2·	15	340	1,690				
50	2,150	108	3·	22	960	2,360		270		270
60	1,370	123	4·1	30	1,870	3,100	70	290	70	560
70	1,000	134	5·	37	2,760	3,870	160	300	230	860
80	810	144	5·7	43	3,530	4,600	170	260	400	1,120
90	700	152	6·3	48	4,140	5,230	160	210	560	1,330
100	650	159	6·7	51	4,630	5,720	140	170	700	1,500

III., or Lowest, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	200			6·67				200	20	20
	550			2·90				550	35	27
	1,100			2·				1,100	55	37
340	1,690	·26	·20	1·30	340	34	9	1,690	59	42
960	2,630	·40	·30	·99	960	62	19	2,630	94	53
1,940	3,390	·51	·38	·84	1,940	98	32	3,660	103	61
2,920	4,170	·56	·42	·78	2,990	105	43	4,730	107	68
3,700	4,860	·57	·43	·74	3,930	94	49	5,720	99	71
4,300	5,440	·57	·43	·72	4,700	77	52	6,560	84	73
4,770	5,890	·57	·43	·71	5,330	63	53	7,220	66	72

YIELD TABLE FOR THE *SILVER FIR*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		10	·5	3		300				
20		35	1·3	9		880				
30		71	2·5	18	500	1,760				
40	1,460	126	4·	30	1,700	3,130	60	290	60	290
50	990	171	5·6	45	3,670	4,940	360	640	420	930
60	660	205	7·5	58	5,700	7,070	570	790	990	1,720
70	460	233	9·6	71	7,960	9,450	1,000	1,260	1,990	2,980
80	340	255	11·7	82	10.350	11,950	1,290	1,570	3,280	4,550
90	250	273	14·2	91	12,550	14,230	1,430	1,710	4,710	6,260
100	200	288	16·2	98	14,290	16,060	1,360	1,570	6,070	7,830
110	180	300	17·5	104	15,780	17,610	860	1,000	6,930	8,830
120	170	310	18·3	109	16,990	18,880	710	830	7,640	9,660
130	160	320	19·1	112	17,850	19,760	600	690	8,240	10,350
140	150	330	20·1	115	18,840	20,840	310	360	8,550	10,710

I., OR BEST, QUALITY OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	300			10·				300	30	30
	880			2·79				880	58	44
500	1,760	·39	·29	1·38	500	50	17	1,760	88	58
1,760	3,420	·45	·34	·83	1,760	126	44	3,420	166	85
4,030	5,580	·48	·36	·64	4,090	210	82	5,870	245	117
6,270	7,860	·48	·36	·59	6,690	260	111	8,790	292	146
8,960	10,710	·48	·36	·57	9,950	326	142	12,430	364	178
11,640	13,520	·49	·37	·57	13,630	**368**	170	16,500	**407**	206
13,980	15,940	·51	·38	·57	17,260	363	192	20,490	399	228
15,650	17,630	·51	·38	·57	20,360	310	204	23,890	340	239
16,640	18,610	·51	·38	·56	22,710	235	**206**	26,440	255	**240**
17,700	19,710	·50	·37	·56	24,630	192	205	28,540	210	238
18,450	20,450	·50	·37	·55	26,090	146	201	30,110	157	232
19,150	21,200	·50	·37	·55	27,390	130	196	31,550	144	225

YIELD TABLE FOR THE *SILVER FIR*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		7	·3	2		200				
20		18	·7	6		500				
30		36	1·3	13		1,000				
40	2,700	75	2·3	22	650	1,900				
50	2,000	127	3·4	33	1,880	3,180	140	320	140	320
60	1,330	159	4·7	44	3,530	4,690	330	530	470	850
70	880	182	6·2	55	5,070	6,350	510	730	980	1,680
80	590	203	7·9	65	6,750	8,130	740	970	1,720	2,550
90	420	220	9·8	73	8,640	10,000	900	1,140	2,620	3,690
100	310	235	11·8	81	10,140	11,650	940	1,180	3,560	4,870
110	250	246	13·4	87	11,470	13,020	950	1,140	4,510	6,010
120	230	265	14·3	92	12,600	14,080	750	870	5,260	6,880
130	210	262	15·1	96	13,180	14,790	570	660	6,830	7,530
140	200	268	15·7	99	13,830	15,480	230	250	6,060	7,780

II., or Average, Quality of Locality.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	200			14·29				200	20	20
	500			4·63				500	30	25
	1,000			2·14				1,000	50	33
650	1,900	·39	·29	1·15	650	65	16	1,900	90	47
2,020	3,500	·45	·34	·76	2,020	137	40	3,500	160	70
3,860	5,220	·50	·37	·67	4,000	175	67	5,540	204	92
5,580	7,080	·51	·38	·63	6,050	205	86	7,930	239	113
7,490	9,100	·51	·38	·62	8,470	242	106	10,680	275	133
9,440	11,140	·52	·39	·62	11,160	**269**	124	13,690	**301**	152
11,080	12,830	·53	·40	·61	13,700	254	137	16,520	283	165
12,420	14,160	·54	·41	·61	15,980	228	145	19,030	251	173
13,250	14,950	·53	·40	·60	17,760	178	**148**	20,960	193	**175**
13,750	15,440	·52	·39	·59	19,010	125	146	22,320	136	172
14,060	15,730	·52	·39	·58	19,890	88	142	23,260	94	166

YIELD TABLE FOR THE *SILVER FIR*,

Age. Years.	Major Part of Wood.						Minor Part of Wood, or Intermediate Yields. Solid Cubic Feet.			
	Stems.		Mean.		Volume. Solid Cubic Feet.		Periodic Yields.		Summary of Yields.	
	Number.	Basal Area. Sq. Ft.	Diam. Inches.	Height. Feet.	Timber over 3 in. diam. at small end.	Timber and Fuel.	Timber over 3 in. diam at small end.	Timber and Fuel.	Timber over 3 in. diam. at small end. Summary of col. *h*.	Timber and Fuel. Summary of col. *i*.
a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*i*	*j*	*k*
10		2	·2	2		100				
20		4	·6	5		220				
30		10	1·	9		440				
40		23	1·5	16	150	990				
50		86	2·4	24	1,000	1,860				
60	2,100	124	3·3	33	1,990	2,930	70	230	70	230
70	1,480	145	4·2	42	3,060	4,090	210	410	280	640
80	1,060	162	5·3	51	4,190	5,300	400	600	680	1,240
90	780	175	6·4	59	5,370	6,570	630	840	1,310	2,080
100	570	188	7·8	66	6,430	7,700	790	930	2,100	3,010
110	430	198	9·2	71	7,270	8,600	860	970	2,960	3,980
120	350	205	10·4	76	7,890	9,280	570	640	3,530	4,620
130	810	209	11·1	79	8,360	9,780	500	570	4,030	5,190
140	290	215	11·7	82	8,800	10,280	280	320	4,310	5,510

III., OR LOWEST QUALITY, OF LOCALITY.

Final Yield. Solid Cubic Feet.		Form Factors.			Total Yield. Solid Cubic Feet.					
Timber.	Timber and Fuel.				Timber only.			Timber and Fuel.		
Sum of columns *f* and *h*.	Sum of columns *g* and *i*.	Timber in the round down to 3 in. diam. at small end.	Timber according to quarter-girth measurement.	Timber and Fuel.	Volume. Sum of columns *f* and *j*.	Current Annual Increment.	Mean Annual Increment.	Volume. Sum of columns *g* and *k*.	Current Annual Increment.	Mean Annual Increment.
l	*m*	*n*	*o*	*p*	*q*	*r*	*s*	*t*	*u*	*v*
	100			25·				100	10	10
	220			11·				220	12	11
	440			4·89				440	22	15
150	990	·41	·31	2·69	150	15	4	990	55	25
1,000	1,860	·48	·36	·90	1,000	77	20	1,860	87	37
2,060	3,160	·49	·37	·72	2,060	106	34	3,160	130	53
3,270	4,500	·50	·37	·67	3,340	128	48	4,730	157	68
4,590	5,900	·51	·38	·64	4,870	153	61	6,540	181	82
6,000	7 410	·52	·39	·64	6,680	181	74	8,650	211	96
7,220	8,630	·52	·39	·62	8,530	185	85	10,710	206	107
8,130	9,570	·52	·39	·61	10,230	170	93	12,580	187	114
8,460	9,920	·51	·38	·60	11,420	119	95	13,900	142	116
8,860	10,350	·51	·38	·59	12,390	97	95	14,970	107	115
9,080	10,600	·50	·37	·58	13,110	72	94	15,790	82	113

PRELIMINARY YIELD TABLES—

Larch.				Alder.				Birch.			
Age. Years.	Quality classes, in solid cubic feet.			Age. Years.	Quality classes, in solid cubic feet.			Age. Years.	Quality classes, in solid cubic feet.		
	I.	II.	III.		I.	II.	III.		I.	II.	III.
10	760	470	170	10	860	510	160	10	670	400	110
20	1,890	1,140	360	20	1,800	1,090	340	20	1,410	820	230
30	3,290	1,940	570	30	2,800	1,700	540	30	2,360	1,360	390
40	4,840	2,790	790	40	3,820	2,330	760	40	3,370	1,940	540
50	6,400	3,640	990	50	4,790	2,930	950	50	4,300	2,470	690
60	7,800	4,400	1,170	60	5,730	3,520	1,130	60	5,120	2,920	790
70	9,060	5,100	1,340	70	6,640	4,050	1,300	70	5,740	3,230	840
80	10,190	5,730	1,500	80	7,530	4,560	1,460	80	6,230	3,460	870
90	11,200	6,290	1,640	90	8,360	5,040	1,600				
100	12,120	6,800	1,760	100	9,100	5,470	1,710				
110	12,950	7,260	1,870								
120	13,680	7,650	1,960								
130	14,310	7,960	2,020								
140	14,860	8,260	2,060								

Coppice of Alder, Poplar, Willow.				Coppice of Oak, Beech, Ash, Birch.			
Age. Years.	Quality classes, in solid cubic feet.			Age. Years.	Quality classes, in solid cubic feet.		
	I.	II.	III.		I.	II.	III.
5	490	260	30	5	310	160	15
10	1,000	510	90	10	640	330	45
15	1,540	860	160	15	980	510	85
20	2,100	1,190	240	20	1,330	700	130
25	2,730	1,510	300	25	1,730	930	175
30	3,360	1,840	350	30	2,120	1,140	220
35	3,940	2,140	400	35	2,460	1,330	260
40	[illegible]	[illegible]	[illegible]	40	2,800	1,510	290

APPENDIX IV.

General Working Plans for the Method of Allotment of Woods to the several Periods of one Rotation.

A. Allotment according to Area.

B. Allotment according to Volume.

GENERAL WORKING PLAN OF A HIGH FOREST OF BEECH

A.—ALLOTMENT OF WOODS TO THE SEVERAL

Com-partment.	Area in Acres.	Present Age.	Final Age *before* Shifting.	Allotment of Woods to Periods **Before** Shifting. I. Period. Acres.	II. Period. Acres.	III. Period. Acres.	IV. Period. Acres.	V. Period, Acres. With Over-wood.	V. Period, Acres. Without Over-wood.
1	19	80	110		19				
2	8	60	110			8			
3	6	60	110			6			
4	11	40	110				11		
5	9	15	105					6	3
6	3	130	140	3					
7	7	10	100					7	
8	9	50	100			9			
9	4	75	105		4				
10	11	75	105		11				
11	9	75	105		9				
12	13	75	105		13				
13	3	80	110		3				
14	7	16	106						7
15	7	16	106						7
16	7	65	95		7				
17	4	70	100		4				
18	5	70	100		5				
19	5	75	105		5				
20	10	82	92	10					
21	3	13	103						3
Total	160			13	80	23	11	13	20
								33	

Total Area = 160 acres. Average per Period = 32 acres.

WITH A MODERATE ADMIXTURE OF OAK, ASH, AND CONIFERS.

PERIODS ACCORDING TO **AREA**.—Rotation = 100 Years.

Compartment.	Final Age *after* Shifting.	Allotment of Woods to Periods After Shifting. I. Period. Acres.	II. Period. Acres.	III. Period. Acres.	IV. Period. Acres.	V. Period. Acres. With Overwood.	V. Period. Acres. Without Overwood.	Remarks.
1	130			19				Shifted, as the wood is of good growth, to relieve II. Period.
2	110			8				
3	130				6			Shifted, to fill up IV. Period.
4	110				11			
5	105					6	3	
6	140	3						
7	100					7		
8	120				9			Shifted, to fill up IV. Period.
9	145				4			
10	105		11					
11	105		9					
12	85	13						Shifted, to provide for I. Period.
13	90	3						Shifted, to provide for I. Period.
14	106						7	
15	106						7	
16	95 & 115		3·5	3·5				
17	100		4					
18	100		5					
19	85	5						Shifted, to provide for I. Period.
20	92	10						
21	103						3	
		34	32·5	30·5	30	13	20	
						33		

LISTS OF WOODS ALREADY UNDER REGENERATION, AND OF

Serial Number.	Compartment.	Area, Acres.	Species.	Age.		Trees to be Left as Standards.	
				Present.	At the Time of Cutting (mean).	Species.	No.
1. *Woods already under*							
1	5	9	Beech	104	107	Oak	16
			Scotch Pine	104	107		
2	7	7	Beech	116	119		
	Total	16					
2. *Woods to be Cut and Regenerated*							
1	6	3	Beech	130	140	Oak	3
2	12	13	Beech	75	85		
			Oak	71	81	Oak	150
			Conifers	70	80		
3	13	3	Beech	80	90	Oak	11
4	19	5	Beech	75	85		
5	20	10	Beech	82	92	Oak	54
			Scotch Pine	82	92		
	Total	34					

WOODS TO BE TAKEN UNDER REGENERATION DURING THE I. PERIOD.

Serial Number.	Yield in Solid Cubic Feet.					Regeneration.				Remarks.
	Estimate.				Actual Result.	Natural, Acres.	Artificial.			
	Present Volume.	Increment.	Total.	Mean per Acre.			Manner of Formation.	Species.	Area. Acres.	
Regeneration.										
1	5,238	151	5,389	625		5	Planting	Oak	4	Increment. $\frac{5238}{104} \times 3 =$ 151 cub. ft.
	232	7	239					Spruce		$\frac{232}{104} \times 3 =$ 7 cub. ft.
2	8,091	209	8,300	1,186		4·50	Planting	Oak Ash Spruce	2·50	$\frac{8091}{116} \times 3 =$ 209 cub. ft.
	13,561	367	13,928			9·50			6·50	
during the I. Period.										
1	19,827	1,525	21,352	7,117		2	Sowing	Oak	1	Increment. $\frac{19,827}{130} \times 10$ $=1,525$ cub. feet.
2	33,237	4,432	37,669	3,350		8	Sowing	Scotch Pine Larch Spruce	5	etc.
	4,444	626	5,070							
	706	101	807							
3	14,575	1,822	16,397	5,466		2	Sowing	Oak	1	
4	30,400	4,053	34,453	6,891		3	Planting	Oak Larch	2	
5	36,279	4,422	40,701	4,225		6	Planting	Larch Spruce Scotch Pine	4	
	1,384	169	1,553							
	140,852	17,150	158,002	4,647		21			13	

CALCULATION OF YIELD FOR THE I. PERIOD.

Sources of Yield.	Solid Cubic Feet. Yield. Detailed.	Yield. Total.	Grand Total.	Mean Annual Yield.
a. Thinnings* say	50,000			
b. Other intermediate yields . .	—	50,000	50,000	2,500
c. Balance in woods already under regeneration	13,928	13,928		
d. Final yield of woods to be regenerated	158,002			
† To be deducted as remaining to be carried over into the II. period .	24,552			
e. Balance of *d* to be cut		133,450		
f. Total of *c* and *e*			147,378	7,369
Total of all yields			197,378	9,868

* See next page.

† The calculation is made as follows:—Regeneration period = 10 years; mean volume per acre of woods in I. period = 4,647 cubic feet. There remain, when the seeding cutting has been made = 4,647 × ·6 = 2,788, say, 2,790 cubic feet. It is assumed that these 2,790 cubic feet are cut away in annually equal instalments of $\frac{1}{10}$th, = 279 cubic feet; hence, the ten coupes, each of $\frac{160}{10}$ = 1·6 acres, will, at the end of the I. period, have volumes per acre equal to:

Coupe 10 = 2,790.	Coupe 9 = 2,511.	Coupe 8 = 2,232.
„ 7 = 1,953.	„ 6 = 1,674.	„ 5 = 1,395.
„ 4 = 1,116.	„ 3 = 837.	„ 2 = 558.
„ 1 = 279.		

thus forming an arithmetical series, the sum of which is = $(2,790 + 279) \times \frac{10}{2}$ = 15,345. This sum must be multiplied by 1·6, the size of the coupe, making the volume to be carried forward into the II. period = 15,345 × 1·6 = 24,552 cubic feet.

LOCAL YIELD TABLE FOR THINNINGS

Age Class.	Yield of Thinnings c' solid per acre.
21— 30	170
31— 40	200
41— 50	230
51— 60	245
61— 70	260
71— 80	230
81— 90	200
91—100	155

This table has been used to calculate the expected yield of thinnings during the next 20 years. The full details have been omitted; the total volume amounts to 50,000 cubic feet in round figures.

Examples.—Taking Compartment 1, now 80 years old, the thinnings would amount, during the next ten years, to

$$19 \times 200 = 3{,}800 \text{ cubic feet.}$$

In the case of Compartment 9, now 75 years old:

$$\text{For the first 5 years} = 4 \times \frac{230}{2} = 460 \text{ cubic feet}$$

$$\text{For the second 5 years} = 4 \times \frac{200}{2} = 400 \quad ,,$$

$$\text{Total} = 860 \text{ cubic feet.}$$

B.—General Working Plan for the Method by **VOLUME**,
The Final Yield has been taken from the Yield

Compartment.	Area in Acres.	Present age.	Final Age *before* Shifting.	Final Yield per Acre *before* Shifting.	Allotment of Final Yield in C' **Before** Shifting.				
					I. Period.	II. Period.	III. Period.	IV. Period.	V. Period.
1	19	80	110	6,590		125,210			
2	8	60	110	6,590			52,720		
3	6	60	110	6,590			39,540		
4	11	40	110	6,590				72,490	
5	9	15	105	6,455					58,095
6	3	130	140	7,300	21,900				
7	7	10	100	6,320					44,240
8	9	50	100	6,320			56,880		
9	4	75	105	6,455		25,820			
10	11	75	105	6,455		71,009			
11	9	75	105	6,455		58,095			
12	13	75	105	6,455		83,915			
13	3	80	110	6,590		19,770			
14	7	16	106	6,482					45,374
15	7	16	106	6,482					45,374
16	7	65	95	6,125		42,875			
17	4	70	100	6,320		25,280			
18	5	70	100	6,320		31,600			
19	5	75	105	6,455		32,275			
20	10	82	92	6,008	60,080				
21	3	13	103	6,401					19,203
Total	160				81,980	515,849	149,140	72,490	212,286

Grand total of yield = 1,031,745 cubic feet.
Average per period = 206,349 „ „

BASED UPON THE DATA GIVEN IN THE TABLE AT PAGE 380.
TABLE FOR BEECH, II. QUALITY, ON PAGES 356, 357.

Compartment.	Final Age *after* Shifting.	Final Yield per Acre *after* Shifting.	Allotment of Final Yield in C' After Shifting.				
			I. Period.	II. Period.	III. Period.	IV. Period.	V. Period.
1	130	7,090			134,710		
2	110	6,590			52,720		
3	130	7,090				42,510	
4	110	6,590				72,490	
5	105	6,455					58,095
6	140	7,300	21,900				
7	100	6,320					44,240
8	120	6,840				61,560	
9	145	7,405				29,620	
10	105	6,455		71,009			
11	105	6,455		58,095			
12	85	5,665	73,645				
13	90	5,930	17,790				
14	106	6,482					45,374
15	106	6,482					45,374
16*	95 & 115	6,125 & 6,715		21,437	23,502		
17	100	6,320		25,280			
18	100	6,320		31,600			
19	85	5,665	28,325				
20	92	6,008	60,080				
21	103	6,401					19,203
Total			201,740	207,421	210,932	206,180	212,286

Grand total of yield = 1,038,559.
Average per period = 207,712.
Average annual yield in I. period = 10,087.
" " " " V. " = 10,614.

* Half of the compartment will be cut in the II. period and the other half in the III. period.

GENERAL WORKING PLAN DRAWN UP ACCORDING

Rotation,

COMPARTMENTS.	DISTRIBUTION OF AGE							
	1—40 years old.		41—60.		61—80.		81—100.	
Number.	Cubic feet.	Acres.	Cubic feet.	Acres.	Cubic feet.	Acres.	Cubic feet.	Acres.
1	48,030	41	70,630	20				
2	200,945	117	169,514	40	192,117	40	15,892	2
3	19,072	34						
4	21,189	38						
5								
6	109,479	87	201,299	37	423,787	67	353,156	49
7	26,487	24	46,617	11	25,074	5	494,418	49
8	28,605	51			35,316	5	42,379	5
9	13,420	48						
Total	467,227	440	488,060	108	676,294	117	905,845	105
Normal state under a rotation of 120 years								
Comparison of real and normal state { + −								

CALCULATION OF THE YIELD.

This is done according to the formula given on page 316:

$$\text{Annual yield} = I_{real} + \frac{G_{real} - G_{normal}}{a}$$

The real increment, I_{real} ...	=	102,696 cubic feet.
The real growing stock, G_{real} ...	=	8,939,771 „ „
The normal growing stock, G_{norm}	=	7,456,200 „ „
The surplus of growing stock ...	=	1,483,571 cubic feet.

Assuming that this surplus is to be removed in the course of 50 years, the yield would be—

$$\text{Annual yield} = 102,696 + \frac{1,483,571}{50} = 102,696 + 29,671$$

$$= 132,367,$$

or, during the first 10 years = 1,323,670 cubic feet.

TO THE AUSTRIAN ASSESSMENT METHOD.

120 Years.

Classes.					Volume per acre, cubic feet.	Increment.			
Compartments.	Over 100 years.		Total.			Annual, per acre.		Total in 10 years.	
Number.	Cubic feet.	Acres.	Cubic feet.	Acres.		Normal.	Real.	Normal.	Real.
1	34,250	4	152,910	65	2.352	85	70	55,250	45,500
2	118.662	12	697,130	211	3,304	85	75	179,350	158,250
3	381.408	37	400,480	71	5,641	70	61	49.700	43,310
4	1,606,861	148	1,628,050	186	8,753	85	71	158,100	132,060
5	1,522,104	133	1,522,104	133	11,444	100	71	133,000	94,430
6	540,329	43	1,628,050	283	5,753	100	78	283,000	220,740
7	365,870	49	958,466	138	6,945	85	85	117,300	117,300
8	459,103	34	565,403	95	5,952	100	71	95,000	67,450
9	1,373,758	124	1,387,178	172	8,365	100	86	172,000	147,920
	6,402,345	584	8,939,771	1,354	6,603		76		1,020,960
			7,456,200		5,507	92		1,242,700	
			1,483,571		1,096		16		215,740

The yield for the next 10 years having been fixed, the forester decides where it is to be cut. He selects in the first place all silvicultural necessities, such as severance cuttings, the removal of shelter trees over young regeneration; next he adds all woods which are poor in increment, especially those which have suffered from natural phenomena; finally he makes up the total yield by adding the oldest woods, with due consideration of a proper distribution of the age classes over the area. In this way, the Special Working Plan on the next two pages has been obtained.

A detailed record of the work done in each compartment is kept (see page 394); from these data, and those on pages 390, 391, the Summary on pages 392, 393 is prepared, which compares the provisions of the working plan with the actual results.

SPECIAL WORKING PLAN.

Compartments.	Description of Cuttings, Cultivation, etc.	Cuttings. Final. Cubic feet.	Cuttings. Intermediate. Cubic feet.	Cultivation. Acres.	Draining, ditches. Feet.	Road construction. Feet.
1.	Final cutting in regenerated part	34,000				
	Filling up blanks with spruce			3		
	Thinning, and cutting cancerous silver firs . .		10,000			
	Total . .	34,000	10,000	3		
2.	*a. Thinning of shelter-wood and partial final cutting.	35,000				
	Filling up blanks with spruce and Scotch pine.			10		
	a & b. Thinning and removal of cancerous trees		53,000			
	Total . .	35,000	53,000	10		
3.	a. Seeding cutting, and partly final cutting .	53,000				
	b & c. Rest.					
	Total . .	53,000				
4.	a. Thinning of shelter-wood, seeding cutting in the fully stocked parts by the removal of cancerous and large trees .	341,000				
	b. Rest.					
	Total . .	341,000				
5.	a Thinning and removal of cancerous trees . .	19,000	19,000			
	b & c. Rest.					
	Construction of an export road to meet the main road					4,900
	Total . .	19,000	19,000			4,900

* a, b, c refer to sub-compartments.

SPECIAL WORKING PLAN—*continued.*

Compartments.	Description of Cuttings, Cultivation, etc.	Cuttings. Final. Cubic feet.	Cuttings. Intermediate. Cubic feet.	Cultivation. Acres.	Draining ditches. Feet.	Road construction. Feet.
6.	*a.* Cutting of all old standards and cancerous trees	45,000				
	Thinning		3,000			
	b. Thinning of shelter-wood and partially final cutting.	198,000				
	Filling up blanks with spruce			12		
	c. Cutting out of old defective trees where young growth exists . .	14,000				
	Construction of an export road to meet the main road					9,500
	Total . .	257,000	3,000	12		9,500
7.	*a.* Thinning and removal of cancerous trees . .	47,000	47,000			
	b. Rest.					
	c. Removal of standards and cancerous trees .	25,000				
	Thinning . . .		15,000			
	Construction of an export road					5,000
	Total . .	72,000	62,000			5,000
8.	In the regeneration area: thinning of shelter-wood and partially final clearing; in the rest seeding cutting	163,000				
	Filling up blanks with spruce			3		
	Construction of an export road					3,500
	Total . .	163,000		3		3,500
9.	Continuation of regeneration cuttings and removal of cancerous trees	195,000				
	Thinning in fully stocked parts		7,000			
	Filling up blanks with spruce and Scotch pine .			8		
	Construction of an export road					3,000
	Total . .	195,000	7,000	8		3,000

SUMMARY OF THE PROVISIONS OF THE

Compartments.	Provisions of Working Plan.					
	Cuttings.			Cultivation. Acres.	Draining. Feet.	Road Construction. Feet.
	Final. Cubic Feet.	Intermediate. Cubic Feet.	Total. Cubic Feet.			
1.	34,000	10,000	44,000	3		
2.	35,000	53,000	88,000	10		
3.	53,000		53,000			
4.	341,000		341,000			
5.	19,000	19,000	38,000			4,900
6.	257,000	3,000	260,000	12		9,500
7.	72,000	62,000	134,000			5,000
8.	163,000		163,000	3		3,500
9.	195,000	7,000	202,000	8		3,000
Total . .	1,169,000	154,000	1,323,000	36		25,900

Note.—The excess was due to heavy windfalls ; it will not derange futur

WORKING PLAN AND OF THE EXECUTION.

Compartments.	Results of Actual Work Done.						Comparison of Proposed and Executed Cuttings.		Remarks.
	Cuttings.			Cultivation. Acres.	Draining. Feet.	Road Construction. Feet.	Cut too much. Cubic Feet.	Cut too little. Cubic Feet.	
	Final. Cubic Feet.	Intermediate. Cubic Feet.	Total. Cubic Feet.						
1.	33,034	12,549	45,583	4·4			1,583		
2.	54,517	75,000	129,517	5·0			41,517		Excess due to windfalls and snow-break.
3.	132,900		132,900	·1			79,900		Excess due to windfalls and snow-break.
4.	177,169		177,169	·1				163,831	Held back, on account of extra fellings in other compts.
5.	86,606	68,301	154,907			5,003	116,907		Excess due to windfalls.
6.	342,444	21,635	364,079	8·4		9,679	104,079		Excess: windfalls and construction of road.
7.	95,852		95,852			5,299		38,148	Thinning held over.
8.	111,049		111,049	·9		3,691		51,951	Held back on account of excess in other compartments.
9.	197,660		197,660			2,953		4,340	
Total .	1,231,231	177,485	1,408,716	18·9		26,625	85,716		

arrangements, as there is a considerable excess of growing stock in the forest.

SAMPLE PAGE OF THE DETAILED CONTROL BOOK.

Compartment 1.

Year.	Description of Cuttings, Cultivation, etc.	Cuttings. Final. Cubic Feet.	Cuttings. Intermediate. Cubic Feet.	Cultivation. Acres.	Draining Ditches. Feet.	Road Construction. Feet.
Provision of Working Plan.						
	Final cutting in regenerated part	34,000				
	Filling up blanks with spruce .			3		
	Thinning and cutting of cancerous silver firs . . .		10,000			
	Total .	34,000	10,000	3		
Execution.						
1884	Final cutting	14,297				
,,	Dry and windfall wood . . .	813				
1885	Windfalls	665				
1886	Final cutting, thinning . . .	6,166	832			
,,	Windfalls	547				
1887	Windfalls	1,363				
1888	Final cutting, thinning . .	7,759	11,717			
,,	Planting			1·7		
,,	Windfalls	82				
1889	Dry wood, windfalls . . .	649				
,,	Planting			2·2		
1890	Windfalls	693				
,,	Planting			·1		
1891	Planting			·2		
1892	Planting			·1		
1893	Planting			·1		
	Total .	33,034	12,549	4·4		

APPENDIX VI.

Notes on the Formulæ for Compound Interest (see page 116).

SUMMATION OF GEOMETRICAL SERIES.

Let a be the first position in a geometrical series, q tho factor with which the first position is multiplied in order to obtain the second position, and so on, and n the number of positions. Then tho sum

$$S = a + aq + aq^2 + aq^3 + \ldots + aq^{n-1}.$$

By multiplying both sides with q, the equation becomes

$$Sq = aq + aq^2 + aq^3 + aq^4 \ldots + aq^n.$$

Deduct the first from the second equation :

$$Sq - S = aq^n - a, \text{ or}$$

$$S(q - 1) = a(q^n - 1).$$

$$S = \frac{a(q^n - 1)}{q - 1} \text{ (for a rising series).}$$

This is the formula for the sum of a rising series. In the case of a falling series ($q < 1$), a more convenient form is obtained by multiplying the formula above and below by -1, which gives the formula—

$$S = \frac{a(1 - q^n)}{1 - q} \text{ (for a falling series).}$$

If now $n = \infty$, then $q^\infty = o$, and

$$S = \frac{a}{1 - q} \text{ (for a falling infinite series).}$$

Formula III.—$C_{mn} = \dfrac{R(1{\cdot}0p^{mn} - 1)}{1{\cdot}0p^m - 1}$ is thus obtained:

$$C_{mn} = R + R \times 1{\cdot}0p^m + R \times 1{\cdot}0p^{2m} + \ldots + R \times 1{\cdot}0p^{(n-1)m}.$$

Here $a = R$ and $q = 1{\cdot}0p^m$; hence,

$$C_{mn} = \frac{R\,(1{\cdot}0p^{mn} - 1)}{1{\cdot}0p^m - 1},\ or = \frac{R\,(1 - 1{\cdot}0p^{mn})}{1 - 1{\cdot}0p^m}.$$

Formula IV.—$C_n = \dfrac{r\,(1{\cdot}0p^n - 1)}{{\cdot}0p.}$

This formula is obtained by introducing $m = 1$ into Formula III.

Formula V.—$C_o = \dfrac{R\,(1{\cdot}0p^{mn} - 1)}{1{\cdot}0p^{mn}\,(1{\cdot}0p^m - 1)}$

$$C_o = \frac{R}{1{\cdot}0p^m} + \frac{R}{1{\cdot}0p^{2m}} + \ldots + \frac{R}{1{\cdot}0p^{mn}}.$$

Here $a = \dfrac{R}{1{\cdot}0p^m}$, and $q = \dfrac{1}{1{\cdot}0p^m}$; hence

$C_o = \dfrac{\dfrac{R}{1{\cdot}0p^m}\left[1 - \left(\dfrac{1}{1{\cdot}0p^m}\right)^n\right]}{1 - \dfrac{1}{1{\cdot}0p^m}}$, which, after reduction, leads to Formula V.

Formula VI.—$C_o = \dfrac{r\,(1{\cdot}0p^n - 1)}{1{\cdot}0p^n \times {\cdot}0p}$ is obtained by introducing $m = 1$ into Formula V.

Formula VII.—$C_o = \dfrac{r}{{\cdot}0p}.$

$C_o = \dfrac{r}{1{\cdot}0p} + \dfrac{r}{1{\cdot}0p^2} + \ldots + \dfrac{r}{1{\cdot}0p^\infty}.$ Here $a = \dfrac{r}{1{\cdot}0p,}$ and $q = \dfrac{1}{1{\cdot}0p}$;

$$\text{hence } C_o = \frac{\dfrac{r}{1{\cdot}0p}}{1 - \dfrac{1}{1{\cdot}0p}} = \frac{r}{{\cdot}0p}.$$

Formula VIII.—$C_o = \dfrac{R}{1{\cdot}0p^n - 1}$

$$C_o = \frac{R}{1{\cdot}0p^n} + \frac{R}{1{\cdot}0p^{2n}} + \ldots + \frac{R}{1{\cdot}0p^\infty}$$

Here $a = \dfrac{R}{1{\cdot}0p^n}$; $q = \dfrac{1}{1{\cdot}0p^n}$; and $n = \infty$; hence,

$$C_o = \frac{\frac{R}{1{\cdot}0p^n}}{1 - \frac{1}{1{\cdot}0p^n}} = \frac{R}{1{\cdot}0p^n - 1}.$$

Formula IX.—$C_o = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1}$

$$C_o = \frac{R}{1{\cdot}0p^m} + \frac{R}{1{\cdot}0p^{m+n}} + \frac{R}{1{\cdot}0p^{m+2n}} + \ldots + \frac{R}{1{\cdot}0p^\infty}.$$

Here $a = \frac{R}{1{\cdot}0p^m}$; $q = \frac{1}{1{\cdot}0p^n}$; and $n = \infty$; hence,

$$C_c = \frac{\frac{R}{1{\cdot}0p^m}}{1 - \frac{1}{1{\cdot}0p^n}} = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1}.$$

Formula X.—$C_o = \frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1}$ is obtained by multiplying Formula VIII. by $1{\cdot}0p^n$; or in the following way:—

$$C_o = R + \frac{R}{1{\cdot}0p^n} + \frac{R}{1{\cdot}0p^{2n}} + \ldots + \frac{R}{1{\cdot}0p^\infty}$$

$$C_o = \frac{R}{1 - \frac{1}{1{\cdot}0p^n}} = \frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1}.$$

Formula XI.—$r = \frac{R}{1{\cdot}0p^n - 1} \times {\cdot}0p$

$$\frac{R}{1{\cdot}0p^n} + \frac{R}{1{\cdot}0p^{2n}} + \ldots \infty = \frac{r}{1{\cdot}0p} + \frac{r}{1{\cdot}0p^2} + \ldots \infty;$$

or, according to Formulas VIII. and VII.,

$$\frac{R}{1{\cdot}0p^n - 1} = \frac{r}{{\cdot}0p} \text{ and } r = \frac{R}{1{\cdot}0p^n - 1} \times {\cdot}0p.$$

Formula XII.—$r = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1} \times {\cdot}0p$

$$\frac{R}{1{\cdot}0p^m} + \frac{R}{1{\cdot}0p^{m+n}} + \frac{R}{1{\cdot}0p^{m+2n}} + \ldots \infty = \frac{r}{1{\cdot}0p} + \frac{r}{1{\cdot}0p^2} + \ldots \infty;$$

or, according to Formulas IX. and VII.,

$$\frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1} = \frac{r}{{\cdot}0p}, \text{ and } r = \frac{R \times 1{\cdot}0p^{n-m}}{1{\cdot}0p^n - 1} \times {\cdot}0p.$$

Formula XIII.—$r = \dfrac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1} \times {\cdot}0p$

$$R + \frac{R}{1{\cdot}0p^n} + \frac{R}{1{\cdot}0p^{2n}} + \ldots \infty = \frac{r}{1{\cdot}0p} + \frac{r}{1{\cdot}0p^2} + \ldots \infty\,;$$

or, according to Formulas X. and VII.

$$\frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1} = \frac{r}{{\cdot}0p}, \text{ and } r = \frac{R \times 1{\cdot}0p^n}{1{\cdot}0p^n - 1} \times {\cdot}0p.$$